

Proof of Proposition 35:

Suppose that $T\bar{v}_i = a_{1i}\bar{v}_1 + a_{2i}\bar{v}_2 + \dots + a_{ni}\bar{v}_n$ (1)

Recall that this is the i th column of the matrix $[T]_{\mathcal{B}}$ for $i = 1, \dots, n$.

Similarly, $U\bar{v}_i = b_{1i}\bar{v}_1 + b_{2i}\bar{v}_2 + \dots + b_{ni}\bar{v}_n$ (2)

is the i th column of the matrix $[U]_{\mathcal{B}}$ for $i = 1, \dots, n$.

Now, substituting from (1), we get :-

$$\begin{aligned} (UT)\bar{v}_i &= U(a_{1i}\bar{v}_1 + a_{2i}\bar{v}_2 + \dots + a_{ni}\bar{v}_n) \\ &= a_{1i}U\bar{v}_1 + a_{2i}U\bar{v}_2 + \dots + a_{ni}U\bar{v}_n \\ &= a_{1i}(b_{11}\bar{v}_1 + b_{21}\bar{v}_2 + \dots + b_{n1}\bar{v}_n) + \dots + a_{ni}(b_{1n}\bar{v}_1 + \dots + b_{nn}\bar{v}_n) \end{aligned}$$

from (2)

In other words, the i th column of $[UT]_{\mathcal{B}}$ is :-

$$\begin{bmatrix} b_{11}a_{1i} + b_{21}a_{2i} + \dots + b_{n1}a_{ni} \\ \vdots \\ b_{1n}a_{1i} + b_{2n}a_{2i} + \dots + b_{nn}a_{ni} \end{bmatrix}$$

$$= i^{\text{th}} \text{ column of } [U]_{\mathcal{B}} [T]_{\mathcal{B}} \text{ since}$$

$$[U]_{\mathcal{B}} = [b_{ij}] \quad \& \quad [T]_{\mathcal{B}} = [a_{ij}]$$