Reflexive

$\mathbf{A} = \mathbf{I_n}^{-1} \mathbf{A} \mathbf{I_n}$ trivially, for all order n square matrices $\mathbf{A}$.

So matrix similarity is reflexive.

$\square$

Symmetric

Let $\mathbf{B} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P}$.

As $\mathbf{P}$ is invertible, we have:

$$\begin{aligned} \mathbf{P} \mathbf{B} \mathbf{P}^{-1} &= \mathbf{P} \mathbf{P}^{-1} \mathbf{A} \mathbf{P} \mathbf{P}^{-1} \\ &= \mathbf{I_n} \mathbf{A} \mathbf{I_n} \\ &= \mathbf{A} \end{aligned}$$

So matrix similarity is symmetric.

$\square$

Transitive

Let $\mathbf{B} = \mathbf{P}_1^{-1} \mathbf{A} \mathbf{P}_1$ and $\mathbf{C} = \mathbf{P}_2^{-1} \mathbf{B} \mathbf{P}_2$.

Then $\mathbf{C} = \mathbf{P}_2^{-1} \mathbf{P}_1^{-1} \mathbf{A} \mathbf{P}_1 \mathbf{P}_2$.

The result follows from the definition of invertible matrix, that the product of two invertible matrices is itself invertible.

So matrix similarity is transitive.

$\square$