

# NCERT Physics 12.7 Q21

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**Question:** Obtain the resonant frequency and Q-factor of a series LCR circuit with  $L = 3.0\text{ H}$ ,  $C = 27\text{ }\mu\text{F}$ , and  $R = 7.4\text{ }\Omega$ . It is desired to improve the sharpness of the resonance of the circuit by reducing its ‘full width at half maximum’ by a factor of 2. Suggest a suitable way.

**Solution:** Given parameters are:

Symbol	Value	Description
$L$	$3.0\text{ H}$	Inductance
$C$	$27\text{ }\mu\text{F}$	Capacitance
$R$	$7.4\text{ }\Omega$	Resistance
$Q$		Quality Factor: ratio of voltage across inductor or capacitor to that across the resistor at resonance
$\omega_0$	$\frac{1}{\sqrt{LC}}$	Angular Resonant Frequency

TABLE I: Given Parameters

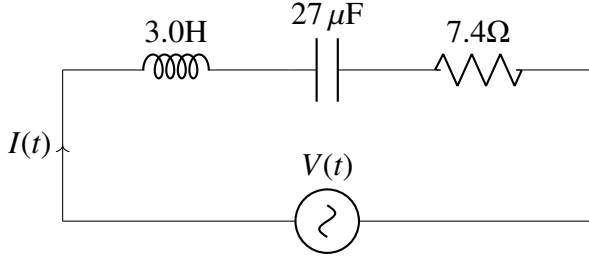


Fig. 1: LCR Circuit

## 1) Frequency Response of the Circuit

From Kirchhoff's Voltage Law (KVL):

$$V(t) = V_R + V_L + V_C \quad (1)$$

Using reactances from Fig. 2,

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \quad (2)$$

$$= I(s) \left( R + Ls + \frac{1}{sC} \right) \quad (3)$$

$$\Rightarrow I(s) = \frac{V(s)}{\left( R + Ls + \frac{1}{sC} \right)} \quad (4)$$

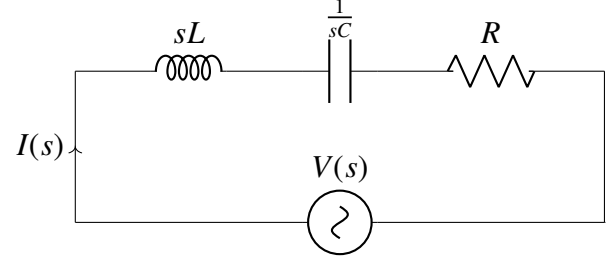


Fig. 2: LCR Circuit

At resonance, the circuit becomes purely resistive. The reactances of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \quad (5)$$

$$\Rightarrow s = j \frac{1}{\sqrt{LC}} \quad (6)$$

$s$  can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \quad (7)$$

Comparing (6) and (7), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (8)$$

## 2) Quality Factor

a) Using voltage across inductor,

$$Q = \left( \frac{V_L}{V_R} \right)_{\omega_0} = \frac{|sLI(s)|}{|RI(s)|} \quad (9)$$

$$= \frac{1}{\sqrt{LC}} \frac{L}{R} \quad (10)$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}} \quad (11)$$

b) Using voltage across capacitor,

$$Q = \left( \frac{V_C}{V_R} \right)_{\omega_0} = \frac{\left| \frac{I(s)}{sC} \right|}{|RI(s)|} \quad (12)$$

$$= \frac{\sqrt{LC}}{RC} \quad (13)$$

$$= \frac{1}{R} \sqrt{\frac{L}{C}} \quad (14)$$

### 3) Plot of Impedance vs Angular Frequency

Impedance is defined as

$$H(s) = \frac{V(s)}{I(s)} \quad (15)$$

Using (4),

$$H(s) = R + sL + \frac{1}{sC} \quad (16)$$

$$\Rightarrow H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \quad (17)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad (18)$$

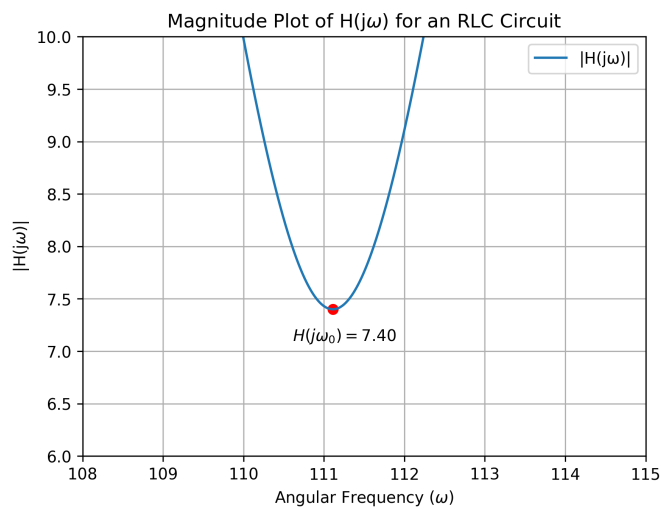


Fig. 3: Impedance vs  $\omega$  (using values in Table I)