NCERT Physics 12.7 Q21

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Question: Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Solution: Given parameters are:

Description	Symbol	Value
Inductance	L	3.0 H
Capacitance	C	27 μF
Resistance	R	7.4 Ω

TABLE I: Given Parameters

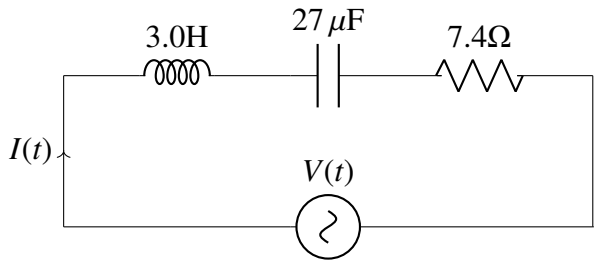

Fig. 1: LCR Circuit

Frequency Response of the Circuit

This is a series LCR circuit, with the elements in series with the voltage source. Applying Kirchhoff's Voltage Law (KVL), we get:

$$V_R + V_L + V_C = V(t) \tag{1}$$

where V_R, V_L and V_C are the voltages across R, L and C respectively and $V(t)$ is the time-varying voltage source.

The circuit can be analysed in the frequency domain instead of the time domain by applying the Laplace Transform. This produces the corresponding impedances of the elements, which allows to solve algebraic equations instead of differential equations.

Elements and their corresponding impedances are given in the table below:

Description	Symbol	Impedance
Resistance	R	R
Inductance	L	sL
Capacitance	C	$\frac{1}{sC}$

TABLE II: Impedances

where s is a complex variable. The circuit can now be redrawn as:

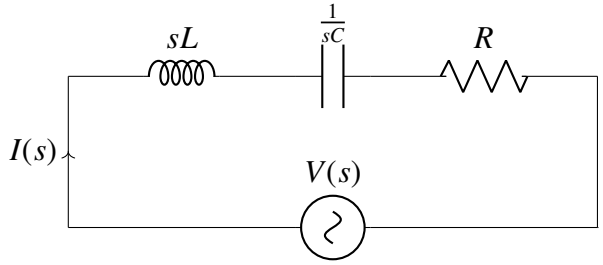

Fig. 2: LCR Circuit

Using the impedances of R, L and C from TABLE II in equation (1), we get

$$V(s) = R \cdot I(s) + sL \cdot I(s) + \frac{1}{sC} \cdot I(s) \tag{2}$$

$$\Rightarrow V(s) = I(s)\left(R + Ls + \frac{1}{sC}\right) \tag{3}$$

$$\Rightarrow I(s) = \frac{V(s)}{\left(R + Ls + \frac{1}{sC}\right)} \tag{4}$$

The term $\frac{I(s)}{V(s)}$ is called the Laplace Admittance Y(s).

$$\Rightarrow Y(s) = \frac{I(s)}{V(s)} = \frac{s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \tag{5}$$