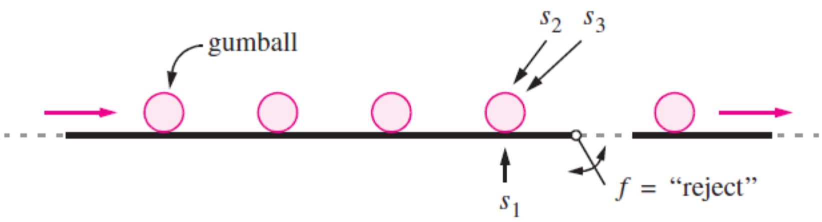
Tutorial 3

Questions:

1. Convert the following decimal numbers into binary
   a. $(17)_{10}$
   b. $(33)_{10}$
   c. $(67)_{10}$
   d. $(130)_{10}$
   e. $(2560)_{10}$
2. What is the minimum number of bits needed to represent the following decimal numbers in binary?
   a. $(270)_{10}$
   b. $(520)_{10}$
   c. $(780)_{10}$
   d. $(1029)_{10}$
3. Prove the validity of the logic equation
   i. $(x_1 + x_3).(\overline{x_1} + \overline{x_3}) = x_1 \cdot \overline{x_3} + \overline{x_1} \cdot x_3$
   ii. $x_1 \cdot \overline{x_3} + x_1 \cdot x_3 + \overline{x_2} \cdot \overline{x_3} + \overline{x_2} \cdot x_3 = x_1 + \overline{x_2}$
4. Implement the following logic function:
   i. $F(x_1, x_2) = x_1 \cdot x_2 + \overline{x_1} \cdot \overline{x_2} + \overline{x_1} \cdot x_2$
   ii. $F(s_1, s_2, s_3) = \overline{s_1} \cdot \overline{s_2} \cdot s_3 + \overline{s_1} \cdot s_2 \cdot s_3 + s_1 \cdot \overline{s_2} \cdot s_3 + s_1 s_2 \overline{s_3} + s_1 s_2 s_3$
5. Use algebraic manipulation to prove following:
   i. $(x + y) \cdot (x + y) = x + y$
   ii. $x \cdot y + y \cdot z + \overline{x} \cdot z = x \cdot y + \overline{x} \cdot z.$
6. Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent the same function.
   (a) $\overline{x}_1 x_3 + x_1 x_2 \overline{x}_3 + \overline{x}_1 x_2 + x_1 \overline{x}_2 = \overline{x}_2 x_3 + x_1 \overline{x}_3 + x_2 \overline{x}_3 + \overline{x}_1 x_2 x_3$
   (b) $x_1 \overline{x}_3 + x_2 x_3 + \overline{x}_2 \overline{x}_3 = (x_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)$
   (c) $(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x}_1 + \overline{x}_3)$
7. Figure 1 depicts a part of a factory that makes bubble gumballs. The gumballs travel on a conveyor that has three associated sensors s_1, s_2, and s_3. The sensor s_1 is connected to a scale that weighs each gumball, and if a gumball is not heavy enough to be acceptable then the sensor sets $s_1 = 1$. Sensors s_2 and s_3 examine the diameter of each gumball. If a gumball is too small to be acceptable, then $s_2 = 1$, and if it is too large, then $s_3 = 1$. If a gumball is of an acceptable weight and size, then the sensors give $s_1 = s_2 = s_3 = 0$. The conveyor pushes the gumballs over a "trap door" that it used to reject the ones that are not properly formed. A gumball should be rejected if it is too large, or both too small and too light. The trap door is opened by setting the logic function f to the value 1. By inspection, we can see that an appropriate logic expression is $f = s_1 s_2 + s_3$. Use Boolean algebra to derive this logic expression from the truth table and implement the logic function using AND, OR and NOT gate.



