

Proof:-

a) The zero vector is unique.

→ Suppose by the way of contradiction that \vec{u} & \vec{v} are 2 distinct zero vectors.

Now: $\vec{u} + \vec{v} = \vec{u}$ ①, since \vec{v} is a zero vector.

Similarly, $\vec{u} + \vec{v} = \vec{v}$ ②, since \vec{u} is a zero vector.

→ From ① & ②, $\vec{u} = \vec{v}$ -

Hence, Proved

c) $-\vec{u} = (-1)\vec{u}$

We know that the addition inverse of \vec{u} is unique (one of the prev. part of proposition 7)

$$\vec{u} + (-\vec{u}) = \vec{0} \quad \text{①}$$

$$0 \cdot \vec{u} = \vec{0} \quad (\text{Part (c) of Proposition 7})$$

$$(1+(-1))\vec{u} = \vec{0}$$

$$\Rightarrow 1 \cdot \vec{u} + (-1)\vec{u} = \vec{0}$$

$$\vec{u} + (-1)\vec{u} = \vec{0} \quad \text{②} \quad (\text{Using axioms})$$

→ by ① & ②

$$-\vec{u} = (-1)\vec{u}$$