

Proof: Suppose $A = PDP^{-1} = PDP^T$ is orthogonally diagonalizable

$$\begin{aligned}\text{Then } A^T &= (PDP^{-1})^T = (PDP)^T \\ &= (P^T)^T D^T P^T \\ &= P D P^T \quad (\because D \text{ is diagonal matrix}) \\ &= A\end{aligned}$$

$$\Rightarrow A^T = A \Rightarrow A \text{ is symmetric}$$