

# NCERT Physics 12.7 Q21

EE23BTECH11009 - AROSHISH PRADHAN\*

**Question:** Obtain the resonant frequency and Q-factor of a series LCR circuit with  $L = 3.0\text{ H}$ ,  $C = 27\text{ }\mu\text{F}$ , and  $R = 7.4\text{ }\Omega$ . It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

**Solution:** Given parameters are:

Symbol	Value	Description
L	3.0 H	Inductance
C	27 $\mu\text{F}$	Capacitance
R	7.4 $\Omega$	Resistance
Q	$\frac{1}{R} \sqrt{\frac{L}{C}}$	Quality Factor
$\omega_0$	$\frac{1}{\sqrt{LC}}$	Angular Resonant Frequency

TABLE I: Given Parameters

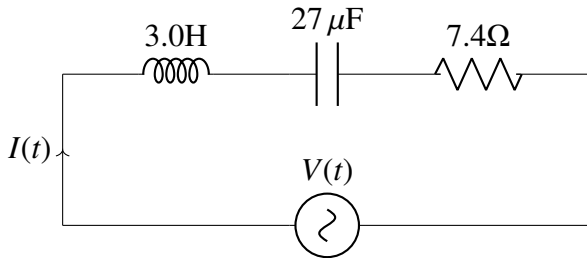


Fig. 1: LCR Circuit

## Frequency Response of the Circuit

Applying Kirchhoff's Voltage Law (KVL), we get:

$$V_R + V_L + V_C = V(t) \quad (1)$$

where  $V_R$ ,  $V_L$  and  $V_C$  are the voltages across R, L and C respectively and  $V(t)$  is the time-varying voltage source.

Elements and their corresponding reactances are given in the table below where  $s$  is a complex variable.

Symbol	Reactance	Description
R	$R$	Resistance
L	$sL$	Inductance
C	$\frac{1}{sC}$	Capacitance

TABLE II: Reactances

The circuit can now be redrawn as:

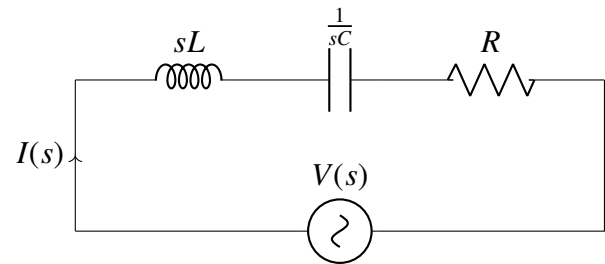


Fig. 2: LCR Circuit

Using the reactances of R, L and C from TABLE II in equation (1), we get

$$V(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) \quad (2)$$

$$\Rightarrow V(s) = I(s) \left( R + Ls + \frac{1}{sC} \right) \quad (3)$$

$$\Rightarrow I(s) = \frac{V(s)}{\left( R + Ls + \frac{1}{sC} \right)} \quad (4)$$

At resonance, the circuit becomes purely resistive. The reactances of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \quad (5)$$

$$\Rightarrow s = j \frac{1}{\sqrt{LC}} \quad (6)$$

$s$  can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \quad (7)$$

Comparing equations (6) and (7), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (8)$$

### Quality Factor

Quality Factor ( $Q$ ) of an LCR circuit is defined as the ratio of voltage across inductor or capacitor to that across the resistor at resonance.

$$Q = \left( \frac{V_L}{V_R} \right)_{\omega_0} = \frac{|sLI(s)|}{|RI(s)|} \quad (9)$$

$$\Rightarrow Q = \frac{1}{\sqrt{LC}} \frac{L}{R} \quad (10)$$

$$\Rightarrow Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (11)$$

### Plot of Impedance vs Angular Frequency

Impedance is defined as

$$H(s) = \frac{V(s)}{I(s)} \quad (12)$$

Using equation (4),

$$H(s) = R + sL + \frac{1}{sC} \quad (13)$$

$$\Rightarrow H(j\omega) = R + j\omega L + \frac{1}{j\omega C} \quad (14)$$

$$\Rightarrow |H(j\omega)| = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2} \quad (15)$$

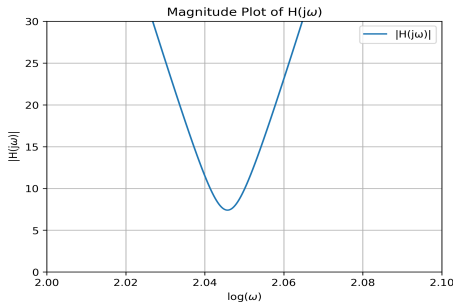


Fig. 3: Impedance vs  $\log \omega$