(a) Conveyor and sensors

Figure 1

8. Design the simplest sum-of-products circuit that implements the function
   i. $f(x, y, z) = \sum m(3,4,6,7)$
   ii. $f(x, y, z) = \sum m(1,3,4,6,7)$
9. Design the simplest product-of-sums circuit that implements the function
   i. $f(x, y, z) = \prod M(0,2,5)$.
   ii. $f(x, y, z) = \prod M(0,1,5,7)$

10. Derive the simplest sum-of-products expression for the function
$$f(w, x, y, z) = w \cdot \bar{y} \cdot \bar{z} + x \cdot \bar{y} \cdot z + w \cdot \bar{x} \cdot \bar{y}$$

11. Derive the simplest product-of-sums expression for the function
$$f(w, x, y, z) = (w + y + z) \cdot (x + y + z) \cdot (w + x + y)$$

12. Design the simplest circuit that has three inputs, x_1, x_2, and x_3, which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output must be 0.

DC

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Q1 a) $(17)_{10}$

2	17	1
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$\Rightarrow (17)_{10} = (10001)_2$

b) $(33)_{10}$

2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
2	1	

$\Rightarrow (33)_{10} = (100001)_2$

c) $(67)_{10}$

2	67	1
2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
	1	

$\Rightarrow (67)_{10} = 1000011$

d) $(130)_{10}$

2	130	0
2	65	1
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	

$\Rightarrow (130)_{10} = (10000010)_2$

e) $(2560)_{10}$

2	2560	0
2	1280	0
2	640	0
2	320	0
2	160	0
2	80	0
2	40	0
2	20	0
2	10	0
2	5	1
2	2	0
	1	

$\Rightarrow (2560)_{10} = (101000000000)_2$

Q2) ~~9 bits~~ ∵ ~~for~~ min no of bits is equal to 2^n where 2^n is just ~~get~~ greater than the number
⇒ n is the no of games.

a) 9 bits

b) 10 bits

c) 10 bits

d) 11 bits

Q3 i) LHS $(x_1+x_3)(\bar{x}_1+\bar{x}_3)$

$x_1\bar{x}_1 + x_1\bar{x}_3 + x_3\bar{x}_1 + x_3\bar{x}_3$ (∵ $a\bar{a}=0$)

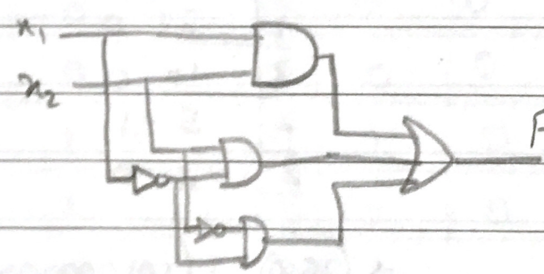
⇒ $x_1\bar{x}_3 + \bar{x}_1 x_3$ = RHS

ii) LHS $x_1\cdot\bar{x}_3 + x_1\cdot x_3 + \bar{x}_2\cdot\bar{x}_3 + \bar{x}_2\cdot x_3$

⇒ $x_1(\bar{x}_3+x_3) + \bar{x}_2(\bar{x}_3+x_3)$ (∵ $\bar{a}+a=1$)

⇒ $x_1+\bar{x}_2$ = RHS

Q4 bi)



~~iii)~~

Q4 ii) $F(s_1, s_2, s_3) = \bar{s}_1 \cdot \bar{s}_2 \cdot s_3 + \bar{s}_1 \cdot s_2 \cdot s_3 + s_1 \cdot \bar{s}_2 \cdot s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3$

$= \bar{s}_1 \cdot s_3 (\bar{s}_2 + s_2) + s_1 \cdot \bar{s}_2 \cdot s_3 + s_1 s_2 (\bar{s}_3 + s_3) \quad (\because a + \bar{a} = 1)$

$= \bar{s}_1 s_3 \cancel{+ \bar{s} + s_2} + s_1 \bar{s}_2 s_3 + s_1 s_2 (1 + s_3)$

