Case 2a)

$$A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 1 & 1 \\ 6 & 4 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \cancel{6}\,(-\lambda)(1-\lambda)^2$$

$\Rightarrow$ eigen values : $\lambda_1 = 1$ (algebraic multiplicity = 2)
$\lambda_2 = 0$ (alg. multi. = 1)

i) Taking $\lambda_1 = 1$, $A - \lambda I = \begin{bmatrix} 3 & 2 & 1 \\ -3 & -2 & 1 \\ 6 & 4 & -2 \end{bmatrix} \xrightarrow{RREF}$

$$\begin{bmatrix} 1 & 2/3 & -1/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore \quad \bar{x} = x_2 \begin{bmatrix} -2/3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1/3 \\ 0 \\ 1 \end{bmatrix}$$

Making apt choices, we get :

$\bar{v}_1 = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$ and $\bar{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ as eigenvectors.

ii) similarly for $\lambda_2 = 0$

$$\vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$ is an eigenvector

In this case we got 3 linearly independent eigenvectors.
So A is diagonizable; This happened because ~~geo. m~~
geo mult. = alg. mult. happened for all λ's