## MTH100B Math 1 (Linear Algebra) Monsoon Semester - 2021-22 Tutorial

## **Exercise for the Week Commencing Monday 20220124**

1. Determine the inverse of the given matrix A *using row reduction*.

$$A = [21 - 1]$$
  
 $|021|$   
 $[52 - 3]$ 

## 2. TRUE or FALSE?

- a) The sum of two invertible matrices (square matrices of the same order) is always invertible.
- b) If matrices A and B commute, then invertibility of A implies invertibility of B.

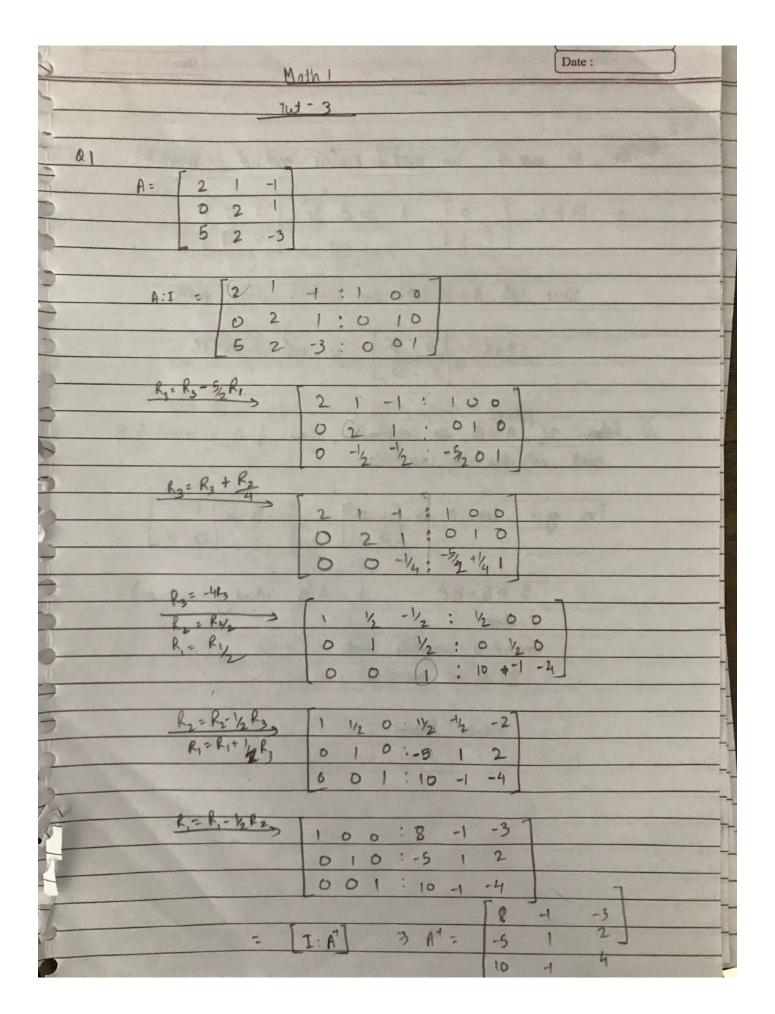
Justify your answer – proof if TRUE or counter-example if FALSE.

- 3. Suppose AB = AC, where B and C are n×p matrices and A is an invertible n×n matrix. Show that B = C. Is this true, in general, when A is not invertible? Justify your answer (proof if true, counter-example if false).
  - 4. **Observation 1 in Invertible Matrices Quick Review** (L07 on Monday 20220117) states that if the inverse of A exists, it is unique. Can you prove this?
- 5. Consider a general 2×2 matrix A = [ a b ] [ c d ]
  - a) Using Theorem 1 (VIT) and Corollary 1.1, show that A is invertible if and only if ad  $-bc \neq 0$ .
  - b) Hence determine an expression (formula) for A<sup>-1</sup>.
- 6. Construct a  $2 \times 2$  matrix A with all non-zero entries such that the solution set of the system  $A\mathbf{x} = \mathbf{0}$  is the line in  $\mathbb{R}^2$  through (5,-3) and the origin. Now find a non-zero vector  $\mathbf{b}$  such that the solution set of  $A\mathbf{x} = \mathbf{b}$  is not a line in  $\mathbb{R}^2$  parallel to the solution set of  $A\mathbf{x} = \mathbf{0}$ . Explain why this does not contradict Observation 6 (see lecture slides for L06)

