

✓ Proof: If  $A$  is invertible, then by VII,  $A$  is row equivalent to  $I$ ,  
 i.e.,  $I = c(e_p, \dots, e_p(A))$  for some sequence of elementary row ops.  
 If  $E_1$  to  $E_p$  are the corresponding elementary matrices, then  
 $I = (E_p \dots E_1)A$ , Each  $E_i$  being invertible,  
 $\Rightarrow A = (E_p \dots E_1)^{-1} I = E_1^{-1} \dots E_p^{-1}$

Hence  $A$  is a product of elementary ops.

Furthermore,  $A^{-1} = (E_1^{-1} \dots E_p^{-1})^{-1} = (E_p \dots E_1) = (E_p \dots E_1)I = c_p(c_p, \dots, c_p(I))$

In other words, the same sequence of row ops that reduce  $A$  to  
 $I$  also reduce  $I$  to  $A^{-1}$ .