Justification (Concise Proof): Any linear dependence relationship between columns of A can be expressed in the form $A\vec{x} = \vec{0}$. When A is row reduced to R, the columns change but the equation $R\vec{x} = \vec{0}$ has the same set of solutions.

In other words, row reduction does not change the dependence relations between the columns. The pivot columns of A must be linearly independent because the pivot columns of R are linearly independent. Also, non-pivot columns are linear combinations of the prev. pivot colum