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Eg:-

$$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} = B$$

char poly. = B det $(B - \lambda I)$

$= \lambda(\lambda - 90)$

Eigenvalues in descending order = $\lambda_1 = 91$, $\lambda_2 = 0$

Putting $\lambda_1 = 90$, $B - \lambda_1 I = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix}$

The solⁿ after getting RREF

$$\bar{x} = x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

After normalizing $\bar{v}_1 = \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \end{bmatrix}$

Putting $\lambda_2 = 0$ & row-reducing, similarly

$$\bar{v}_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

NB: $\bar{v}_1, \bar{v}_2$ are orthonormal as ~~usual~~ expected.

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$\therefore \quad V = \begin{bmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ -1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$

& singular values $\sigma_1 = \sqrt{90} = 3\sqrt{10}$
& $\sigma_2 = 0$

Now we need to compute U

$$\bar{u}_1 = \frac{1}{\sigma_1} A\bar{v}_1 = \frac{1}{3\sqrt{10}} \begin{bmatrix} -10/\sqrt{10} \\ 20/\sqrt{10} \\ 20/\sqrt{10} \end{bmatrix} = \begin{bmatrix} -1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

However, $A\bar{v}_2 = \bar{0}$. We need to extend $\bar{u}_1$ to an orthonormal basis of $\mathbb{R}^3$.

This is equivalent to finding $W^\perp$ where $W = \text{Span}\{\bar{u}_1\}$

This means solving the linear system.

$\bar{u} \cdot \bar{x} = 0$

$u^T \bar{x} = 0$ or $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{3}{2} \end{bmatrix} \bar{x} = \bar{0}$

$\Rightarrow x_1 = 2x_2 + 2x_3$

$\Rightarrow \bar{x} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ as the general sol^n.

But we need orthonormal vectors.

$\Rightarrow$ Use G-S process

We can get two orthonormal solⁿ by inspection :-

Putting $x_2 = \frac{2}{3}$ & $x_3 = -\frac{1}{3}$

$$\bar{u}_2 = \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix}$$

& $x_2 = -\frac{1}{3}$, $x_3 = \frac{2}{3}$

$$\bar{u}_3 = \begin{bmatrix} 2/3 \\ -1/3 \\ 2/3 \end{bmatrix}$$

Check $\bar{u}_2 \cdot \bar{u}_3 = 0$

So finally.

$A = U \Sigma V^T$

$$A = \underset{3\times 3}{\begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & 2/3 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 3/\sqrt{10} & -1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}$$