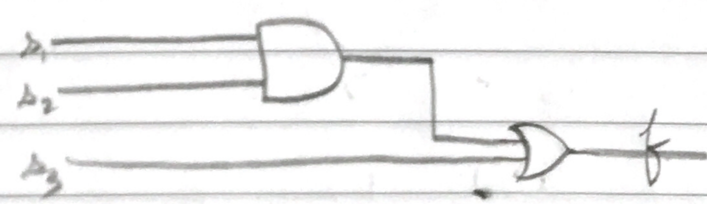
$= \bar{s}_1 s_3 + s_1 s_3 (\bar{s}_2 + s_2) + s_1 s_2$

$= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2$

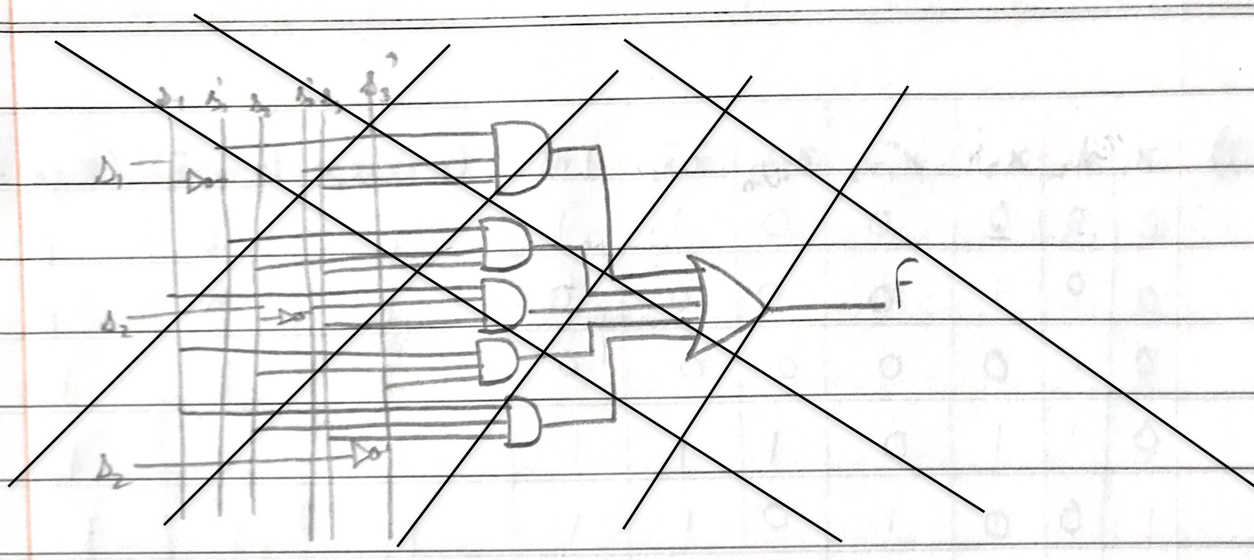
$= s_3 (s_1 + \bar{s}) + s_1 s_2$

$= s_3 + s_1 s_2$

$\Rightarrow$



ii)



Q5 i) $^{LHS}(x+y)\cdot(x+y)$ $\quad$ $(\because a\cdot a = a)$

$= (x+y) = RHS$

ii) $^{LHS}\; xy + yz + x'z = xy + yz(x+x') + x'z = xy + xyz + x'yz + x'z$

$= xy(1+z) + x'z(y+1) = xy + x'z = RHS$

Q6 i)

x_1	x_2	x_3	$\bar{x}_1x_3$	$x_1x_2\bar{x}_3$	$\bar{x}_1x_2$	$x_1\bar{x}_2$	LHS	$\bar{x}_2x_3$	$x_1\bar{x}_3$	$x_2\bar{x}_1$	$\bar{x}x_2x_3$	RHS
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	1	0	0	0	1
0	1	0	0	0	1	0	1	0	0	1	0	1
0	1	1	1	0	1	0	1	0	0	0	1	1
1	0	0	0	0	0	1	1	0	1	0	0	1
1	0	1	0	0	0	1	1	1	0	0	0	1
1	1	0	0	1	0	0	1	0	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0	0	0

