Proof:

a) [$\Rightarrow$] Obviously, if $\bar{v} \perp$ every vector in W, then $v \perp$ every vector in a spanning set.

[$\Leftarrow$] Let $S = \{\bar{w}_1, \dots, \bar{w}_p\}$ be a spanning set for W, and suppose $\bar{v} \perp \bar{w}_i$ holds for all i,

Let $\bar{w} \in W$

then $\bar{w} = c_1\bar{w}_1 + \dots + c_p\bar{w}_p$

$$\therefore \langle \bar{w}, \bar{v} \rangle = \langle c_1\bar{w}_1 + \dots + c_p\bar{w}_p, \bar{v} \rangle$$
$$= c_1\langle \bar{w}_1, \bar{v} \rangle + \dots + c_p\langle \bar{w}_p, \bar{v} \rangle$$
$$= 0 \quad \text{as req.}$$

v) $W^\perp$ is a subspace of V, and $W \cap W^\perp = \{0\}$

Using prop 8.

(i) Clearly $0 \in W^\perp$

ii) Suppose $\bar{v}_1, \bar{v}_2 \in W^\perp$, and $\bar{w} \in W$

Then: $\langle \bar{v}_1 + \bar{v}_2, \bar{w} \rangle = \langle \bar{v}_1, \bar{w} \rangle + \langle \bar{v}_2, \bar{w} \rangle = 0$

$(\because \bar{v}_1, \bar{v}_2 \in W^\perp)$

iii) If $c \in F$, then $\langle c\bar{v}_1, \bar{w} \rangle = c \langle \bar{v}_1, \bar{w} \rangle = 0$

$\Rightarrow c\bar{v}_1 \in W^\perp$

Finally, suppose $\bar{w} \in W \cap W^\perp$

then $\langle \bar{w}, \bar{w} \rangle = 0 \Leftrightarrow \cancel{\bar{w} \in} \ \bar{w} = \bar{0}$