NCERT Math 11.9.2 Q8

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Question: If the sum of n terms of an AP is $(pn + qn^2)$, where p and q are constants, find the common difference.

Solution:

Symbol	Value	Description
$s(n)$	$(pn + qn^2)$	Sum of n terms
$x(n)$		n^{th} term of AP
d	$x(n+1) - x(n)$	Common Difference

TABLE I: Given Parameters

Sum of n terms, as a discrete signal:

$$s(n) = (pn + qn^2)u(n) \tag{1}$$

Taking the Z-Transform,

$$s(n) \overset{z}{\longleftrightarrow} S(z) \tag{2}$$

$$\implies S(z) = \sum_{n=-\infty}^{\infty} s(n)z^{-n} \tag{3}$$

$$= \sum_{n=-\infty}^{\infty} (pn + qn^2)u(n)z^{-n} \tag{4}$$

$$= p\sum_{n=-\infty}^{\infty} nu(n)z^{-n} + q\sum_{n=-\infty}^{\infty} n^2u(n)z^{-n} \tag{5}$$

$$= p\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) + q\left(\frac{z^{-1}(1+z^{-1}).}{(1-z^{-1})^3}\right) \tag{6}$$

where $\{z \in \mathbb{C} : |z| > 1\}$
Now,

$$s(n) = x(n) * u(n) \tag{7}$$

$$\implies S(z) = X(z)U(z) \tag{8}$$

$$\implies X(z) = \frac{S(z)}{U(z)} \tag{9}$$

where,

$$U(z) = \mathcal{Z}(u(n)) \tag{10}$$

$$= \frac{1}{1-z^{-1}} \tag{11}$$

for $\{z \in \mathbb{C} : |z| > 1\}$

Using (11) in (9),

$$X(z) = p\left(\frac{z^{-1}}{(1-z^{-1})}\right) + q\left(\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^2}\right) \tag{12}$$

Simplifying using partial fractions, we get:

$$X(z) = (q-p) + \frac{p-3q}{1-z^{-1}} + \frac{2q}{(1-z^{-1})^2} \tag{13}$$

$$= (q-p) + \frac{(p-q)}{1-z^{-1}} + \frac{2qz^{-1}}{(1-z^{-1})^2} \tag{14}$$

Taking the inverse Z-Transform,

$$x(n) = (q-p)\delta(n) + (p-q)u(n) + 2qnu(n) \tag{15}$$

To simplify, use $n = 0$:

$$s(0) = x(0) = 0 \tag{16}$$

$$\implies (q-p)\delta(0) + (p-q)u(0) + 2qnu(0) = 0 \tag{17}$$

$$\implies p = q \tag{18}$$

because $\delta(0) \to \infty$
$\therefore$ rewriting (15):

$$x(n) = 2qnu(n) \tag{19}$$

Common difference is given by:

$$d = x(n+1) - x(n) \tag{20}$$

$$= 2q(n+1)u(n+1) - 2qnu(n) \tag{21}$$

$$= 2q \tag{22}$$

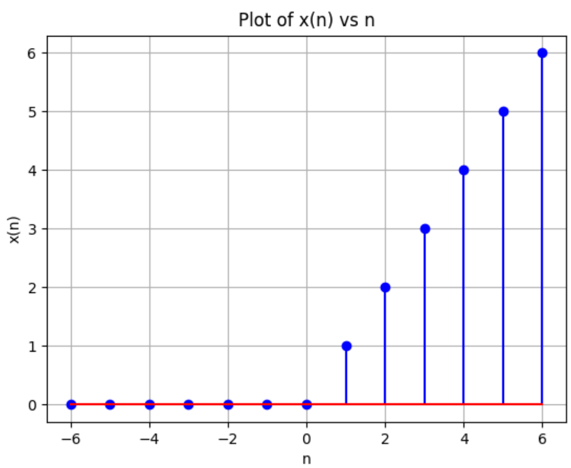

Fig. 1: Plot of x(n) vs n