Eg:- Find Basis for $W^{\perp}$ given Basis of $W = \{\bar{v}_1, \bar{v}_2\}$

$\bar{v}_1 = (1,1,1,1)$; $\bar{v}_2 = (2,2,4,6)$

Suppose $\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4) \in W^{\perp}$

$\therefore$ $\bar{v}_1 \cdot \bar{x} = 0 \Rightarrow x_1 + x_2 + x_3 + x_4 = 0$

$\bar{v}_2 \cdot \bar{x} = 0 \Rightarrow 2x_1 + 2x_2 + 4x_3 + 6x_4 = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 4 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\Rightarrow$ 2 free variables

$\Rightarrow \dim W^{\perp} = 2$

$\Rightarrow \dim W + \dim W^{\perp} = 4$

~~Find to non homogeneous system to find the basis.~~

Vectors in the basis of Nul R forms the basis for W⊥

~~W~~

Note 4: $\bar{y}^{\wedge}$ is called projection of $\bar{y}$ on W, written $\mathrm{proj}_W \bar{y}$