GATE 2021 NM Q24

EE23BTECH11009 - AROSHISH PRADHAN*

Question: The sum of the infinite geometric series

$$1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+...$$

(rounded off to one decimal place) is _____. (GATE 2021 BT Q20)

Solution:

Symbol	Value	Description
$x(n)$		$(n+1)^{th}$ term of series
$x(0)$	1	1^{st} term of series
r	$\frac{1}{3}$	Common ratio
$y(n)$		Sum of $(n+1)$ terms

TABLE I: Given Parameters

General term:

$$x(n) = x(0)r^n u(n) \tag{1}$$

$$\implies X(z) = \frac{1}{1-rz^{-1}} \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$\implies Y(z) = X(z)U(z) \tag{4}$$

$$= \frac{1}{(1-rz^{-1})(1-z^{-1})} \tag{5}$$

$$= \frac{1}{r-1}\left(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}}\right) \tag{6}$$

$$\tag{7}$$

Taking inverse Z-transform:

$$y(n) = \frac{1}{r-1}\left(r(r^n u(n)) - u(n)\right) \tag{8}$$

$$= \left(\frac{r^{n+1}-1}{r-1}\right)u(n) \tag{9}$$

$$= \left(\frac{1-r^{n+1}}{1-r}\right)u(n) \tag{10}$$

For infinite terms:

$$y(\infty) = \lim_{n\to\infty}\left(\frac{1-r^{n+1}}{1-r}\right)u(n) \tag{11}$$

$$= \frac{1}{1-r} \tag{12}$$

$$= \frac{3}{2} \tag{13}$$

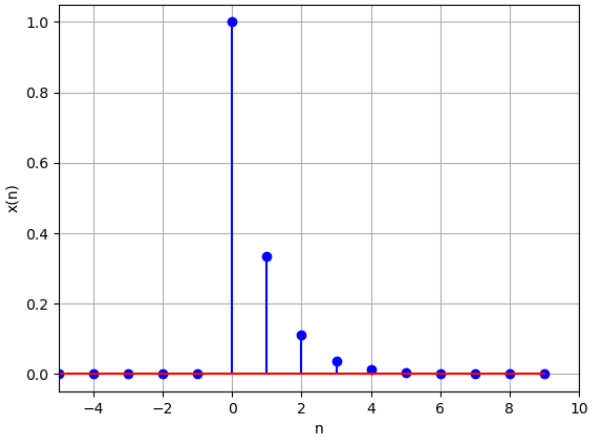

Fig. 1: Plot of $x(n)$ vs n

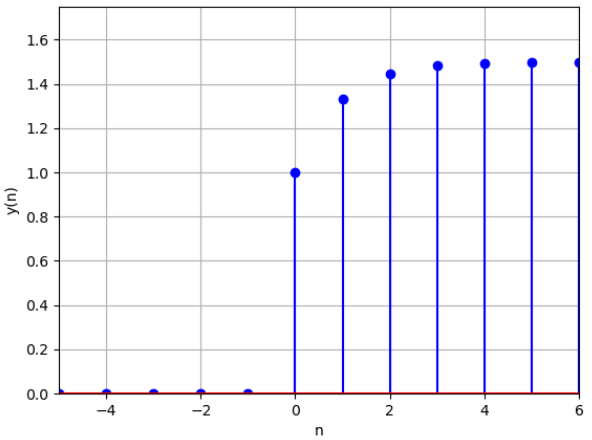

Fig. 2: Plot of $y(n)$ vs n