Proof →

(a) For any functions f and g from V to W and any scalar c, we define the functions $(f+g)$ and (cf) by:

$$*\ (f+g)(\vec{u}) = f(\vec{u}) + g(\vec{u}) \text{ for all } \vec{u} \text{ in } V$$

$$*\ (cf)(\vec{u}) = c f(\vec{u}) \text{ for all } \vec{u} \text{ in } V.$$

⇒ Because W is a vector space over F, W^V becomes a vector space over F

(b) To show it's a subspace →

(i) 0 function is a LT, hence belongs to $L(V,W)$

(ii) $$T+U(\vec{u}+\vec{v}) = T(\vec{u}+\vec{v}) + U(\vec{u}+\vec{v})$$
$$= T(\vec{u}) + T(\vec{v}) + U(\vec{u}) + U(\vec{v})$$
$$= (T+U)(\vec{u}) + (T+U)(\vec{v})$$

Similarly, $$(T+U)(c\vec{u}) = c(T(\vec{u}) + U(\vec{u}))$$
$$= c(T+U)(\vec{u})$$

(iii) $$(cT)(\vec{u}+\vec{v}) = cT(\vec{u}+\vec{v}) = c(T\vec{u} + T\vec{v})$$
$$= (cT)(\vec{u}) + (cT)(\vec{v})$$