**Variance Gamma**

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**Abstract:**

We examine Monte Carlo methods for simulation of Variance Gamma process. We implement algorithms for fitting Variance Gamma density, distribution and quantile functions. We use these to test the empirical evidence of non-normality in financial data. We develop algorithms for pricing European options and geometric Asian options where the underlying evolves as a Variance Gamma process using sequential Monte Carlo methods and benchmark our price against prices from Fourier transform techniques developed in papers based on Variance Gamma process. To reduce variance in our estimates we examine Control Variate, Importance sampling Bridge sampling techniques for Asian options. Finally we use pathwise estimation to develop Greeks for European options.

The paper is organized as follows. In section 1 we motivate the need for Variance Gamma process. In section 2 we briefly describe the notation and results used in the paper. In section 3 we present the algorithms developed and finally we present the results.

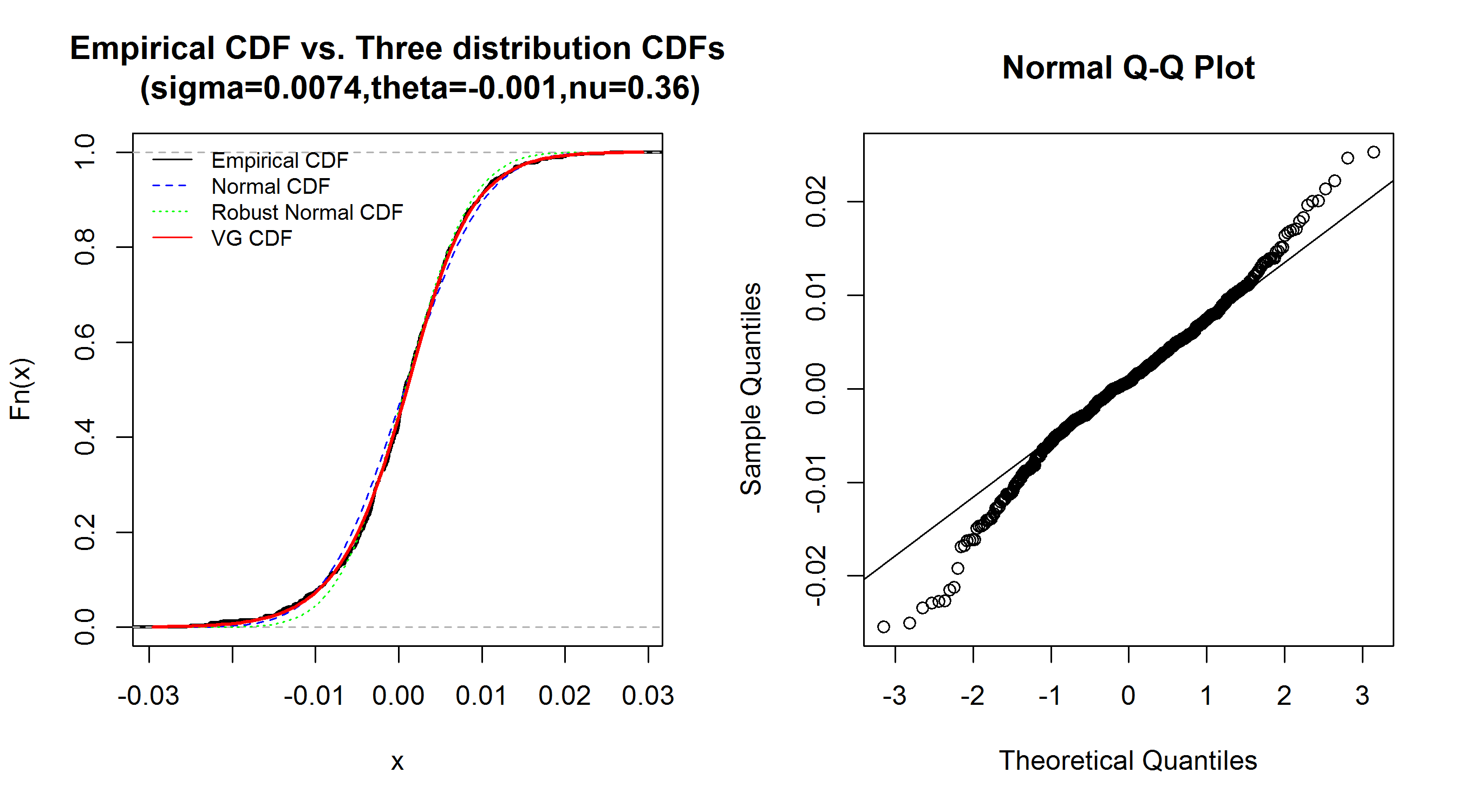
1. **Introduction**

Black-Scholes model for option pricing assumes that the underlying stock price process is log-normally distributed. However, implied stock price density suggests fat tail and skewed behavior. To counter this problem researchers have proposed Variance Gamma distribution as an underlying model for evolution of stock price process.

We developed functionality to fit a  density to data by optimizing log-likelihood. Since we do not have a closed form solution for the density we use the following result



Here  is a characteristic function. We then use root finding to develop the quantile function. We use these functions to fit real data and use the fitted density to construct the following graphs.

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The adjacent figure shows a QQ plot. By examining the tails we can conclude non-normality. We can also see deviation from typical behavior from the overlayed distributions

1. **Notation**

A random variable which is distributed with respect a Variance Gamma distribution has the following density



The above form shows that we can also obtain the density by conditioning a Normal density on Gamma density. We exploit this relationship to construct the density of a Variance Gamma random variable. Its characteristic function has the following form



The characteristic function severs an important role for pricing contracts using Fourier transform technique as suggested in [4]. We list the main result here



where is the characteristic function of the risk-neutral distribution. We use this technique to price European Calls and puts with Variance Gamma as the underlying. We further use results from [5] to price discretely monitored Geometric Asian options. These prices serve as a benchmark for all our pricing results from Monte-Carlo simulation.

1. **Methods**
   1. Simulation using time changed Brownian motion
   2. Importance Sampling
   3. Control Variate
   4. Gamma Bridge Sampling
2. **Results**

# **References**

1. Madan, Dilip B.; Seneta, Eugene (1990). "The Variance Gamma (V.G.) Model for Share Market Returns". [Journal of Business](http://en.wikipedia.org/wiki/Journal_of_Business) 63 (4): 511–524
2. Fu, MichaelC. "Variance-Gamma and Monte Carlo." In Advances in Mathematical Finance, edited by MichaelC Fu, RobertA Jarrow, Ju-YiJ Yen and RobertJ Elliott, 21-34: Birkhäuser Boston, 2007.
3. Dilip Madan, Peter Carr, Eric Chang (1998). ["The Variance Gamma Process and Option Pricing"](http://www.math.nyu.edu/research/carrp/papers/pdf/VGEFRpub.pdf). European FinanceReview 2: 79–105
4. Carr, P., and Madan, D. (1999). "Option valuation using the fast Fourier transform", The Journal of Computational Finance 2(4), 61–73
5. Fusai G, Meucci, A. (2008), “Pricing Discretely Monitored Asian Options under Levy Processes”, Journal of Banking and Finance, 32(10), p.2076-2088