

An example of case 1:

$$A = \begin{bmatrix} 42 & -33 \\ 22 & -13 \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} 42 - \lambda & -33 \\ 22 & -13 - \lambda \end{bmatrix} = 180 - 29\lambda + \lambda^2 \\ &= (20 - \lambda)(9 - \lambda) \end{aligned}$$

\Rightarrow the ~~two~~ eigen values are distinct, $\lambda_1 = 20, \lambda_2 = 9$

Finding corresponding eigen vector

$$1) \text{ For } \lambda_1 = 20; A - \lambda_1 I = \begin{bmatrix} 22 & -33 \\ 22 & -33 \end{bmatrix} \rightarrow \text{RREF}$$

$$\begin{bmatrix} 1 & -3/2 \\ 0 & 0 \end{bmatrix}$$

so we can take any scalar multiple of the an eigen vector, $\Rightarrow \vec{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$$\Rightarrow A\vec{v}_1 = \lambda_1 \vec{v}_1 = 20 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Similarly for, λ_2