

Proof \rightarrow (a) Given : $\dim(V) = n$

\therefore Any basis has exactly n elements.

We know that the elements of a basis span V and by Steinitz exchange lemma, any L.I set has $\leq n$ elements.

- The $\dim(V) = n$ and thereby if B is a L.I set with n elements, it's a maximal L.I set or a minimal spanning set and thus a basis.

(b) Given : $\dim(V) = n$

\therefore Any basis has exactly n elements.

We know that elements of basis are L.I and by Steinitz exchange lemma, any spanning set has $\geq n$ elements.

- The $\dim(V) = n$ and thereby if B is a spanning set with n elements, it's a ~~minimal~~ ^{maximal} L.I set or a minimal spanning set and thus a basis.