

Observation (Proof): The statements a) & b) of the Rank Theorem follow from our discussion of finding the basis for  $\text{Col } A$  and  $\text{Row } A$  resp. In each case, the no of basis vectors corresponded to the number of pivot elements in RREF matrix  $R$  of a given matrix  $A$ .

- For statement c), ~~Assumption that~~ observe that pivot column of  $R$  correspond to basis vectors of  $\text{Col } A$  (leading variables of the homogeneous system), whereas the remaining columns corresponds to basis vectors of  $\text{Nul } A$  (free variables of homogeneous system).

Since, the total no of columns = no of variables =  $n$

$$n = \text{rank}(A) + \text{nullity}(A), \text{ as required}$$