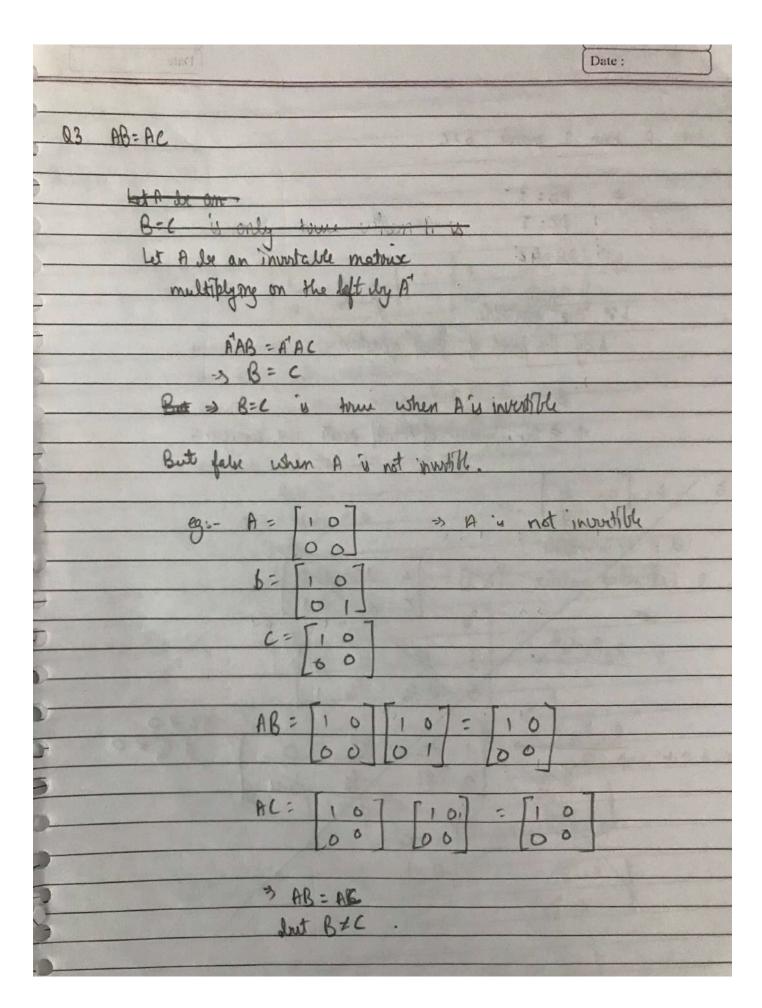
- on Thursday 20220113).
- 7. Prove Proposition 5 in the general case, i.e. for any row operation e and any matrix A. (NB: The three cases of scaling, replacement and interchange require separate proofs.)

