

# NCERT Math 11.9.2 Q8

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**Question:** An input voltage in the form of a square wave of frequency  $1\text{ kHz}$  is given to a circuit, which results in the output shown schematically below. Which one of the following options is the CORRECT representation of the circuit?

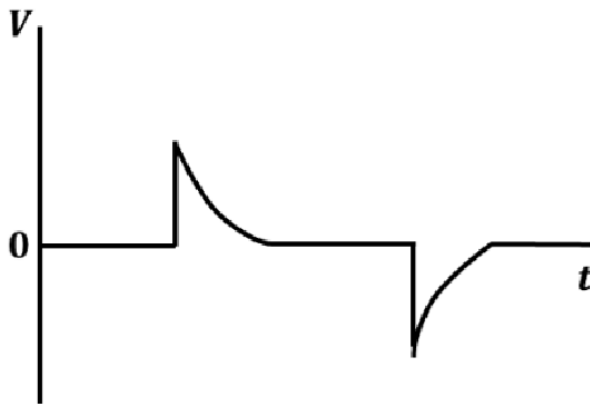
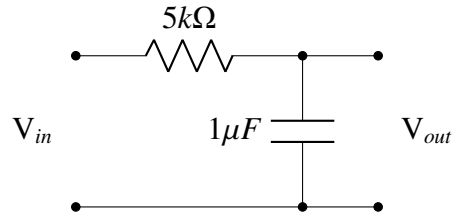


Fig. 1



(d)

**Solution:**

Symbol	Value	Description
$V_{in}(t)$		Input Voltage
$\mathcal{V}_{in}(f)$		Fourier Transform of $V_{in}(t)$
$V_{out}(t)$		Output Voltage
$\mathcal{V}_{out}(f)$		Fourier Transform of $V_{out}(t)$
$f$	$1000\text{Hz}$	Input Wave Frequency
$T$	$\frac{1}{f} = 10^{-3}\text{s}$	Input Wave Time Period
$R$	(a) $0.5\text{k}\Omega$ (b) $5\text{k}\Omega$	Resistance
$C$	(a) $0.1\mu\text{F}$ (b) $1\mu\text{F}$	Capacitance
$\tau$	$RC$	Time Constant
$Z$	$R + \frac{1}{sC}$	Impedance
$H(f)$	$\frac{V_{out}}{V_{in}}$	General Transfer Function
$H_R(f)$	$\frac{V_{R,out}}{V_{in}}$	Transfer Function for Resistor
$H_C(f)$	$\frac{V_{C,out}}{V_{in}}$	Transfer Function for Capacitor

TABLE I: Given Parameters

Input waveform is a square wave (Fig. 3), so we take its Fourier Transform as shown in Fig. 4

$$V_{in}(t) = 2 \left( 2 \left[ \frac{\left( t - \frac{T}{4} \right)}{T} \right] - \left[ \frac{2 \left( t - \frac{T}{4} \right)}{T} \right] \right) + 1 \quad (1)$$

$$V_{in}(t) \xleftrightarrow{\mathcal{F}} \mathcal{V}_{in}(f) \quad (2)$$

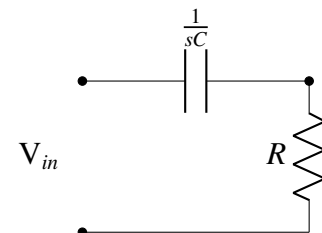
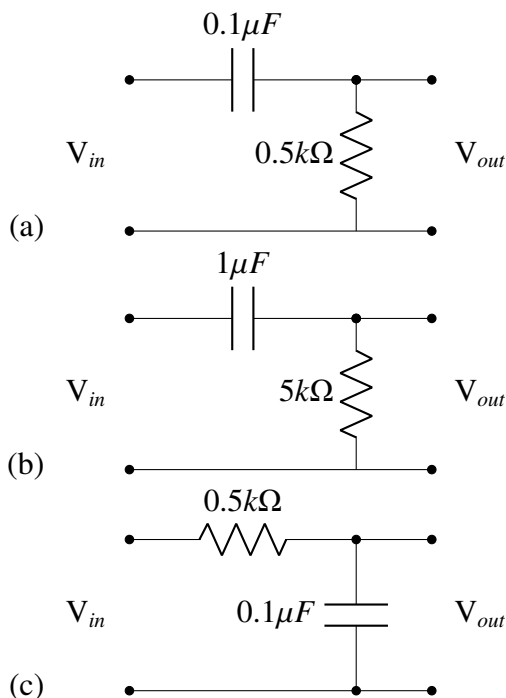


