Eg:-

Consider old basis $\alpha = \{\bar{e}_1, \bar{e}_2\}$ & new basis $\beta = \{\bar{u}_1, \bar{u}_2\}$, where $\bar{u}_1 = (2,1)$ & $\bar{u}_2 = (5,3)$

Step 1: Construct the matrix Q which has the vectors of β in terms of old basis as columns.

$$\text{So, } Q = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \qquad \begin{pmatrix} u_1 = 2e_1 + 1e_2 \\ u_2 = 5e_1 + 3e_2 \end{pmatrix}$$

Step 2: Then change the basis matrix

$$P = P_{\alpha \to \beta} = Q^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

To illustrate, let us determine $[\bar{v}]_\beta$ for any specific vector, say ~~$\bar{v} = (1,2)_\alpha$~~ $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_\alpha$

$$\text{Then } [\bar{v}]_\beta = P[\bar{v}]_\alpha = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \end{bmatrix}_\beta$$