~~Proof~~: As noted before, we only need to prove the forward direction of this result. So we have only to show that if $A$ is invertible, then each $A_i$ is invertible. We will first show that the last matrix in the product, i.e., $A_n$ is invertible. Let $\bar{y}$ be any sol$^n$ of the homogeneous system $A_n\bar{x} = \bar{0}$. So $A_n\bar{y} = \bar{0}$

Multiplying on the left ~~Both sides~~ ~~on the left~~ by $A_1 A_2 \cdots A_{n-1}$

$A_1 A_2 \cdots A_{n-1} A_n \bar{y} = \bar{0}$ or $A\bar{y} = \bar{0}$. Since $A$ is invertible, multiplying on the left ~~on the~~ by $A^{-1}$

we get $A^{-1}A\,\bar{y} = \bar{0} \Rightarrow I\bar{y} = \bar{0} \Rightarrow \bar{y} = \bar{0}$

In short the homogeneous system $A_n\bar{x} = \bar{0}$ has only trivial sol$^n$. Hence, by VIT, $A_n$ is invertible

Now putting $A_1 A_2 \ldots A_{n-1} A_n = A$ & multiplying on the right by $A_n^{-1}$ we get,

$$A_1 A_2 \ldots A_{n-1} = A A_n^{-1} = B \text{ (say)}$$

The matrix $B$ being a product of invertible matrices is invertible. Thus by what we have shown above, $A_{n-1}$ is invertible.

By repeating this step, we get that each $A_i$ is invertible