

Proposition 5

Let e be a \mathbf{e} -op.

Let A be a $m \times n$ matrix.

$\Delta e(I_m) = E$, where E is a $m \times m$ matrix.

\Rightarrow If e is scaling $R_R \rightarrow \alpha R_R$

$$\Rightarrow E = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \dots & \alpha & 0 \\ 0 & 0 & \dots & \dots & 1 \end{bmatrix}$$

$$\Rightarrow e(A) = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \dots & \dots & a_{mn} \end{bmatrix}$$

$$\text{Let } EA = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{1n} \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = e(A)$$

$$\Rightarrow e(A) = EA$$

Hence Proved

Similarly, for replace and interchange