Design Project 4

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Due Date: February 16, 2019

ABSTRACT

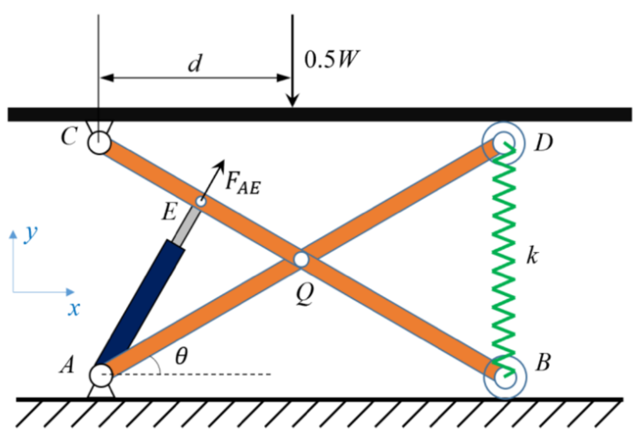
The objective of this project is to investigate the mechanics of a scissor hydraulic lift system used to lift up cars when different amounts of weights are applied to points in the system. This is done through the use of MATLAB, in which the results are calculated and plotted in a graph for better visualization.

INTRODUCTION

The hydraulic lift system below consists of a horizontal rigid plate supported by two crossing linkages on which hydraulic cylinders exert equal force (only one linkage and one cylinder on one side are shown). The weight of the car, W, is evenly distributed at each of the four wheels. The horizontal distance between the center of gravity and point C is represented by length d. The weight of the hydraulic lift itself can be neglected. The 3D model of the system can be seen in the diagram labeled A below.

In the system, there are the members AD and BC, which each have a length L, the point E which is the midpoint of CQ, and the point A on which the hydraulic cylinder is pinned. On BD, there is a linear elastic spring with spring constant of k = 0.2W(N-m) which has an uncompressed length of L. The diagram labeled B provides an in-detailed depiction the system:

(A)



(B)

Additionally, we are instructed to assume that θ = 10° when the lift is at its lowest position and θ = 70° when the lift is at its highest position (10° < θ < 70°).

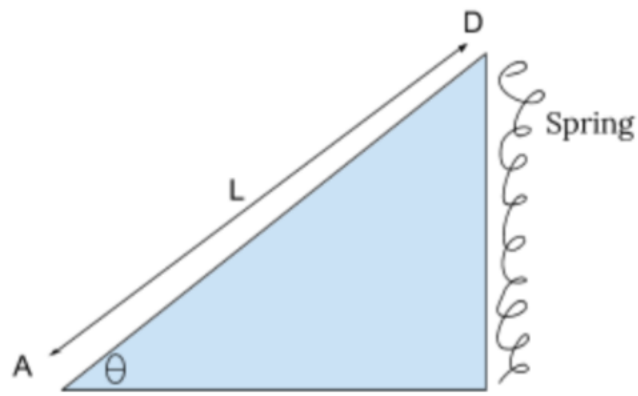
RESULTS

Question 1

The question asks us to show that the normalized force exerted by the cylinder, F_{AE}/W can be expressed as a function of θ and L in the following form:

$$\frac{F_{AE}}{W} = (\frac{1}{4} - \frac{L}{10}(1 - \sin(\theta))\frac{\sqrt{1 + 8\sin^2\theta}}{\sin(\theta)}$$

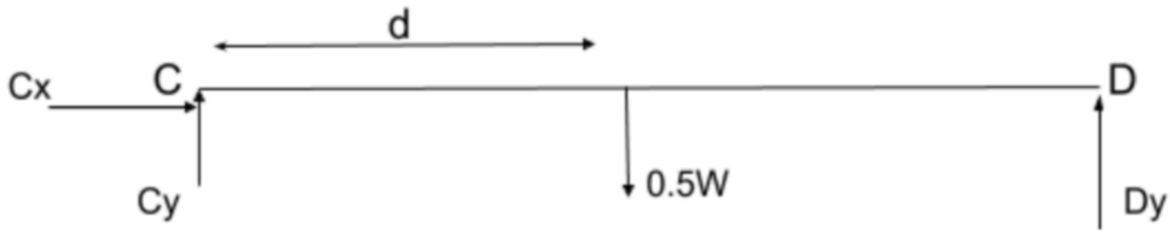
To do so, we must break the system down into components and evaluate the forces. First, we can resolve the forces in the spring on the line BD. This can be done by drawing the following diagram of the forces acting within the spring system:



From this diagram, we can see that the length of the spring can be represented by the trigonometric ratio in the diagram: Lcos(θ). Therefore, the equation for the force of the spring can be represented in the following manor:

$$F_s = -k * x = -0.2(Lcos(\theta) - L)$$

Next, we can resolve the forces acting on the upper horizontal bar that directly supports the weight of the car. This can be done by drawing the following free body diagram:



From the free body diagram, we can use the equilibrium equations and the moment equation about point C to solve for the forces in the Y direction. Understanding that each of the equilibrium equations and the moment equation equal zero, we can state the following:

$$\sum \boldsymbol{F_X} = 0 = \boldsymbol{C_x}$$

$$\sum \boldsymbol{F_Y} = 0 = -0.5W + \boldsymbol{D_Y} + \boldsymbol{C_Y}$$

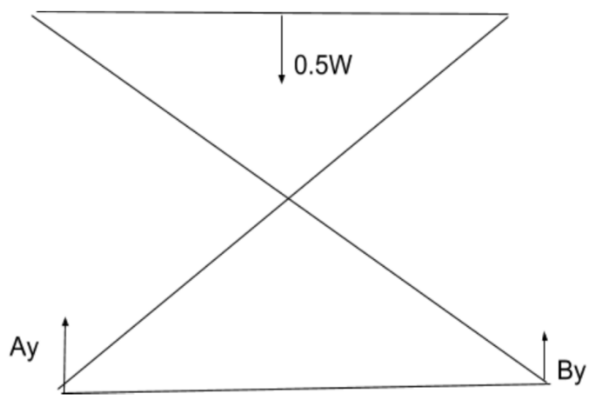
$$\sum \boldsymbol{M_C} = 0 = Lcos(\theta)\boldsymbol{D_Y} + 0.5Wd$$

Ignoring the x-components which equal zero, we can use the equations to first solve for the y-component forces on D and then on C. The results are the following:

$$\boldsymbol{D_Y} = \frac{0.5Wd}{Lcos(\theta)}$$

$$\boldsymbol{C_Y} = 0.5W(1 - \frac{d}{Lcos(\theta)})$$

Then we can resolve all the forces that are external to the system. These are the forces from the weight and the normal forces that counteract the weight at points A and B. They can be seen in the free body diagram below:



From this diagram, we can establish a new set of equilibrium equations and moment equation to solve for the forces. Because there are no forces in the x-direction, we can simply ignore it for this calculation.

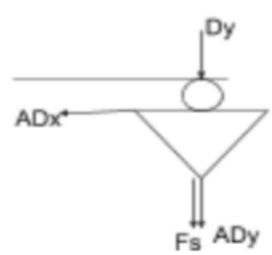
$$\sum \boldsymbol{F_Y} = 0 = -0.5W + \boldsymbol{A_Y} + \boldsymbol{B_Y}$$

$$\sum \boldsymbol{M_A} = 0 = \boldsymbol{B_Y}(Lcos(\theta) - 0.5Wd)$$

Solving for the forces, we can see that

$$\boldsymbol{B_Y} = \frac{0.5Wd}{Lcos(\theta)}$$

Next, we can resolve the forces on the roller support at point D. The following free body diagram can be used to show the forces acting on the roller:



From this diagram, we can establish yet another a new set of equilibrium equations to solve for the forces. No moment calculation is needed to solve these forces.