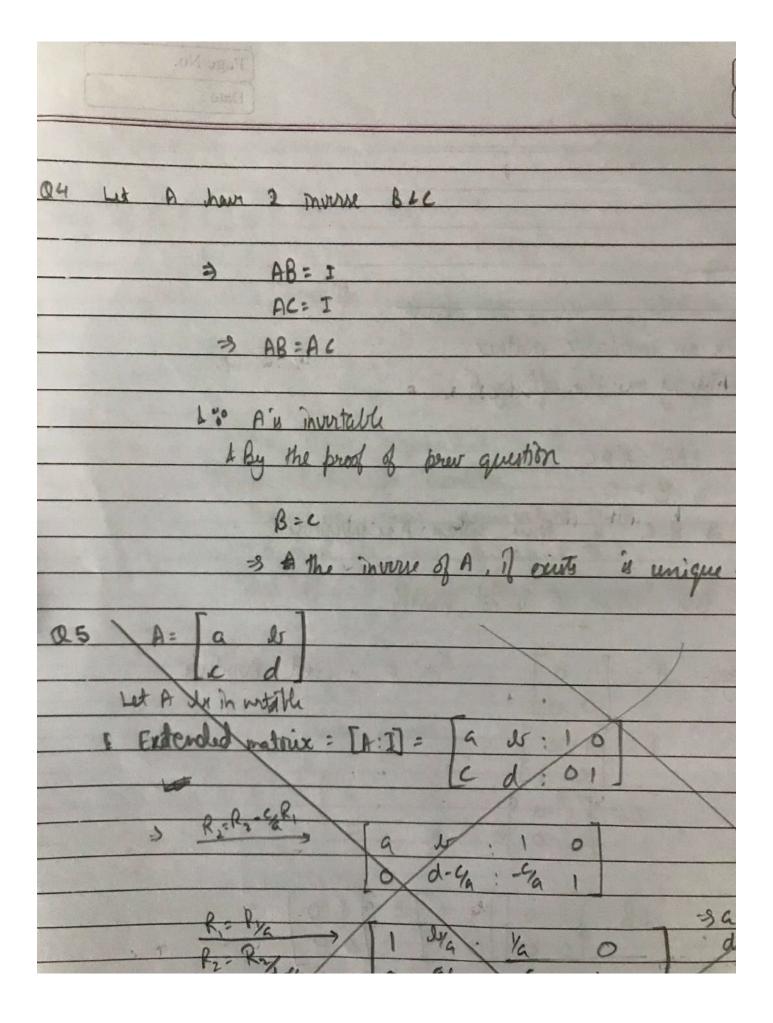
**Hint:** Recall that if  $\mathbf{u} = (u_1, u_2, \dots, u_n)$  and  $\mathbf{v} = (v_1, v_2, \dots, v_n)$  are vectors in  $\mathbb{R}^n$ , then the dot product  $\mathbf{u} \cdot \mathbf{v}$  is the scalar (real number) given by  $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 u_2 v_2 \dots u_n v_n$ . If A is an m×n matrix and B is an n×p matrix, then the rows of A and columns of B are vectors in  $\mathbb{R}^n$ . The general i,j-th term of the product C = AB is  $c_{i,j} = dot$  product of i-th row of A with j-th column of B. This tip is useful while doing the algebraic calculations required in proofs of results involving multiplication of matrices.

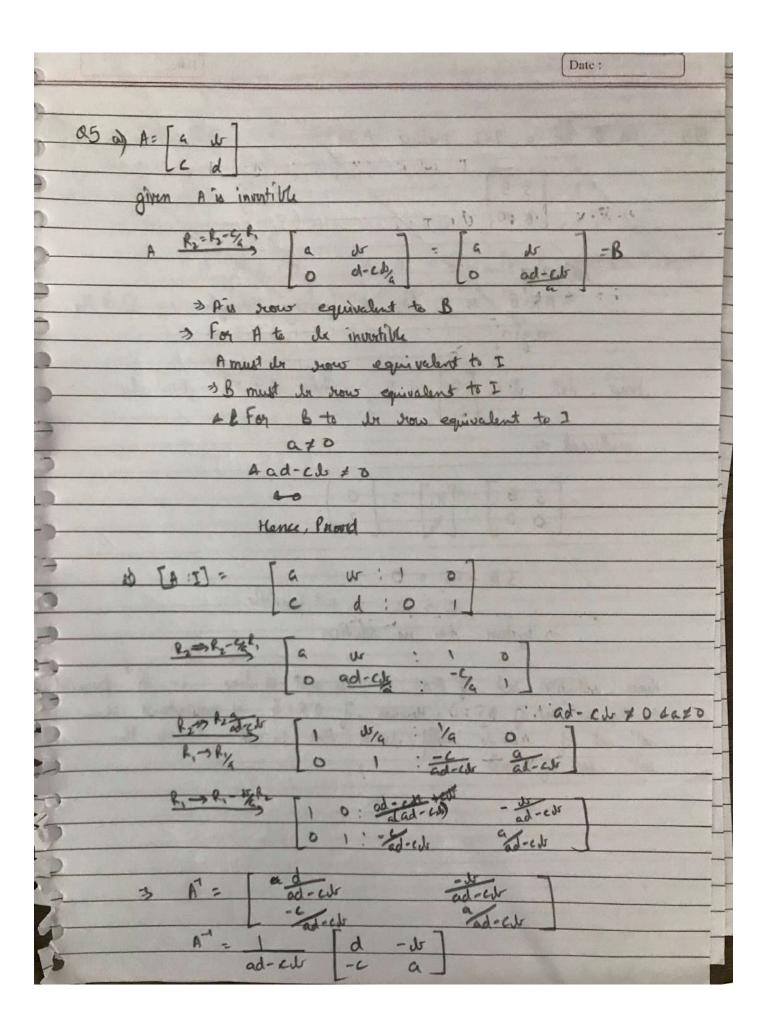
8. Given an m×n matrix A and an n×p matrix B, the product AB is given by the rule AB =  $[Av_1 Av_2 ..... Av_p]$  in column form where B =  $[v_1 v_2 ..... v_p]$  in column form. Construct an example to illustrate this rule. The matrix A in your example should be at least 3×3 and B should be at least 3×2. Then prove the rule in the general case.



