Proof: suppose $P = [\bar{v}_1 \ \bar{v}_2 \ \dots \ \bar{v}_n]$ with the $\bar{v}_i$ being orthonormal column vectors. Then:

$$P^T P = \begin{bmatrix} \bar{v}_1^T \\ \bar{v}_2^T \\ \vdots \\ \bar{v}_n^T \end{bmatrix} \begin{bmatrix} \bar{v}_1 & \bar{v}_2 & \dots & \bar{v}_n \end{bmatrix} = \begin{bmatrix} \bar{v}_1^T \bar{v}_1 & \bar{v}_1^T \bar{v}_2 & \dots & \bar{v}_1^T \bar{v}_n \\ \vdots & & & \\ \bar{v}_n^T \bar{v}_1 & \bar{v}_n^T \bar{v}_2 & \dots & \bar{v}_n^T \bar{v}_n \end{bmatrix}$$

Since $\bar{v}_i^T \bar{v}_j = \bar{v}_i \cdot \bar{v}_j = \delta_{ij}$ (Kronecker delta),

we get $P^T P = I$.