GATE 2023 PH Q37

EE23BTECH11009 - AROSHISH PRADHAN*

Question: In the circuit shown below, the switch S is closed at $t = 0$. The magnitude of the steady state voltage, in volts, across the 6Ω resistor is _____.(*round off to two decimal places*)

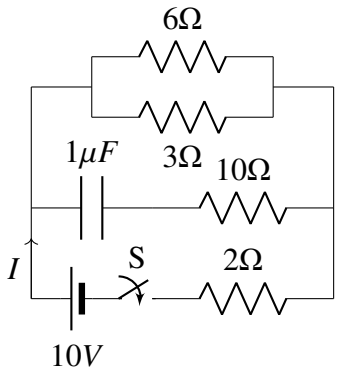

Fig. 1

Solution: Consider a sinusoidal input source of angular frequency ω.

Symbol	Value	Description
ω	0 for D.C.	Angular Frequency
C	$1\mu F$	Capacitance
$V_{in}(t)$	$10\cos(\omega t)$	Input Voltage
$V_{out}(t)$		Output Voltage across 6Ω
$V_{out}(j\omega)$	$H(j\omega)V_{in}(j\omega)$	Output in Frequency Domain
$H(j\omega)$		Transfer Function

TABLE I: Given Parameters

Using KCL and KVL, we can calculate:

$$I(j\omega) = \frac{V_{in}}{\left(\frac{2\left(10+\frac{1}{j\omega C}\right)}{12+\frac{1}{j\omega C}} + 2\right)} \tag{1}$$

$$\implies V_{out}(j\omega) = 2\left[\left(\frac{10+\frac{1}{j\omega C}}{12+\frac{1}{j\omega C}}\right) I(j\omega)\right] \tag{2}$$

$$= 2\left[\left(\frac{10+\frac{1}{j\omega C}}{12+\frac{1}{j\omega C}}\right) \frac{V_{in}(j\omega)}{\left(\frac{2\left(10+\frac{1}{j\omega C}\right)}{12+\frac{1}{j\omega C}} + 2\right)}\right] \tag{3}$$

$$\implies H(j\omega) = \frac{1 + 10\,j\omega C}{2(1 + 11\,j\omega C)} \tag{4}$$

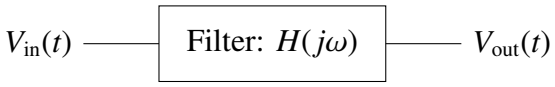

Fig. 2: Filter Equivalent of Circuit

$$H(j\omega) = \left(\frac{\sqrt{1+100\omega^2C^2}}{2\sqrt{1+121\omega^2C^2}}\right) e^{j(\tan^{-1}(10\omega C)-\tan^{-1}(11\omega C))} \tag{5}$$

$$= \left(\frac{\sqrt{1+100\omega^2C^2}}{2\sqrt{1+121\omega^2C^2}}\right) e^{j\tan^{-1}\left(\frac{-\omega C}{1+110\omega^2C^2}\right)} \tag{6}$$

$$\therefore V_{out}(t) = 10\,|H(j\omega)|\cos(\omega t + \angle H(j\omega)) \tag{7}$$

$$= \frac{5\sqrt{1+100\omega^2C^2}}{\sqrt{1+121\omega^2C^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega C}{1+110\omega^2C^2}\right)\right) \tag{8}$$

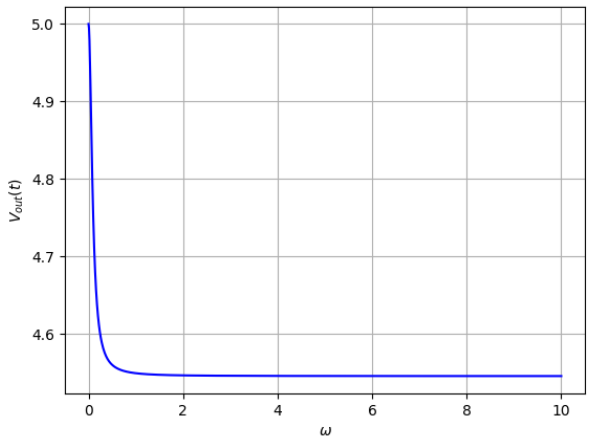

Fig. 3: Plot of $V_{out}(t)$ at $t = 0$ w.r.t ω

As $\omega \to 0$, $V_{in}(t)$ approaches being a D.C. input source $(10V)$.

$\therefore$ substituting $\omega = 0$, we get:

$$V_{out}(t) = 5V \tag{9}$$