NCERT Math 11.9.2 Q8

EE23BTECH11009 - AROSHISH PRADHAN*

Question: The vertices of ΔPQR are $P(2,1)$, $Q(-2,3)$ and $R(4,5)$. Find equation of the median through the vertex R.

Solution:

Vertex	Coordinate
P	$(2,1)$
Q	$(-2,3)$
R	$(4,5)$

TABLE I: Given Parameters

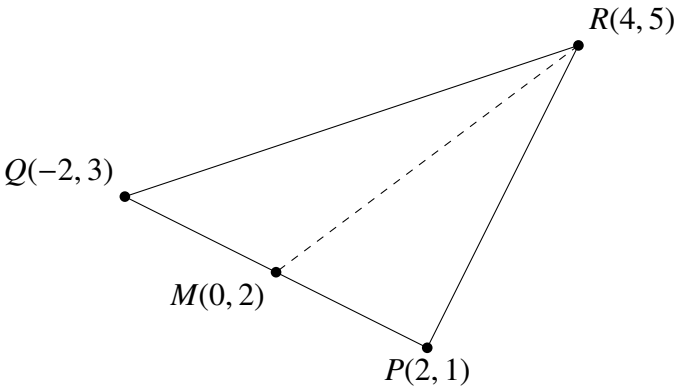

Fig. 1: ΔPQR

Coordinates of mid-point M of PQ are given by:

$$(x_M, y_M) = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2}\right) \tag{1}$$

$$= (0,2) \tag{2}$$

$\therefore$ equation of median RM:

$$y - 2 = \left(\frac{5-2}{4-0}\right)(x-0) \tag{3}$$

$$\implies y = \frac{3}{4}x + 2 \tag{4}$$

1) Plotting the discrete signal:

$$x(n) = \left(\frac{3}{4}n + 2\right)u(n) \tag{5}$$

where,

$$u(n) = \begin{cases} 0 & : n < 0 \\ 1 & : n \geq 0 \end{cases} \tag{6}$$

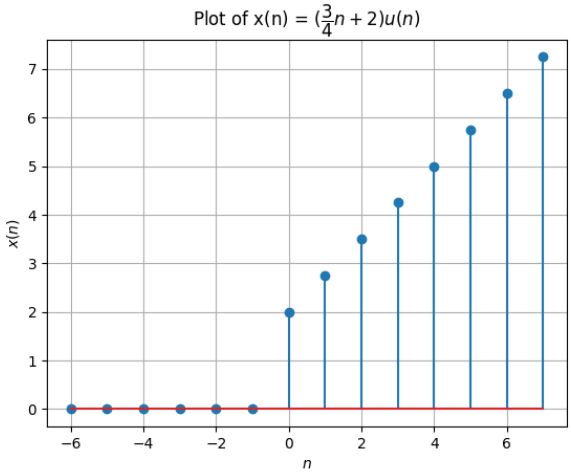

Fig. 2: Plot of x(n)

2) Z-Transform of $x(n)$

$$x(n) \overset{z}{\longleftrightarrow} X(z) \tag{7}$$

$$\implies X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{8}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{3}{4}n + 2\right) u(n) z^{-n} \tag{9}$$

$$= \frac{3}{4}\sum_{n=0}^{\infty} n z^{-n} + 2\sum_{n=0}^{\infty} z^{-n} \tag{10}$$

$$= \frac{3}{4}S_2 + 2S_1 \tag{11}$$

Solving the summations:

a) S_1

$$S_1 = \sum_{n=0}^{\infty} z^{-n} \tag{12}$$

$$= \frac{1}{1 - z^{-1}} \tag{13}$$

b) S_2

From (12) and (13),

$$\sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}} \tag{14}$$

$$\implies \frac{d}{dz}\left(\sum_{n=0}^{\infty} z^{-n}\right) = \frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right) \tag{15}$$

$$\implies \sum_{n=0}^{\infty} -nz^{-n-1} = \frac{-z^{-2}}{1-z^{-1}} \tag{16}$$

$$\implies S_2 = \sum_{n=0}^{\infty} nz^{-n} = \frac{z^{-1}}{(1-z^{-1})^2} \tag{17}$$

Using (13) and (17) in (11)

$$X(z) = \frac{3}{4}\frac{z^{-1}}{(1-z^{-1})^2} + \frac{2}{1-z^{-1}} \tag{18}$$

where $\{z \in \mathbb{C} : |z| > 1\}$