

Proof of Prop 36

Let us assume that $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p$ are linearly dependent.

Let m be the smallest number such that $\vec{v}_1, \dots, \vec{v}_m$ are linearly independent & \vec{v}_{m+1} is a linear combination of the preceding vectors.

$$\text{Then, } c_1 \vec{v}_1 + \dots + c_m \vec{v}_m = \vec{v}_{m+1} \quad (1)$$

Left multiplication by A

$$c_1 A \vec{v}_1 + c_2 A \vec{v}_2 + \dots + c_m A \vec{v}_m = A \vec{v}_{m+1}$$

$\therefore v_i$'s are eigen vectors

$$c_1 \lambda_1 \vec{v}_1 + c_2 \lambda_2 \vec{v}_2 + \dots + c_m \lambda_m \vec{v}_m = \lambda_{m+1} \vec{v}_{m+1} \quad (2)$$

Multiplying (1) by λ_{m+1} & subtracting from (2), we get

$$c_1 (\lambda_1 - \lambda_{m+1}) \vec{v}_1 + \dots + c_m (\lambda_m - \lambda_{m+1}) \vec{v}_m = 0 \quad (3)$$

However, $\vec{v}_1, \dots, \vec{v}_m$ are lin. indep. so all the coeff in (3) are

$$0 \Rightarrow c_1 (\lambda_1 - \lambda_{m+1}) = 0,$$

$\Rightarrow c_i = 0 : \lambda$'s are given to be distinct

$$\text{Similarly } c_2 = c_3 = \dots = c_m = 0$$

But from (1)

$$\text{we get, } \vec{v}_{m+1} = 0$$

But this is not possible as all the \vec{v} 's are eigenvectors.

\Rightarrow By contradiction that $\{\vec{v}_1, \dots, \vec{v}_p\}$ are linearly independent