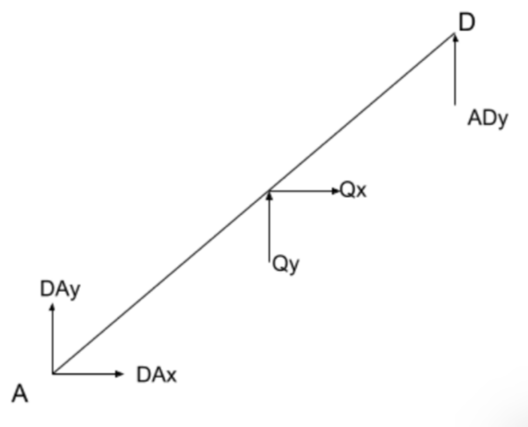
$$\sum \boldsymbol{F_X} = 0 = A * \boldsymbol{D_x}$$

$$\sum \boldsymbol{F_Y} = 0 = -\boldsymbol{F_s} - A * \boldsymbol{D_Y} - \boldsymbol{D_Y}$$

Solving this system of equations, and cross-referencing with the result for the force of the spring, we can find that:

$$A * \boldsymbol{D_Y} = \frac{-0.5Wd}{Lcos(\theta)} - KL(1 - \sin(\theta))$$

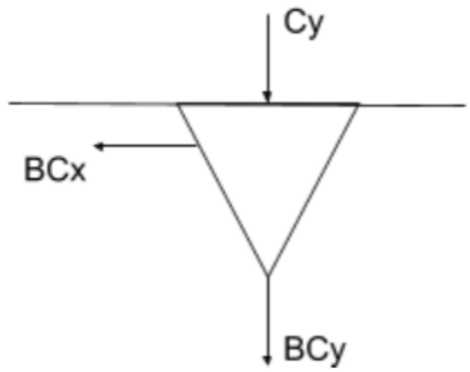
Next, we can resolve the forces on the bar from point A to point D. The following free body diagram can be used to show the forces acting on the bar:



From this free body diagram, we can see that Q_x and Q_y are the reactions occurring on the part at the midpoint Q. Knowing that the system is at equilibrium, we can say that Q_x and $D*A_x$ are equal and opposite in magnitude, thus resolving the x-components. Then, solving for the y-components we can see from our previous findings in other free body diagrams that:

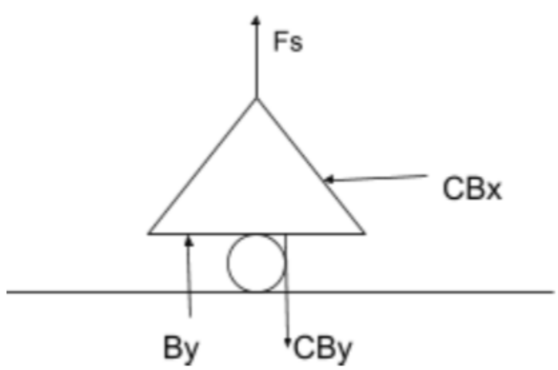
$$D * \boldsymbol{A_Y} = \frac{-0.5Wd}{Lcos(\theta)} - KL(1 - \sin(\theta) - Q_y)$$

From there, we can resolve the forces in bar from point B to point C. First, we should resolve the pin support system at C with the following free body diagram:



Resolving the forces on the pin at point C through the forces in the equilibrium expressions, we can find that $BC_x = 0$ and that $BC_y = -C_y$

Then we can resolve the roller system at point B with the following free body diagram:

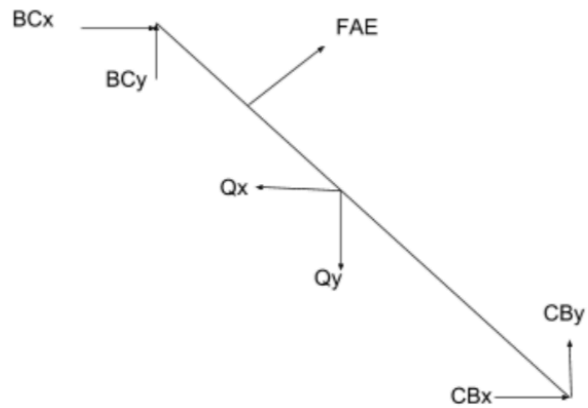


Resolve the forces on the roller support, we can see that $CB_x = 0$ because the system is at equilibrium and it is the only force in the x direction. Solving for CB_y, we can see that:

$$\sum \boldsymbol{F_Y} = 0 = \boldsymbol{F_s} - \boldsymbol{B_Y} - C\boldsymbol{B_Y}$$

$$C * \boldsymbol{B_Y} = 0.2W(Lcos(\theta) - L) + \frac{0.5Wd}{Lcos(\theta)}$$

Hence, we can draw the overall free body diagram of the bar from point B to point C as the following. All of the forces in the system have already been resolved, and thus, no expressions need to be derived from this diagram:



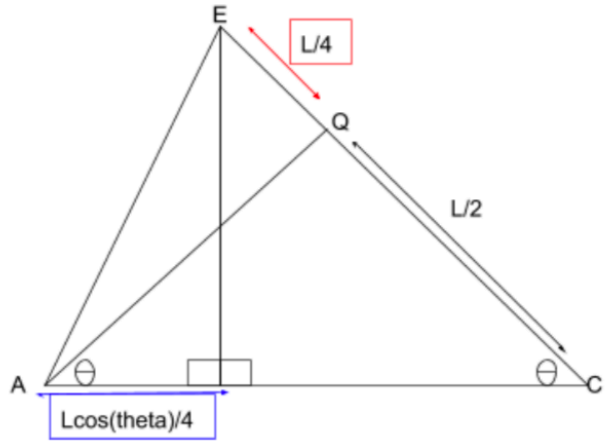
With all the free body diagrams solved, the only unknown values in our system that remain are the components of Q and the force F_{AE}. To solve for these quantities, we should set up a moment about point Q that includes all of the forces expressed previously. The moment can be expressed and reduced in the following manor:

$$\sum \boldsymbol{M_Q} = 0 = C * \boldsymbol{B_Y}(distance) - B * \boldsymbol{C_Y}(distance) - \boldsymbol{F_{AE}}(distance)$$

$$\sum \boldsymbol{M_Q} = 0 = \frac{Lcos(\theta)}{2}(\frac{-0.5Wd}{Lcos(\theta)} - KL(1-\sin(\theta)) - \left(0.5W\left(1-\frac{d}{Lcos(\theta)}\right)\right) - \boldsymbol{F_{AE}}(distance))$$

$$\sum \boldsymbol{M_Q} = 0 = \frac{-L^2cos(\theta)k}{2}(1-\sin(\theta)) + \frac{Lcos(\theta)W}{4} - (\boldsymbol{QE} \times \boldsymbol{F_{AE}}))$$

To solve these equations, we must make use of the fact that the points CE, AE and AC form a triangle, with several other triangles within it. These triangles can be seen in the following diagram:



From the diagram, we can express the lengths of the horizontal and vertical components of the vectors from A to E and Q to E. Thus, we can say:

$$\boldsymbol{AE} = \frac{3Lcos(\theta)}{4}\underset{i}{\rightarrow} + \frac{3Lcos(\theta)}{4}\underset{j}{\rightarrow}$$

$$\boldsymbol{QE} = -\frac{Lcos(\theta)}{4}\underset{i}{\rightarrow} + \frac{Lcos(\theta)}{4}\underset{j}{\rightarrow}$$

To find the force $F_{AE,}$ we must calculate the magnitude of the vector AE and multiply it by the unit vector in that direction, which can be found by dividing the vector AE above by the magnitude. Executing that procedure will result in getting the following expression for the force F_{AE}

$$\boldsymbol{F_{AE}} = \frac{Fcos(\theta)}{\sqrt{1+8\sin^2(\theta)}}\underset{i}{\rightarrow} + \frac{3Fcos(\theta)}{\sqrt{1+8\sin^2(\theta)}}\underset{j}{\rightarrow}$$

