Proof of Prop 37:

λ is an eigenvalue of A

$\Leftrightarrow$ there is a non zero vector $\bar{v}$ such that $A\bar{v} = \lambda\bar{v}$ (by definition)

$\Leftrightarrow$ the system $(A-\lambda I)\bar{x} = \bar{0}$ has a non trivial solution

$\Leftrightarrow$ the matrix $(A-\lambda I)$ is not invertible (UIT)

$\Leftrightarrow$ $\det(A-\lambda I) = 0$ (by VII)

$\Leftrightarrow$ λ is a root of the characteristic eq^n