Proof of Prop 30

We use the fact that if P is the change of basis matrix from α to β, then P^{-1} is the change of basis matrix from β to α. Let $A = [T]_\alpha$

$\Rightarrow$ for any $\bar{v} \in V$:

$$
\begin{aligned}
(PAP^{-1})[\bar{v}]_\beta &= (PA)P^{-1}[v]_\beta \\
&= (PA)[v]_\alpha \\
&= P(A[v]_\alpha) \\
&= P(T_\alpha [v]_\alpha) \\
&= P[Tv]_\alpha \\
&= [Tv]_\beta = [T]_\beta [v]_\beta =
\end{aligned}
$$

$\therefore$ The above holds for all vectors $\bar{v} \in V$, it follows that $PAP^{-1} = [T]_\beta = B$