Using this vector, we can now calculate the cross product $(\boldsymbol{QE} \times \boldsymbol{F_{AE}})$ to obtain the following expression:

$$\boldsymbol{F_{AE}} \times \mathbf{QE} = 0\underset{i}{\rightarrow} + 0\underset{j}{\rightarrow} + -\frac{LF_{AE}cos\theta sin\theta}{\sqrt{1+8\sin^2(\theta)}}\underset{k}{\rightarrow}$$

Thus, the moment equation can be rewritten as:

$$\sum \boldsymbol{M_Q} = 0 = \frac{-L^2cos(\theta)k}{2}(1-\sin(\theta)) + \frac{Lcos(\theta)W}{4} - \frac{LF_{AE}cos\theta sin\theta}{\sqrt{1+8\sin^2(\theta)}})$$

From this equation, we are able to solve for the normalized force exerted by the cylinder, F_{AE}/W as a function of θ and L. We can do so by performing simple algebra to reduce the equation and rearrange it into the equation we want. The procedure for doing so can be seen below:

$$\frac{-L^2 cos(\theta)k}{2}(1-\sin(\theta))+\frac{Lcos(\theta)W}{4}=\frac{LF_{AE}cos\theta sin\theta}{\sqrt{1+8\sin^2(\theta)}})$$

$$\frac{LW}{10}(1-\sin(\theta))+\frac{W}{4}=\frac{F_{AE}sin\theta}{\sqrt{1+8\sin^2(\theta)}})$$

Thus, we have:

$$\frac{F_{AE}}{W}=(\frac{1}{4}-\frac{L}{10}(1-\sin(\theta))\frac{\sqrt{1+8\sin^2\theta}}{\sin(\theta)}$$

ANALYSIS AND DISCUSSION

Question 2

Question 2 asks us to plot a contour plot of the magnitude of the normalized force with respect to θ and L. Using the equation found in Question 1, and MATLAB, we can generate the following plot:

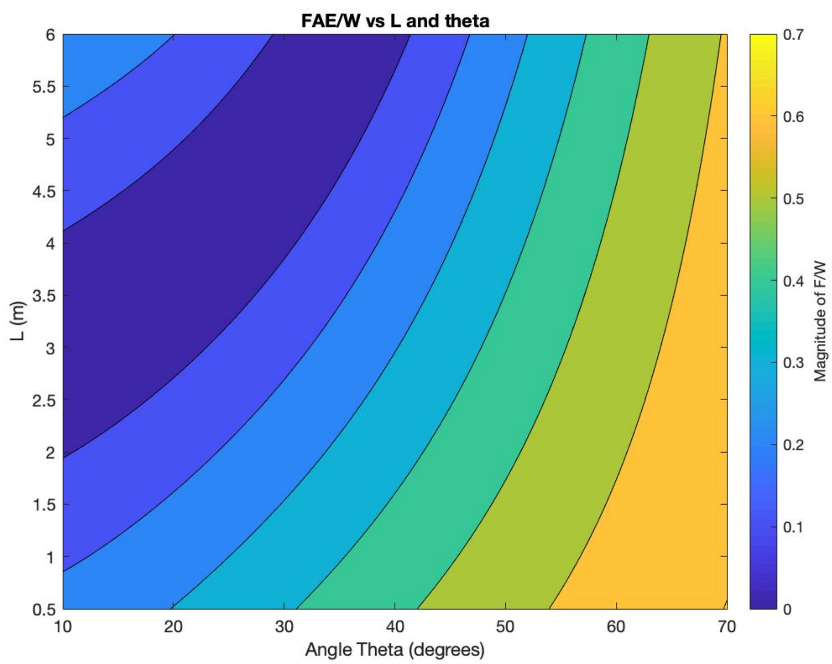


Figure 1: Plot of F_{AE} versus the Length L and Angle Theta

From this plot, we are able to see the range of magnitudes produced by the components L and theta, and assess where the optimal range of values are for minimizing the magnitude of the F_{AE}

