Proof: Let $\bar{u}_1$ & $\bar{u}_2$ be eigenvectors corresponding to different ~~to~~ eigenvectors λ_1 and λ_2. Then:

$$\lambda_1(\bar{u}_1 \cdot \bar{u}_1) = (\lambda_1 \bar{u}_1 \cdot \bar{u}_2)$$
$$= (\lambda_1 \bar{u}_1)^T \bar{u}_2$$ (definition of dot product)
$$= (A\bar{u}_1)^T \bar{u}_2$$
$$= \bar{u}_1^T A^T \bar{u}_2$$
$$= \bar{u}_1^T A \bar{u}_2$$ ($\because$ A is symmetric)
$$= \bar{u}_1^T (\lambda_2 \bar{u}_2)$$
$$= \cancel{\lambda}(\bar{u}_1 \cdot \lambda_2 \bar{u}_2)$$
$$= \lambda_2(\bar{u}_1 \cdot \bar{u}_2)$$

$\because \lambda_1 \neq \lambda_2$

$\Rightarrow \bar{u}_1 \cdot \bar{u}_2 = 0$

H.P.

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