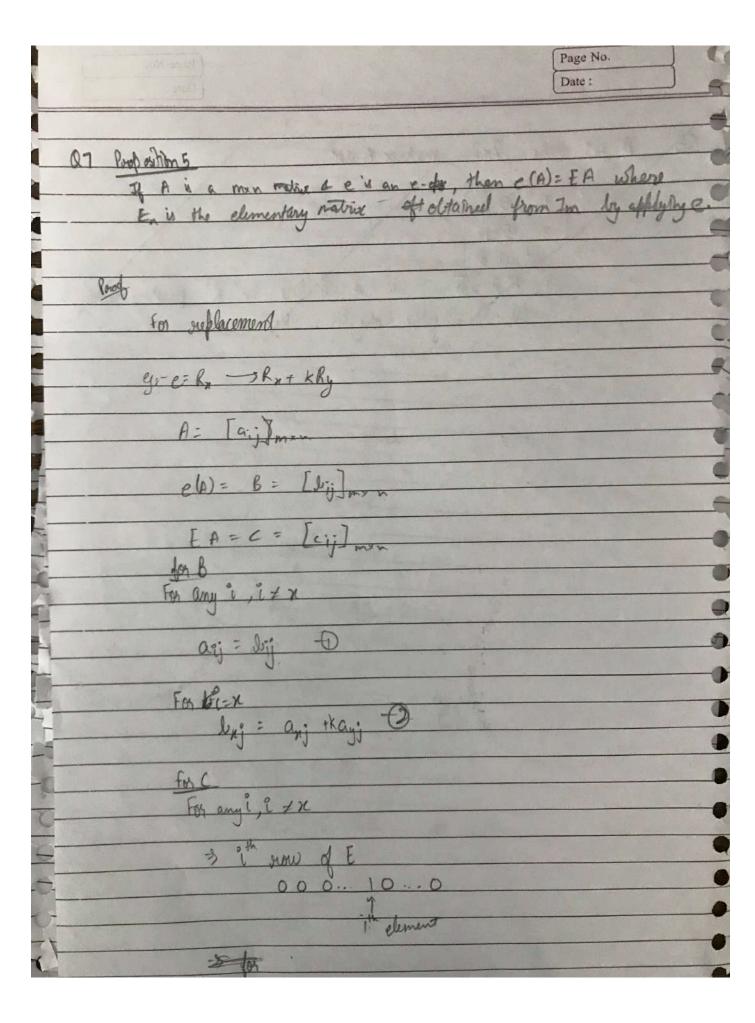
Page No. Date: a) False the sum of two murtille matrix need not in invertable g:- WAln 4 da B dr Since leth A & B loth are invitable dut APB 'y not investable. A+R = natrices D) Falso, If A & B so commute, then if A is invotible B need not also be invertible 4 B dr For eg: Let A in [ , o] 3 AB=BA (: A'ss identify notice 800 And A is investible enot 8 is not involvible Herei, Provid