Question 3

The question asks us to redraw the plot from *Figure 1* to show what values of theta and L satisfy the condition that the maximum magnitude of W be less than 0.8W. The following graph displays this condition below:

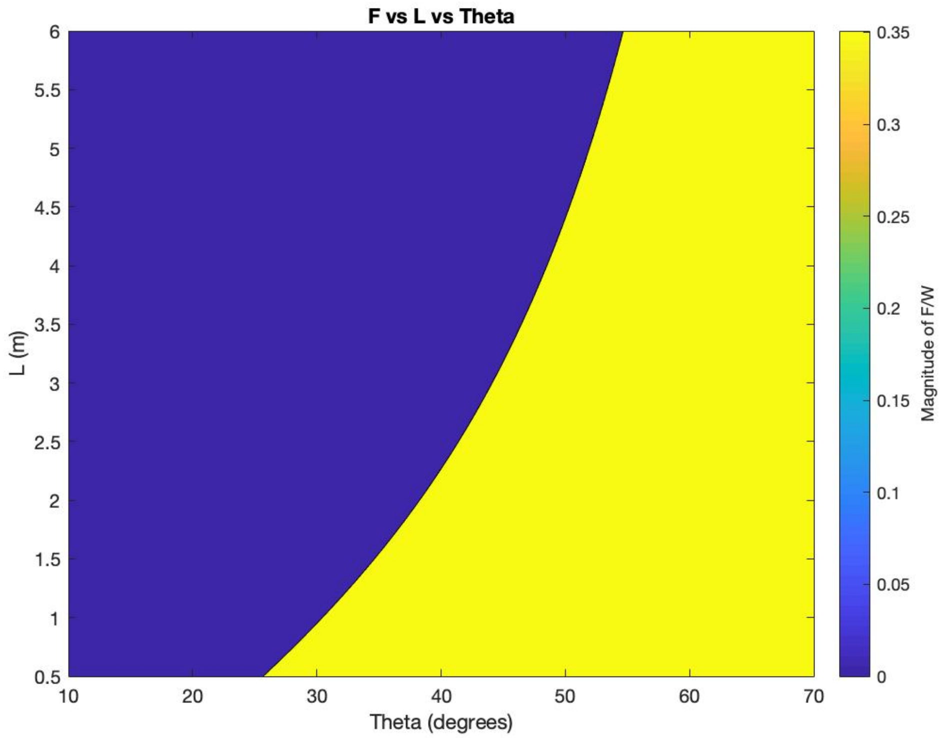


Figure 2: Range of acceptable values for Figure 1

CONCLUSION

This project highlights the relevance of using moment equilibrium expressions with equilibrium in mechanical systems, and how engineers can use them to break down complex systems into different force components. Specific attention was brought to the mechanical interactions in different support systems, and how they might determine the optimal specifications of a system.

APPENDIX

MATLAB Code

```
% EA2 Design Project 4

%Graph 1
theta=[10:1:70];
Ls=[0.5:0.1:6];
F=[]
F(1:56,1:61)=zeros;
for i=1:61;
    for j=1:56;
        thet=theta(i);
        L=Ls(j);
        FAEW=(sqrt(1+8*(sind(thet)^2)))*((1/4)-(L/10)*(1-sind(thet)));
        Fmagnitude=norm(FAEW);
        F(j,i)=Fmagnitude;
    end
end
F;
figure;
contourf(theta,Ls,F);
title('FAE/W vs L and theta');
xlabel('Angle Theta (degrees)');
ylabel('L (m)');
c=colorbar;
c.Label.String='Magnitude of F/W'

%Figure 2
figure;
contourf(theta,Ls,F,1);
xlabel('Theta (degrees)');
ylabel('L (m)');
zlabel('Force of AE (N)');
title('F vs L vs Theta');
c=colorbar;
c.Label.String='Magnitude of F/W'
```