Proof: Case 1: Suppose A has ~~an~~ a left inverse; then there exists a matrix C such that $CA = I$. Now consider the homogeneous system $A\bar{x} = \bar{0}$. If $\bar{y}$ is any solⁿ, then $A\bar{y} = \bar{0}$.

$$\Rightarrow (CA)\,y = C\bar{0}$$

$$\Rightarrow I\bar{y} = \bar{0} \Rightarrow \bar{y} = \bar{0}$$

In short, ~~An~~ homogenous system $A\bar{x} = \bar{0}$ has only trivial sol^n

Hence, by IVT, A is invertible. ~~B~~ Furthermore, $I = CA = A^{-1}A$, so multiplying on the right by A^{-1}, we get $C = A^{-1}$

Case 2: Suppose A has a right inverse; then there exists a matrix D such that $AD = I$. In other words D has a left inverse & so is invertible by case 1. Hence

$$(AD)\,D^{-1} = I\,D^{-1} \text{ or } A = D^{-1}$$

Thus, A being the inverse of invertible matrix, is itself invertible & $A^{-1} = D$