ECE111: Digital Circuits

Practice Problem IV

1. A Boolean function F1 and F2 of four binary variables *A*, *B*, *C*, *D* has the following SOP and POS expressions:

   F1 (A, B, C) = A'.B + A.B' + C'.D and F2 (A, B, C) = (A + B).(C' + D').(C + D).

   a. Obtain the expression for *F1* in the SOP with the minimum number of MINTERMS.

   b. Obtain the expression for *F1* in the POS with the minimum number of MAXTERMS.

   c. Convert the expression obtained in part (b) into the SOP form and compare the expression so obtained with the expression obtained in part (a).

   Repeat the same process for function *F2* as well.

2. Using postulates,

   a) Find minimum SOP expression for the function,

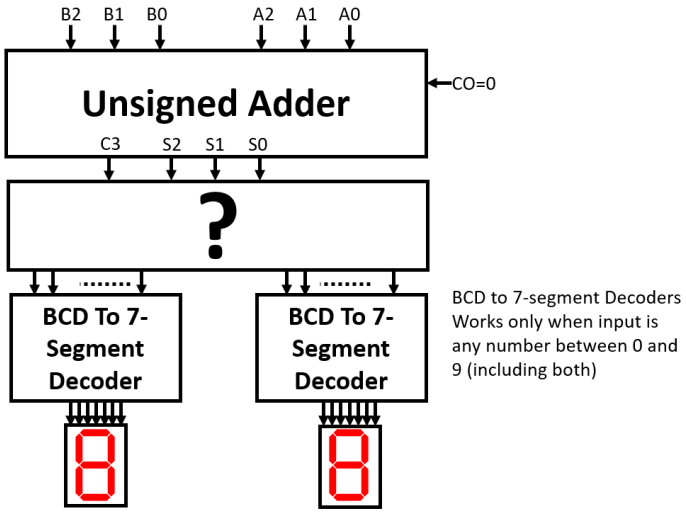
   $$F(A, B, C) = A \cdot C + A \cdot B' + A' \cdot B \cdot C + A' \cdot B' \cdot C'$$

   b) Prove that $B \oplus (A \cdot B + B \cdot C + A' \cdot C) = A' \cdot (B \oplus C)$

5. A treasure box is continuously monitored by a logic circuit, which has three keys (x1, x2 and x3). The circuit can open the treasure box whenever two or more keys are used. Implement this circuit functionality using only NAND gates with minimal cost.

6. Implement 4-bit BCD to Excess-3 converter using minimum number of NAND gates.

7. Consider the circuit shown below where the output of a 3-bit adder needs to be displayed on the two digits of the 7-segment display. For example, if the adder output is 9, then circuit should display 09 where 0 should be displayed on left display and 9 on right display. Existing BCD-to-7 segment decoder works only when input number is between 0 and 9 and we have to use it since it has been hardwired to 7-segment display board. You need to design combinational circuit between Adder and 7-segment decoder (shown as question mark in the Figure below) so that the adder output is correctly displayed on the 7-segment display.



8. Design a Combinational Circuit with 5 inputs and 1 output. The output is '1' when the binary value of the input is odd.
9. Design a Combinational Circuit with 5 inputs and 1 output. The output is '1' when the binary value of the input is even.
10. Find the prime implicants and essential prime implicants for the following expressions:
    a. $F(A,B,C,D,E) = \sum m(1,2,3,5,7,11,13,17,19,23,29,31)$
    b. $F(A,B,C) = \sum m(1,3,5,6,7)$
    c. $F(A,B,C,D) = \sum m(1,4,5,6,7, 9,14,15)$
    d. $F(A,B,C) = \prod m(0,1,3,4,5)$
    e. $F(A,B,C,D) = \prod m(0,2,5,7,8,10,13,15)$
11. A robot has four permitted directions of movement and three possible speed settings.

    Let the direction control and the speed control be applied through two bits each:

    D_1D_0 = 00(forward) / 11(reverse) / 01(right) / 10(left), and

    S_1S_0 = 00(zero) / 01(low) / 10(medium) / 11(high).

    It is desired to have an electronic protection system to ensure that the robot can move at high speed for forward movement only, and reverse only at low speed. This will have to be achieved by generating an output bit P which should go HIGH if any of these two conditions is violated, and then using P to shut off the power to the robot. Write down the Boolean expression for the output P in the sum of products form in terms of D_1, D_0, S_1, S_0.
12. The control unit of a chemical process is required to control the temperature and the pressure inside a reactor by binary (ON/OFF) control of a Heater (H) and an inlet Valve (V) according to the following logic:

    Heater is ON if the Temperature is Low and the Pressure is not HIGH.

    Valve is Open if Pressure is Low and Temperature is not LOW.

    In addition, the control unit has also to sound an Alarm (A) if the temperature and the pressure are either both LOW and both HIGH. Assign two binary variables TL and TH to represent the three possible ranges – LOW, NORMAL and HIGH – of temperature and two binary variables PL and PH to represent the three possible ranges of pressure. Obtain Boolean expressions for the output variables H, V and A in terms of TL, TH, PL and PH.
13. An economist proposed the following technique for making money in the stock market:
    a. If the dividends paid on the stock exceed those paid on a bond, buy the stock.
    b. If the dividends paid on the bond exceed those paid on a stock, buy the bond unless the growth rate of the stock has been at least 25% annually for the past 5 years, in which case the stock should be purchased.

    Design a circuit with NAND gates to give two outputs one going high for buying the stock and the other going high for buying the bond.
13. Buses leave the terminal every hour on the hour unless there are fewer than 10 passengers or the driver is late. If there are fewer than 10 passengers the bus will wait for 10 minutes or the number of passengers increases to 10. If the bus leaves on time it can travel at

60kmph. If the bus leaves late or if it rains the bus can travel only at 30kmph. Under what condition will the bus travel at 60kmph? Construct a circuit to get an output 1 for the bus to travel at 60kmph using NAND gates.

14. In an examination consisting of three papers A, B and C, a student can pass (i) if he/she gets 60% marks in at least one of them and more than 50% in the rest, (ii) if he/she gets 60% marks in two of them and greater than 40% in the third, (iii) if he/she gets greater than 80% in one of them and greater than 40% in one of the other two and greater than 35% in the other. Realize the logic using basic gates to get an output 1 when the student passes the examination.