$\Rightarrow$ LHS = RHS

1)

x_1	x_2	x_3	$x_1\bar{x}_3$	x_2x_3	$\bar{x}_2\bar{x}_3$	LHS	$(x_1+\bar{x}_2+x_3)$	$(x_1+x_2+\bar{x}_3)$	$(\bar{x}_1+x_2+\bar{x}_3)$	RHS
0	0	0	0	0	1	1	1	1	1	1
0	0	1	0	0	0	0	1	0	1	0
0	1	0	0	0	0	0	0	1	1	0
0	1	1	0	1	1	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1	1
1	0	1	0	0	0	0	1	1	0	0
1	1	0	1	0	0	1	1	1	1	1
1	1	1	0	1	0	1	1	1	1	1

$\Rightarrow$ LHS = RHS

2)

x_1	x_2	x_3	(x_1+x_3)	$(\bar{x}_1+\bar{x}_2+\bar{x}_3)$	$(\bar{x}_1+x_2)$	LHS	$(x_1\oplus x_2)$	$(x_2\oplus x_3)$	$(\bar{x}_1+\bar{x}_3)$	RHS
0	0	0	0	1	1	0	0	0	1	0
0	0	1	1	0	1	0	0	1	1	0
0	1	0	0	0	1	0	1	1	1	1
0	1	1	1	0	1	0	1	1	1	1
1	0	0	1	0	0	0	1	0	1	0
1	0	1	1	0	0	0	1	1	0	0
1	1	0	1	0	1	0	1	1	1	1
1	1	1	1	0	1	0	1	1	0	0

$\Rightarrow$ LHS $\neq$ RHS

Q7

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\Rightarrow f = \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3$$

$$= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1\bar{s}_2 s_3 + s_1 s_2(\bar{s}_3 + s_3) \quad \because (a + \bar{a} = 1)$$

$$= \bar{s}_1 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2(1 + s_3)$$

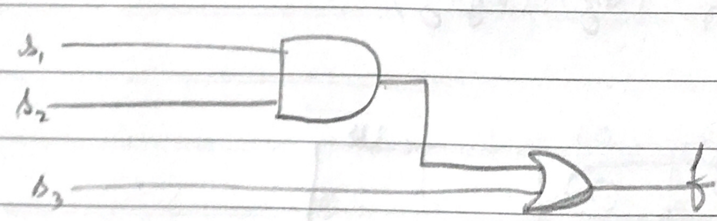
$$= \bar{s}_1 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 + s_1 s_2 s_3$$

$$= \bar{s}_1 s_3 + s_1 s_2 + s_1 s_3(s_2 + \bar{s}_2)$$

$$= \bar{s}_1 s_3 + s_1 s_2 + s_1 s_3$$

$$= s_3(s_1 + \bar{s}_1) + s_1 s_2$$

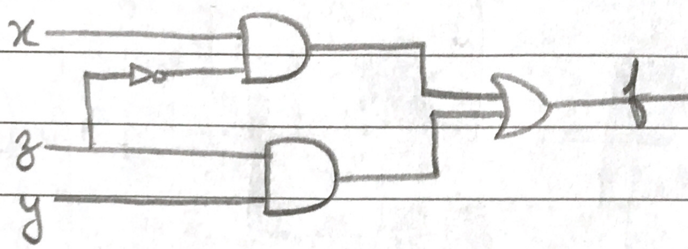
$$= s_1 s_2 + s_3$$



Q8 i)

