Design Project 3

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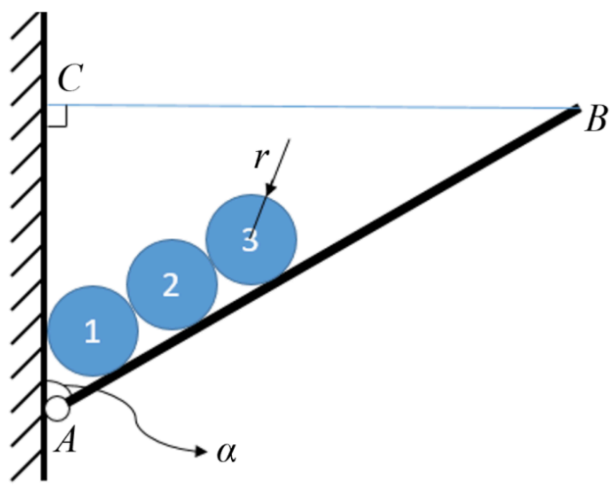
ABSTRACT

The objective of this project is to investigate the mechanics of a beam-cable support system with a cable perpendicular to the wall when additional weight is applied to the system. This is done through the use of MATLAB, in which the results are calculated and plotted in a 3D graph for better visualization.

INTRODUCTION

A beam-cable support system is introduced with the beam AB being fixed to a wall at point A. The cable BC is connected to the wall by the point C, which is moveable. The length of the beam, L_0, is equal to 1 meter. The balls labled 1, 2 and 3 are at rest on the beam AB. Each of the balls has a mass of 1kg and a radius of 0.1 meters. Additionally, the angle between the beam AB and the wall is represented by the angle α. There is no frictional forces acting on the system and the weights of the beam and cable are negligible. The following diagram depicts the support system:



RESULTS

Question 1

The question asks us to assume that the cable BC remains perpendicular to the wall. We are asked to find the optimal angle α_{opt} such that the tension on BC has the minimum magnitude. To solve this question, we must relate the downward gravitational force created by the balls and the tension force created by the beam-cable support system. To do so, we can create a moment around point A.

The moment around point A is composed of the normal forces on the balls as well as the tension force acting on the cable BC. To find the moment, we first must consider the distance between each of the balls. We can do this by

Then, we must find the normal force on the balls. To do so, we must look at each ball individually, as each ball has different forces acting on it. The easiest to resolve is the upper most ball. Ball 3 has two forces acting on it: the normal force from the cable and the normal force from the contact with ball 2. These forces for ball 3 can be expressed as:

$N_3 = (mg) * \sin(\alpha) \quad F_3 = (mg) * \cos(\alpha)$

From these expressions, the forces acting on ball 2 and ball 1 can be derived. Ball 2 has the same normal force acting on it:

$N_2 = (mg) * \sin(\alpha)$

In addition, ball 2 makes contact with ball 1 creating a normal force with that interaction as well. This normal force will be twice as a large as the force on ball 3, as it carries the downward force from both ball 3 and ball 2. This normal force can be expressed as

$F_2 = 2 * (mg) * \cos(\alpha)$

Then to solve for the forces acting on ball 1, the forces from ball 3 and ball 2 must be taken into account as well as the normal force acting on ball 1. Additionally, there is a contact force that ball 1 makes with the wall. Breaking these forces down into their component form such that they are parallel to and normal to the beam AB results in the following two expressions:

$N_1 = (mg)\sin(\alpha) + F_{wall}\cos(\alpha)$

$3(mg)\cos(\alpha) = F_{wall} * \sin(\alpha)$

By rearranging and combining these equations, we can see that:

$N_1 = (mg)\sin(\alpha) + F_{wall}\left(\cos^2(\alpha)/\sin(\alpha)\right)$

Next, we must determine the straight-line distances between each of the forces and point A, in order to set up a moment. Each of the distances can be expressed in the following way:

$$D_1 = \frac{0.1m}{\sin(\alpha)} + 0.1(\cot(\alpha))$$

$$D_2 = \frac{0.1m}{\sin(\alpha)} + 0.2 + 0.1(\cot(\alpha))$$

$$D_3 = \frac{0.1m}{\sin(\alpha)} + 0.4 + 0.1(\cot(\alpha))$$

With the above distances and the forces, we are able to set up an equation for the moment about the point A with respect to the diagram. Taking into account the counterclockwise motion from the tension on BC, and the clockwise motion from the normal and gravitational components found above, we can derive the following equation:

$$\begin{aligned} T_{BC}\cos(\alpha) = (mg)(\sin(\alpha))\left(\frac{0.1m}{\tan\left(\frac{\alpha}{2}\right)} + 0.4 + 0.1(\cot(\alpha))\right) \\ + (mg)(\sin(\alpha))\left(\frac{0.1m}{\tan\left(\frac{\alpha}{2}\right)} + 0.2 + 0.1(\cot(\alpha))\right) + ((mg)\sin(\alpha) \\ + F_{wall}(\cos^2(\alpha)/\sin(\alpha))\left(\frac{0.1m}{\tan\left(\frac{\alpha}{2}\right)} + 0.1(\cot(\alpha))\right)) \end{aligned}$$

Plugging this equation in to MATLAB, we can accurately visualize and draw conclusions about the system with regards to the optimal angle α_{opt}

Question 2

The question asks us to assume that the cable BC is adjustable, and that the angle between the cable and the wall β changes such that $20° \leq \beta \leq 90°$. We are also asked to assume the same range of angles for α as in question 1. We are asked to find how the angles α and β affect the tension on cable BC.