Fig. 2: Series RC Circuit in s-domain

$$s = j2\pi f \quad (3)$$

$$\Rightarrow Z = R + \frac{1}{sC} \quad (4)$$

$$= R + \frac{1}{j2\pi fC} \quad (5)$$

$$H(f) = \frac{V_{out}}{V_{in}} \quad (6)$$

Across C,

$$H_C(f) = \frac{1}{R + \frac{1}{j2\pi fC}} \quad (9)$$

$$\Rightarrow \mathcal{V}_{out}(f) = H_C(f)\mathcal{V}_{in}(f) \quad (10)$$

$$\mathcal{V}_{out}(f) \xleftrightarrow{\mathcal{F}} V_{out}(t) \quad (11)$$

$\mathcal{V}_{in}(f)$  was input into all four circuits and Inverse Fourier Transform was taken of the response. All responses are plotted below:

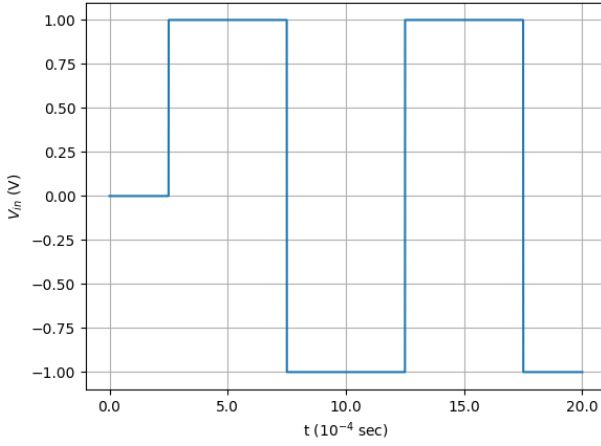


Fig. 3: Input Square Waveform ( $V_{in}(t)$ )

