Proof: Since W is a proper subspace, it contains a vector $\overline{w}_1 \neq 0$. If $\overline{w}_1$ spans W, then W is finite dimensional. If not, there's a vector $\overline{w}_2$ ~~outside~~ in ~~$\overline{w}_2$~~ W outside span$\{\overline{w}_1\}$, & by adjoining $\overline{w}_2$ to $\{\overline{w}_1\}$, we still have a linearly independent set (by proposition 14).

Continuing in the way, we get a basis of W with at most $\dim V$ elements (Proposition 12). Hence, W is finite dimensional & $\dim W \leq \dim V$. Since W is a proper ~~subset~~ subspace, there is a vector $\overline{v}$ outside W. ~~Adjoining $\overline{v}$ to~~

Hence, $\dim W$ must be (strictly) less than $\dim V$.