MTH 100B_ 20220402_SAT_ L37
NOTES

Proof of Prop. [redacted] 43 : We need to show that if $\bar{y} \in W$, and $\bar{y} = c_1 \bar{u}_1 + \cdots + c_p \bar{u}_p$ (1),

where the $\bar{u}_i$'s form an orthogonal basis of W,

then $$c_j = \frac{\langle \bar{y}, \bar{u}_j \rangle}{\langle \bar{u}_j, \bar{u}_j \rangle} \quad \text{for } j = 1, \ldots, p. \qquad (2)$$

→ Using (1), we take the inner product of $\bar{y}$ with $\bar{u}_j$:-

$$\langle \bar{y}, \bar{u}_j \rangle = \langle c_1 \bar{u}_1 + \cdots + c_p \bar{u}_p, \bar{u}_j \rangle$$

$$= c_1 \langle \bar{u}_1, \bar{u}_j \rangle + c_2 \langle \bar{u}_2, \bar{u}_j \rangle + \cdots + c_p \langle \bar{u}_p, \bar{u}_j \rangle$$

$$= c_j \langle \bar{u}_j, \bar{u}_j \rangle \Rightarrow c_j = \langle \bar{y}, \bar{u}_j \rangle / \langle \bar{u}_j, \bar{u}_j \rangle$$