x \ yz	00	01	11	10
0			1	
1	1		1	1

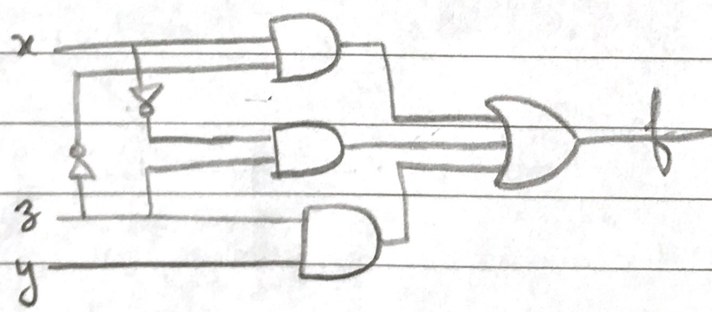
$$f = x\bar{z} + yz$$



ii)

x \ yz	00	01	11	10
0		1	1	
1	1		1	1

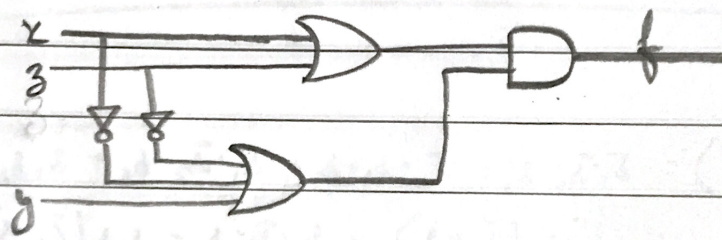
$$f = yz + x\bar{z} + \bar{x}z$$



Q9 i) $f(x,y,z) = \prod M(0,2,5)$

$x \backslash yz$	00	01	11	10
0	0 $_0$	$_1$	$_3$	0 $_2$
1	$_4$	0 $_5$	$_7$	$_6$

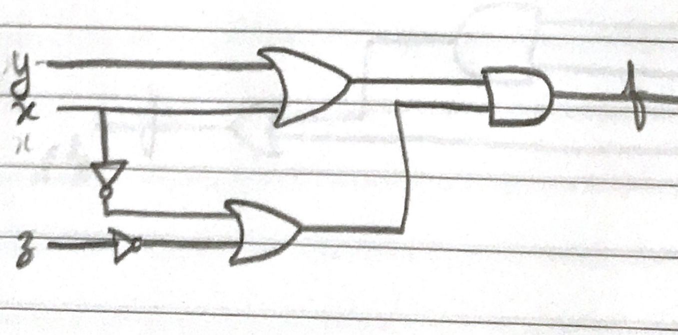
$\Rightarrow f = (x+z) \cdot (\bar{x} + y + \bar{z})$



ii) $f(x,y,z) = \prod M(0,1,5,7)$

$x \backslash yz$	00	01	11	10
0	0 $_0$	0 $_1$	$_3$	$_2$
1	$_4$	0 $_5$	0 $_7$	$_6$

$\Rightarrow f = (x+y) \cdot (\bar{x} + \bar{z})$



Q10 $f(w,x,y,z) = w\cdot\bar{y}\cdot\bar{z} + x\cdot\bar{y}\cdot z + w\cdot\bar{x}\cdot\bar{y}$

$= w\cdot\bar{y}\cdot\bar{z}(x+\bar{x}) + x\cdot\bar{y}\cdot z(w+\bar{w}) + w\bar{x}\cdot\bar{y}(z+\bar{z})$

$= w\cdot x\cdot\bar{y}\cdot\bar{z} + w\bar{x}\bar{y}\bar{z} + wx\bar{y}z + \bar{w}x\bar{y}z + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z}$

$\Rightarrow f(w,x,y,z) = \Sigma m(12, 8, 13, 5, 9)$

wx \ yz	00	01	11	10
00	0	1	3	2
01	4	1 (5)	7	6
11	1 (12)	1 (13)	15	14
10	1 (8)	1 (9)	11	10