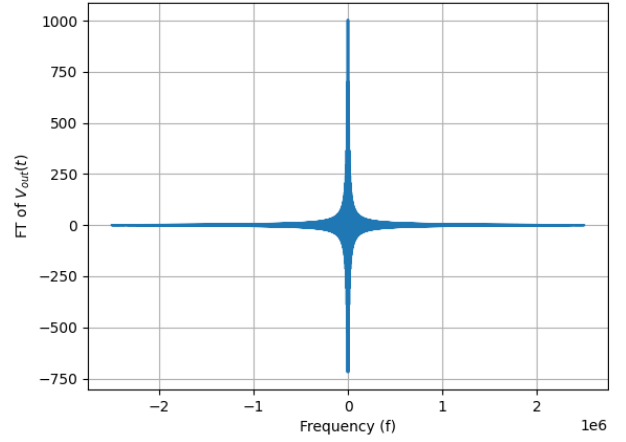


Fig. 5: Opt A: Fourier Transform of  $V_{out}(t)$

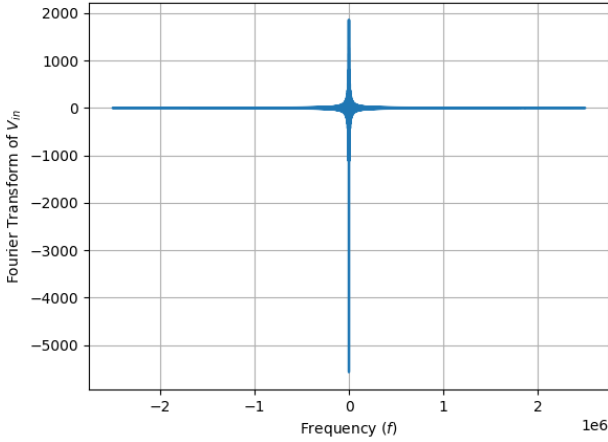


Fig. 4:  $\mathcal{V}_{in}(f)$  (Fourier Transform of  $V_{in}(t)$ )

Across R,

$$H_R(f) = \frac{R}{R + \frac{1}{j2\pi fC}} \quad (7)$$

$$\Rightarrow \mathcal{V}_{out}(f) = H_R(f)\mathcal{V}_{in}(f) \quad (8)$$

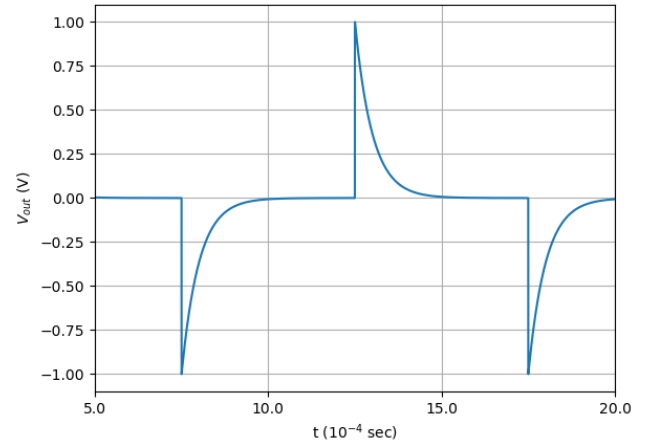


Fig. 6: Opt A: Response

As Fig. 6 resembles question Fig. 1, option (a) is the correct answer.

