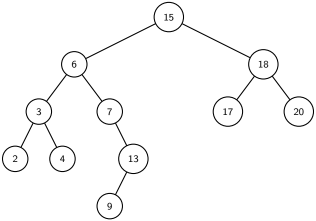
Successor

- The structure of BST allows us to determine the successor of a node without ever comparing keys!



Successor

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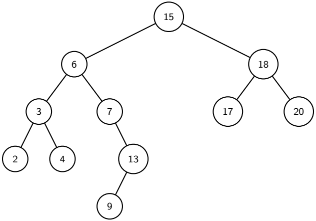
- Two cases may arise:
  - **Case 1:** The right sub-tree of a node x is non-empty.
    - **Successor of x:** The leftmost node in the right sub-tree.
    - $\therefore$ call $\textsc{Tree-Minimum}(\textit{right}[x])$.



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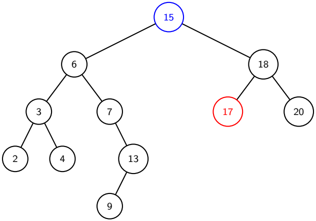
        - **Example:** Successor of 15 is 17.



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- Two cases may arise:
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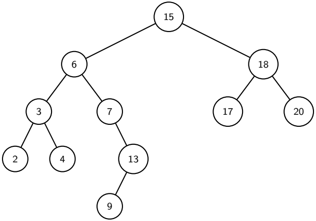

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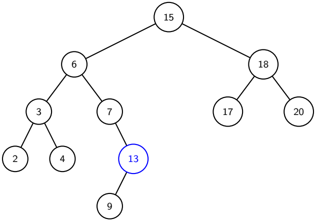
    - Go up the tree from x until we encounter a node that is the left child of its parent.
    - **Example:** Consider the node 13.



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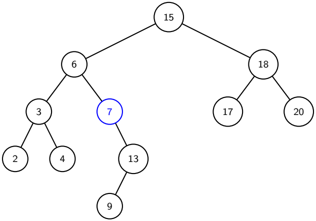

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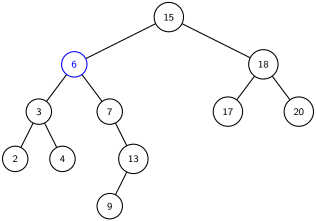

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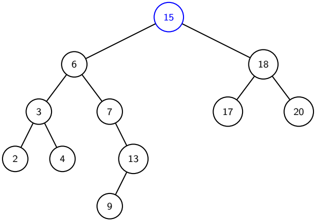

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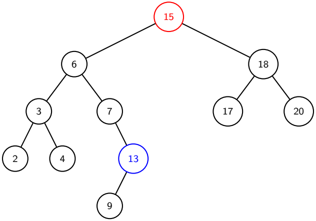
    - Then this parent is the successor.
    - **Example:** The successor of 13 is 15.

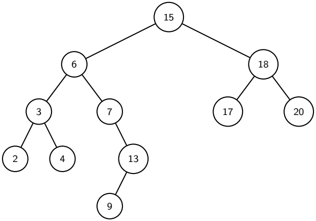


Tree-Successor

```
Tree-Successor($x$)
  I/P: A node $x$ whose successor we need to find.
  O/P: The successor of $x$.
  Begin
    if ($right[x] \neq \textbf{nil}$)
      return Tree-Minimum($right[x]$);
    $y \leftarrow parent[x]$;
    while ($y \neq \textbf{nil}$) and ($x = right[y]$)
      $x \leftarrow y$;
      $y \leftarrow parent[y]$;
    return $y$;
  End
```

Predecessor

- The structure of BST allows us to determine the Predecessor of a node without ever comparing keys!



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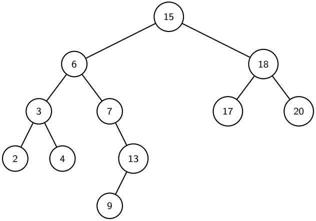
- Two cases may arise:
  - **Case 1:** The left sub-tree of a node x is non-empty.
    - **Predecessor of x:** The rightmost node in the left sub-tree.
    - $\therefore$ call $\textsc{Tree-Maximum}(\textit{left}[x])$.



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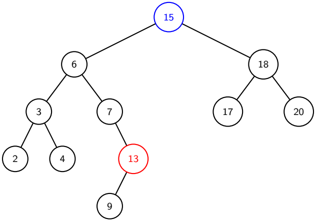
    - **Example:** Predecessor of 15 is 13.



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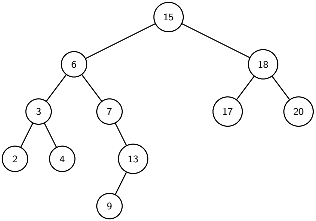

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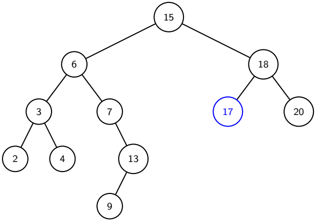

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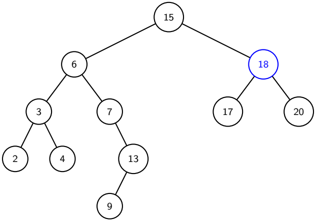

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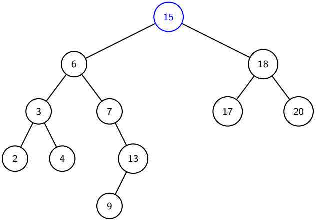

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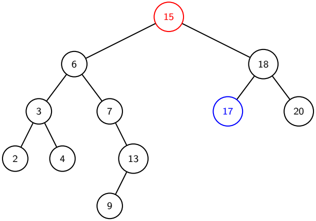
    - **Example:** The Predecessor of 17 is 15.



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