Proof: Suppose $S = \{\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_p\}$ is an orthogonal set of non zero vectors and suppose $c_1\bar{u}_1 + c_2\bar{u}_2 + \cdots + c_p\bar{u}_p = \bar{0}$.
Taking ~~the~~ the inner product with $\bar{u}_1$, we get:

$$c_1\langle \bar{u}_1, \bar{u}_1\rangle + \cdots + c_p\langle u_1, u_p\rangle = 0$$

Since $\langle \bar{u}_1, \bar{u}_i\rangle = 0$ for $i = 2, \ldots, p$ this forces $c_1\langle \bar{u}_1, \bar{u}_1\rangle = 0$

$\because$ $\langle \bar{u}_1, \bar{u}_1\rangle > 0$, we get $c_1 = 0$

Similarly,

$c_2 = c_3 = \cdots = c_p = 0$

Hence, the set S is linearly independent, as was to be proved.