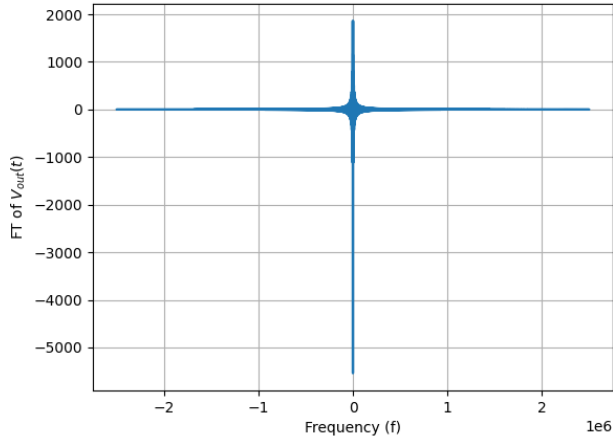


Fig. 7: Opt B: Fourier Transform of  $V_{out}(t)$

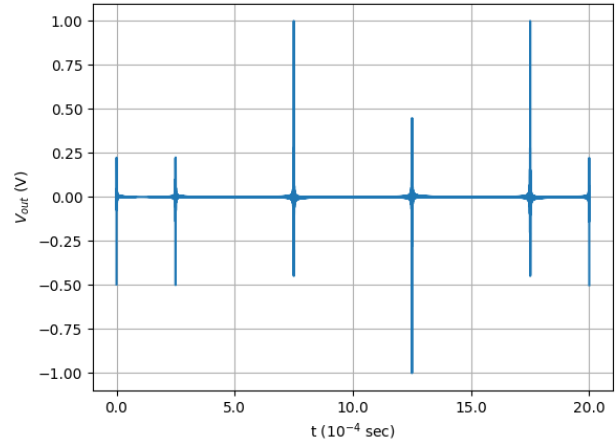


Fig. 10: Opt C: Response

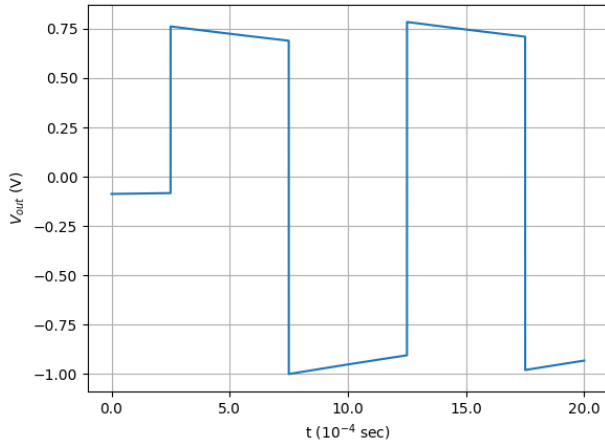


Fig. 8: Opt B: Response

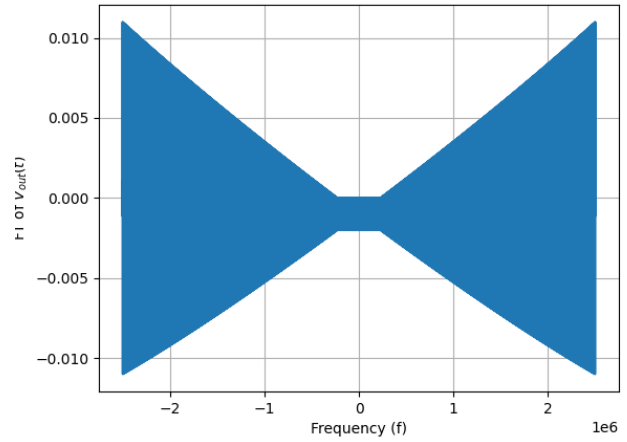


Fig. 11: Opt D: Fourier Transform of  $V_{out}(t)$

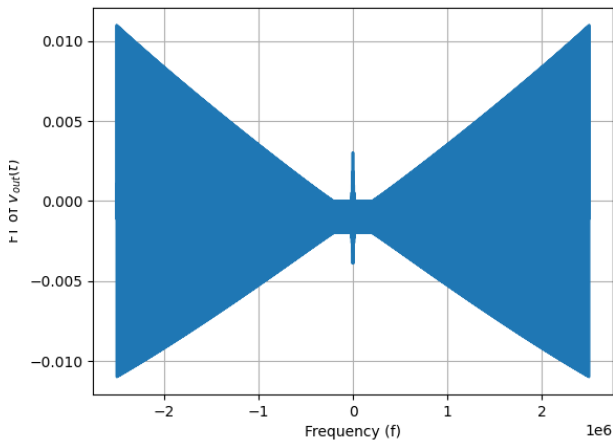


Fig. 9: Opt C: Fourier Transform of  $V_{out}(t)$

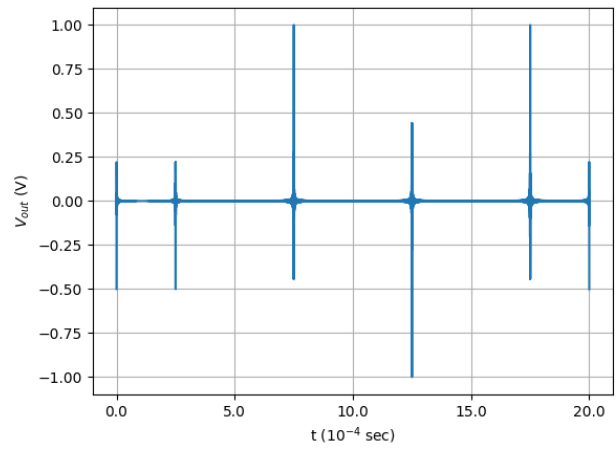


Fig. 12: Opt D: Response