To do this, we can follow the same procedure as in question 1, however, we instead express the vertical and horizontal components of the tension on BC to obtain an equilibrium expression for the moment. We do this by expressing these components in terms of the angles alpha and beta. Using the same distances and normal forces found in Question 1, we can set up the moment in terms of both alpha and beta. The moment can be expressed in the following manor:

$$T_{BC}(\sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)) = (mg)\left(\frac{0.3}{\tan\left(\frac{\alpha}{2}\right)} + 0.6\right) + 3(mg)(\cot(\alpha))\left(\frac{0.1}{(\tan\left(\frac{\alpha}{2}\right))}\right)$$

The equation can subsequently be rearranged and plotted in MATLAB to see the relationship between alpha, beta, and the tension on the cable.

ANALYSIS AND DISCUSSION

Question 1

The question asks us to make a two-dimensional plot of the tension as a function of the angle α within the ranges of $20° \leq \alpha \leq 70°$, as found in Question 1. The plot of the function found in the results can be seen below:

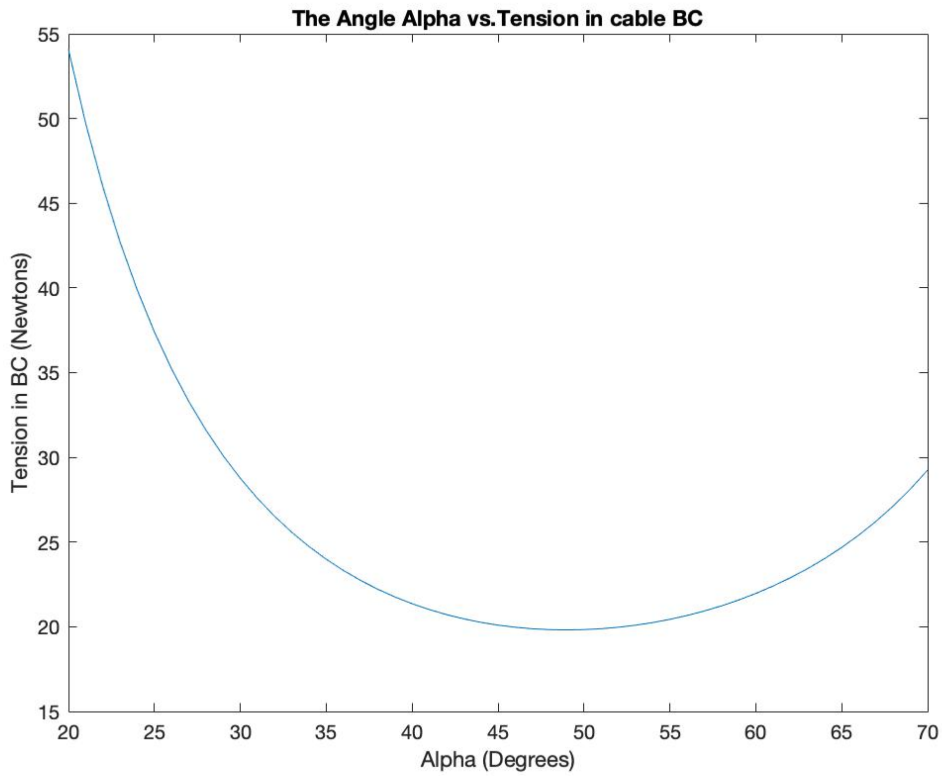


Figure 1: Plot of Tension of Cable BC versus Angle Alpha

As can be seen in *figure 1* (as well as calculated precisely by the MATLAB code in the appendix), the smallest tension value occurs at 49.0°. The magnitude of tension at this point is 19.8N.

Question 2

We are asked to create a 3-Dimensional plot of the dependence relationship between beta, alpha, and the tension on BC found in Question 2. The plot of the function can be shown below:

