Proof: Suppose $\bar{v}$ is not in Span S, & consider

$$c_1\bar{v}_1 + \dots + c_n\bar{v}_n + c\bar{v} = \bar{0} \quad \text{— (1)}$$

If $c \neq 0$, we can write $c\bar{v} = -c_1\bar{v}_1 - \dots - c_n\bar{v}_n$

or $c^{-1}c\,\bar{v} = -c^{-1}c_1\bar{v}_1 + \dots + -c^{-1}c_n\bar{v}_n = \bar{v}$

which contradicts our assumption. Hence, $c = 0$

Then (1) becomes

$c_1\bar{v}_1 + \dots + c_n\bar{v}_n = \bar{0}$, since S is linearly independent, $c_i = 0$ for all i. ~~Thus~~ ~~So~~

Thus, $S \cup \{\bar{v}\}$ is linearly independent.