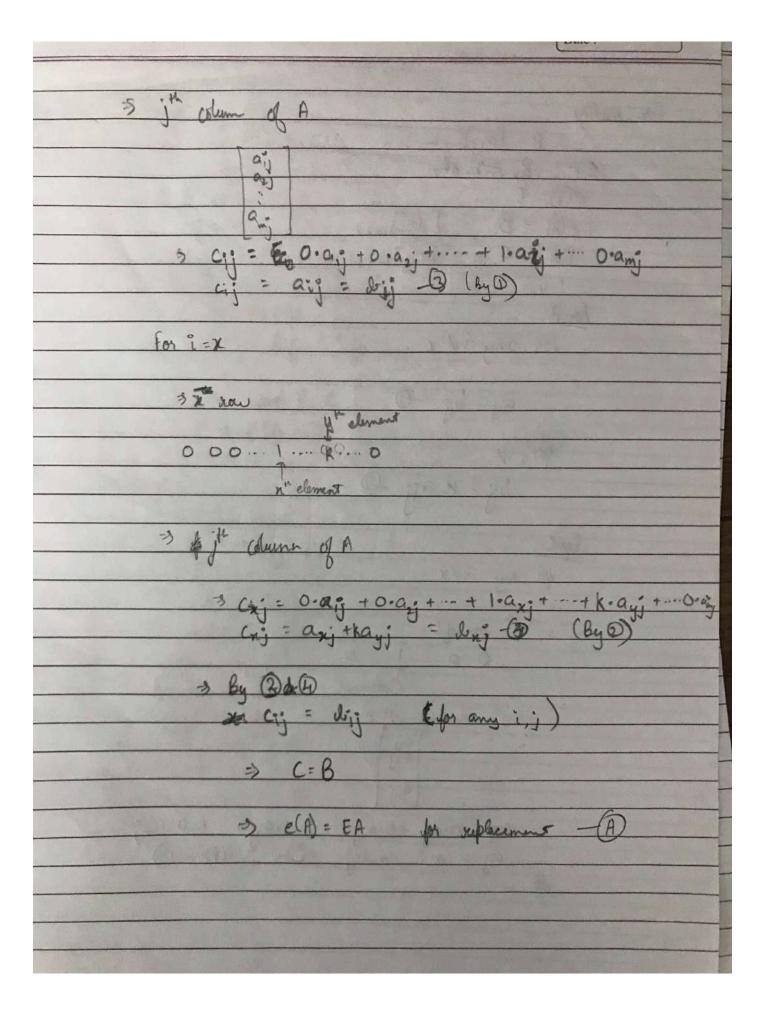






Page No. Date: the a 2×2 metric A dr Q6 Ld 4 6 10 sol set of is a line in IR2 passing through (5,-3) 4 the 3 n, +5n, = 0 0 = 2 = not posite system has no solution Hence, solution set of An = ar is not a line in  $IR^2$  be to the solution set of  $A\bar{n} = \bar{0}$ . However, if  $A\bar{n} = \bar{0}$  is consistent the solution set of  $A\bar{n} = \bar{0}$  will be a line in  $IR^2$  11 to the solution set of  $A\bar{n} = \bar{0}$ 





Date	Date:	
For scaling 0- [-]		
ti- lajlman		
e:- Rx → khx		
e(1) = £		
= Dela = B = [dij]min EA = C [cij]min		
EA = C [Gi]nx		
ForB		
for any v/x		
1 000		
aij = bij -D		
40.1=x		
bij= kaij D		
For C  for any ixx		
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0010		
it demont		
> jt column of A		4
		-
anj		
Cin = 0.an + 0.an + + 1.an + + 0	·anj	
Cij = aij = bij - (FArom(		0
4		9
		A

0-aj + 0-aj + ... + k-aj + ... + 0 am kazj = dzj & (Forom@) From 3 45 = Bij (for any i, j for scaling e(A) = EA For interchange FM B