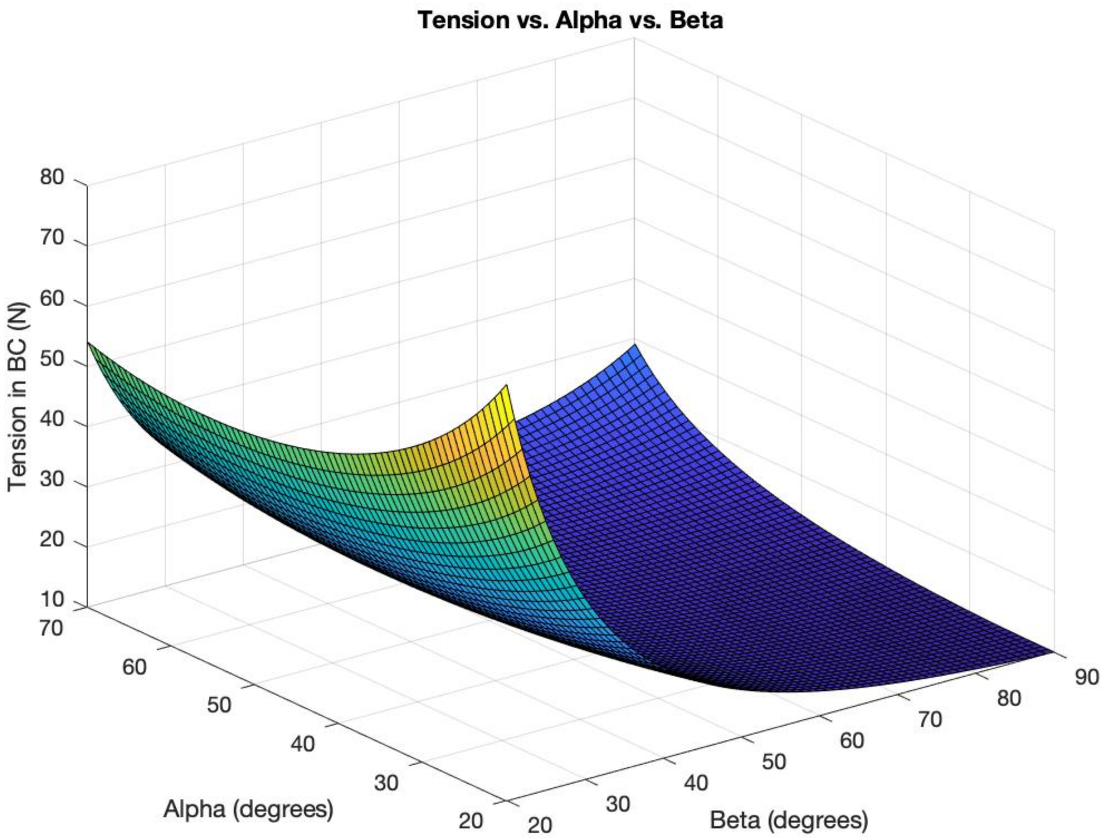


Figure 2: Plot of Tension of Cable BC versus Angle Alpha versus Angle Beta

From this graph we can see the tension is maximized when alpha and beta both have very low angles, and the tension is minimized when beta is maximized, and alpha is minimized. Thus, to minimize the tensional force acting on BC, the most ideal strategy is to minimize alpha and maximize beta.

CONCLUSION

This project highlights the relevance of moment equilibrium expressions in mechanical systems, and how engineers can use them to break down complex systems into different force components. Specific attention was brought to the mechanical interactions between separate objects and how they might determine the optimal specifications of a system.

APPENDIX

MATLAB Code

```
%%%%Problem 1

%Initialize variables
gravity = 9.81;
m = 1;
Lo = 1;
rad = 0.1 * Lo;
Tbc = zeros(1,51);
set = 20:70;

for i = 1:length(Tbc)
    % Distance components
    x1 = rad/tand(set(i)/2);
    x2 = rad/tand(set(i)/2) + 2*rad;
    x3 = rad/tand(set(i)/2) + 4*rad;
    % Normal force vector components
    v1 = (3*m*gravity*cosd(set(i))^2)/sind(set(i)) + m*gravity*sind(set(i));
    v2 = m*gravity*sind(set(i));
    v3 = v2;
    % Resulting Tension
    % Note: The sign (+/-) of the tension should be inverted
    Tbc(i) = (x1*v1 + x2*v2 + x3*v3)/(Lo*sind(90-set(i)));
end

%Find Min
minimum = min(Tbc);
ind = find(Tbc == minimum,1);
fprintf('The smallest tension is %.1fN at %.1f degrees.\n', minimum,
set(ind));

%Plot
figure;
plot(set,Tbc);
title('The Angle Alpha vs.Tension in cable BC');
xlabel('Alpha (Degrees)');
ylabel('Tension in BC (Newtons)');


%%%%Problem 2

%Initialize Variables
i = 0;
j = 0;
grav = 9.81;
Lo = 1;
m = 1;
rad = 0.1 * Lo;
Tbc = zeros(71,51);

for B = 20:90
    i = i + 1;
    j = 0;
    for a = 20:0.71321341242132321321312:70
```

```
        j = j + 1;
        % Distance from A
        x1 = rad/tand(a/2);
        x2 = rad/tand(a/2) + 2*rad;
        x3 = rad/tand(a/2) + 4*rad;
        % Normal force
        N1 = (3*m*grav*cosd(a)^2)/sind(a) + m*grav*sind(a);
        N2 = m*grav*sind(a);
        N3 = N2;
        % Tension
        Tbc(i, j) = (x1*N1 + x2*N2 + x3*N3)/(Lo*sind(180-B-a));
    end
end

%Plot
B = 20:90;
a = 20:0.7123124123214213211:70;
figure;
surf(B,a,Tbc);
title('Tension vs. Alpha vs. Beta');
xlabel('Beta (degrees)');
ylabel('Alpha (degrees)');
zlabel('Tension in BC (N)');
```