

Proof \rightarrow

let us take an ordered matrix $\alpha = \alpha(v_1, v_2, \dots, v_n)$ for V

and $\beta = \beta(w_1, w_2, \dots, w_m)$ for W

let T be any linear transformation in $L(V, W)$.
Then we can find the matrix of T with respect to bases α and β and let us call it $[T]_{\alpha \rightarrow \beta}$

The mapping $\phi : L(V, W) \rightarrow F^{m \times n}$ which takes a linear transformation T to its matrix $[T]_{\alpha \rightarrow \beta}$ is an isomorphism!

Note: ϕ depends on choice of α and β