NCERT Physics 12.7 Q21

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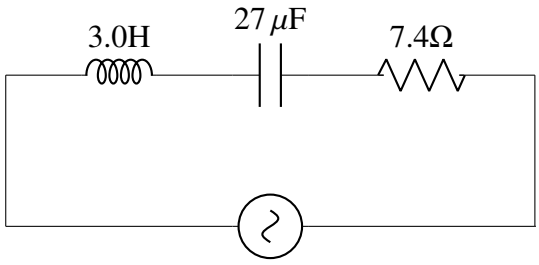
Question: Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Solution: Given parameters are:

L = 3.0 H
C = 27 μF
R = 7.4 Ω



Resonance Frequency (ω) is given by:

$$\omega = \frac{1}{\sqrt{LC}}$$

Substituting values of L and C gives:

$$\omega = \frac{1}{\sqrt{3 \cdot 27 \times 10^{-6}}} \tag{1}$$

$$= \frac{10^3}{9}\, s^{-1} \tag{2}$$

$$= 111.1\, s^{-1} \tag{3}$$

Quality Factor (Q) is given by:

$$\mathrm{Q} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$$

Substituting values of R, L and C gives:

$$\mathrm{Q} = \frac{1}{7.4} \cdot \sqrt{\frac{3}{27 \times 10^{-6}}} \tag{4}$$

$$= \frac{10^3}{22.2} \tag{5}$$

$$\approx 45 \tag{6}$$

To reduce the full width at half maximum by a factor of 2, the quality factor needs to be doubled. One way of doing this is to reduce the resistance by a factor of 2.

$$\mathrm{R'} = \frac{R}{2} \tag{7}$$

$$= \frac{7.4}{2}\Omega \tag{8}$$

$$= 3.7\Omega \tag{9}$$