$f = w\bar{y} + x\bar{y}z$

Q11 $f(w,x,y,z) = (w+y+z)\cdot(x+y+z)\cdot(w+x+y)$

$= [(w+y+z)+(x\cdot\bar{x})]\cdot[(x+y+z)+(w\cdot\bar{w})]\cdot[(w+x+y)+(z\cdot\bar{z})]$

$= (w+x+y+z)\cdot(w+\bar{x}+y+z)\cdot(w+x+y+z)(\bar{w}+x+y+z)(w+x+y+z)(w+x+y+\bar{z})$

$= (w+x+y+z)\cdot(w+\bar{x}+y+z)\cdot(\bar{w}+x+y+z)\cdot(w+x+y+\bar{z})$

$f(w,x,y,z) = \Pi M(0,4,8,1)$

~~wx \ yz~~	~~00~~	~~01~~	~~11~~	~~10~~
~~00~~	~~0 (0)~~	~~0 (1)~~	~~3~~	~~2~~
~~01~~	~~0 (4)~~	~~5~~	~~7~~	~~6~~
~~10~~	~~12~~	~~13~~	~~15~~	~~14~~
~~11~~	~~0 (8)~~	~~9~~	~~11~~	~~10~~

$\cancel{f(w,x,y,z) = (\bar{w}+\bar{x}+\bar{y})\cdot(\bar{w}+\bar{y}+\bar{z})\cdot(w+x+y+\bar{z})}$

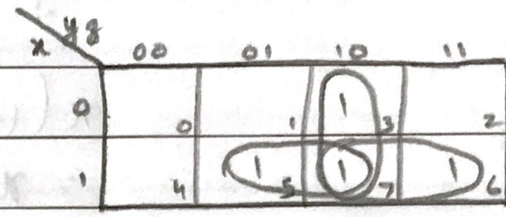
wx \ yz	00	01	11	10
00	0	0		
01	0			
11				
10	0			

$$f(w,x,y,z) = (w+y+z)\cdot(x+y+z)\cdot(w+x+y)$$

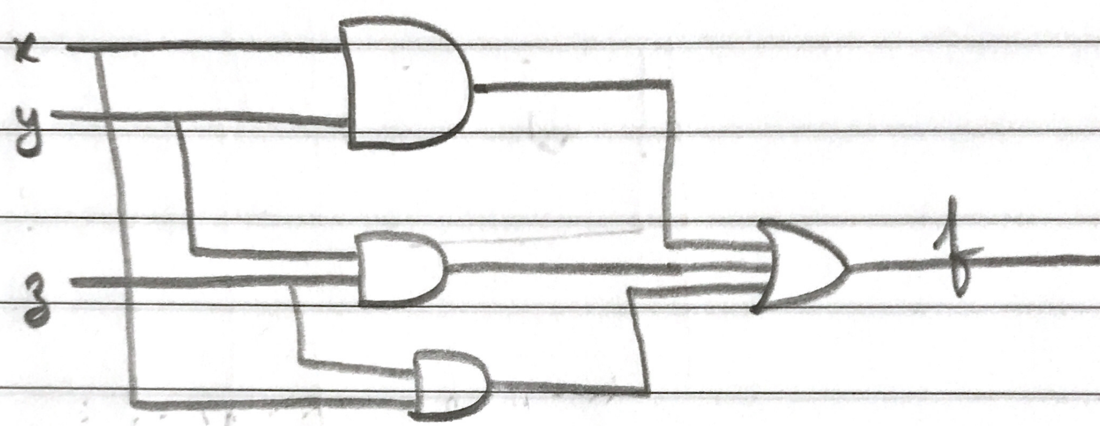
Q12

x_1	x_2	x_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$f = \sum m(3,5,6,7)$$



$$f = xy + yz + xz$$


x
y
z
f