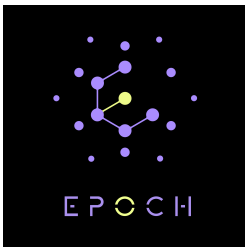

Matrix Theory Revisit
Epoch 1: Image Compression

31st July 2024

Background

Singular Value Decomposition (SVD) is a powerful mathematical tool used in various applications like dimensionality reduction, image compression, and collaborative filtering in recommendation systems. Understanding SVD can provide deep insights into the structure of data and the underlying patterns.

Objective

The goal of this epoch is to implement SVD from scratch and apply it to a problem to solidify your understanding of the concept.

Problem Statement

Imagine you are part of a team that is responsible for enhancing the quality of images in a large digital museum. The museum's digital collection contains thousands of images of artworks, many of which are old and of low resolution. Your task is to implement SVD and use it for image compression and enhancement, making these digital artworks look more appealing and preserving their details.

Tasks

1. **Understanding SVD:**
   - Research and understand the mathematical formulation of SVD.
   - Write a concise summary explaining the theory behind SVD, including the decomposition of a matrix into three matrices: U, Σ, and V^T.
2. **Implementation:**
   - Implement SVD from scratch in Python without using any libraries that directly perform SVD (e.g., NumPy's `numpy.linalg.svd`).
   - Verify your implementation by decomposing a given matrix and reconstructing it using the obtained U, Σ, and V^T matrices.
3. **Image Compression:**
   - Apply your SVD implementation to compress images.
   - Select a few high-resolution images of artworks (you can use public domain images for this).
   - Compress the images by retaining only the top k singular values and vectors, where k is significantly smaller than the original dimensions.
4. **Quality Enhancement:**
   - Compare the quality of the original images with the compressed ones.
   - Experiment with different values of k and observe how the image quality changes.
   - Write a brief report on your findings, discussing the trade-offs between compression and image quality.
5. **Creative Application:**
   - Extend your SVD implementation to create an interactive tool where users can upload an image, choose the level of compression, and see the results in real-time.
   - Optionally, incorporate a feature that allows users to enhance the image by adjusting the singular values manually.

Deliverables

1. **Code:** A well-documented Jupyter notebook or Python script containing your SVD implementation, image compression, and quality enhancement tasks.
2. **Report:** A report summarizing your understanding of SVD, the results of your experiments with image compression, and the performance of your interactive tool.
3. **Interactive Tool:** If implemented, provide a link to your interactive tool or instructions on how to run it locally.

Checkpoint Questions

1. Explain the role of the matrices U, Σ, and V^T in SVD.
2. What are the advantages and limitations of using SVD for image compression?
3. How does the choice of k singular values affect the compressed image quality?
4. Describe a real-world scenario, other than image compression, where SVD can be applied effectively.