Design Project 1

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ABSTRACT

The objective of this project is to investigate the mechanics of spring systems with a snap-through instability mechanism in regard to the critical load and critical angle of instability. This is done through the use of MATLAB, in which the results are calculated and plotted in a 3D graph for better visualization

INTRODUCTION

A system of two identical springs consists of anchor points A and B - as shown in Figure 1. Each spring is defined by the undeformed length L_o and the deformed length L_f, by which the system become increasingly unstable, and the angle α and θ that each spring makes with the horizontal plane.

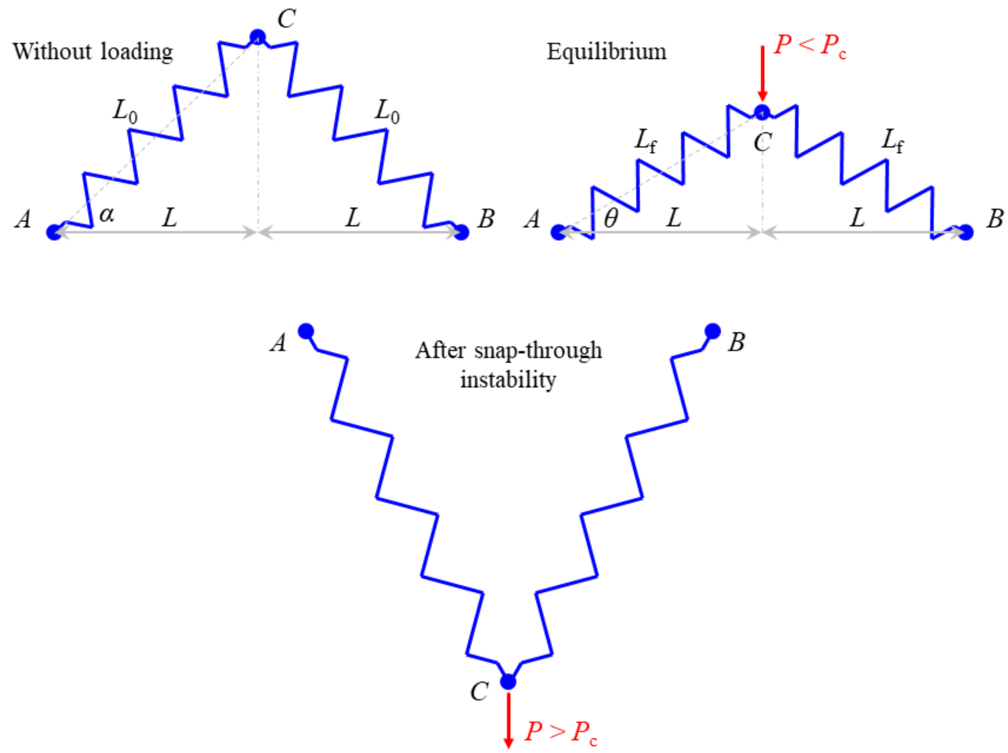


Figure 1

The design specification for the mechanisms are as follows:

1. The horizontal projection length of each spring is fixed at L = 1m
2. The activation force P_c for the structure should be between 250 and 500 N
3. The longest springs that can be produced are 2m
4. The stiffest springs that can be produced are 750 N/m

RESULTS

Question 1

The question asks us to express L_0 as a function of α and L. Looking at the overall system, this can be done using the trigonometric ratios of the triangle made by points A and C and the vertical projection from C to the horizontal plane.

$$\cos(\alpha) = \frac{L}{L_0}$$

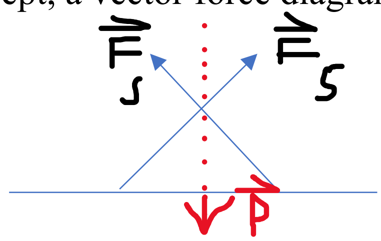
As shown above, this can be done first by expressing the sin of angle α as a function of the quotient of L and L_0. By rearranging the expression, we can see that:

$$L_0 = \frac{L}{\cos(\alpha)}$$

Question 2

The question asks us to express the load P in terms of the parameters k, L, α, and θ. To do this, the load P must first be understood as a vertical, downward force acting on the point B. As the force increases, the more compressed the spring will become more and more compressed, proportional to the load P.

The second step is to understand that in order for the system to be in equilibrium, the force P must be equal to the upward force exerted by the springs. In order to fully visualize this concept, a vector force diagram must be drawn:



From the free body diagram, the follow expressions can be developed in regard to the net force in the X and Y directions:

$$\sum \boldsymbol{F}_x = 0$$

$$\sum \boldsymbol{F}_y = 2 * \boldsymbol{F}_S * \sin(\theta)$$

For the system to be in equilibrium, there must be no net movement, meaning that the net upward force must equal the net downward force in the y-direction. It then follows that:

$$\boldsymbol{P} = -(2 * \boldsymbol{F_S} * \sin(\theta))$$

This is the expression for the load P. The last step of this problem is to ensure that P is written in terms of the parameters k, L, α, and θ. This can be one by representing the spring force F_S in terms of those variables. From mechanics, the force of a spring can be represented by the following expression:

$$\boldsymbol{F_S} = k(L_0 - L_F)$$

To get this expression in terms of the parameters k, L, α, and θ as asked in the question, the trigonometric ratio in *figure 1* can be exploited. Thus, F_S can be rewritten and simplified in the following manor:

$$\boldsymbol{F_S} = kL(\frac{1}{\cos{(\alpha)}} - \frac{1}{\cos{(\theta)}})$$

Finally, the full expression for the load P in terms of the parameters k, L, α, and θ can be written as:

$$\boldsymbol{P} = kL\left(\frac{1}{\cos(\alpha)} - \frac{1}{\cos(\theta)}\right) * 2\sin(\theta)$$

Question 3

The question asks us to find the critical angle θ_c corresponding to the maximum load the can be carried. We are told to do this by using the given initial condition $\frac{dP}{d\theta} = 0$. Hence, the first step to solve this problem is to take the derivative with respect to θ of the load found in question 2. The derivative can be expressed as:

$$\frac{d\boldsymbol{P}}{d\theta} = 2LK\left(\cos(\theta) * \left(\frac{1}{\cos(\alpha)} - \sec(\theta)\right) - \sec(\theta)\tan(\theta)\sin(\theta)\right) = 0$$

This expression can be reduced to:

$$\frac{d\boldsymbol{P}}{d\theta} = 2LK\left(\cos(\theta) * \left(\frac{1}{\cos(\alpha)} - \frac{1}{\cos(\theta)}\right) - \tan(\theta)^2\right) = 0$$

The 2LK component can be ignored because they are constants with regards to the overall expression. They can be multiplied out to 0. The goal at this point is to isolate and solve for θ, as that will be equivalent to the critical angle θ_c. By distributing the cos(θ) term, and adding $\tan(\theta)^2$ to both sides, the expression can become solvable. The following expression is the result:

$$\left(\frac{\cos(\theta)}{\cos(\alpha)} - 1\right) = \tan(\theta)^2$$

Because of the trigonometric identity "$\tan(\theta)^2 = \sec(\theta)^2 - 1$", the expression can be rewritten and simplified as:

$$\left(\frac{\cos(\theta)}{\cos(\alpha)}\right) = \sec(\theta)^2$$

The trigonometric identity that "$\sec(x)^2 = 1/\cos(x)^2$" can further reduce the expression to show that:

$$\left(\frac{\cos(\theta)}{\cos(\alpha)}\right) = \frac{1}{\cos(\theta)^2}$$

and rearranged to show that:

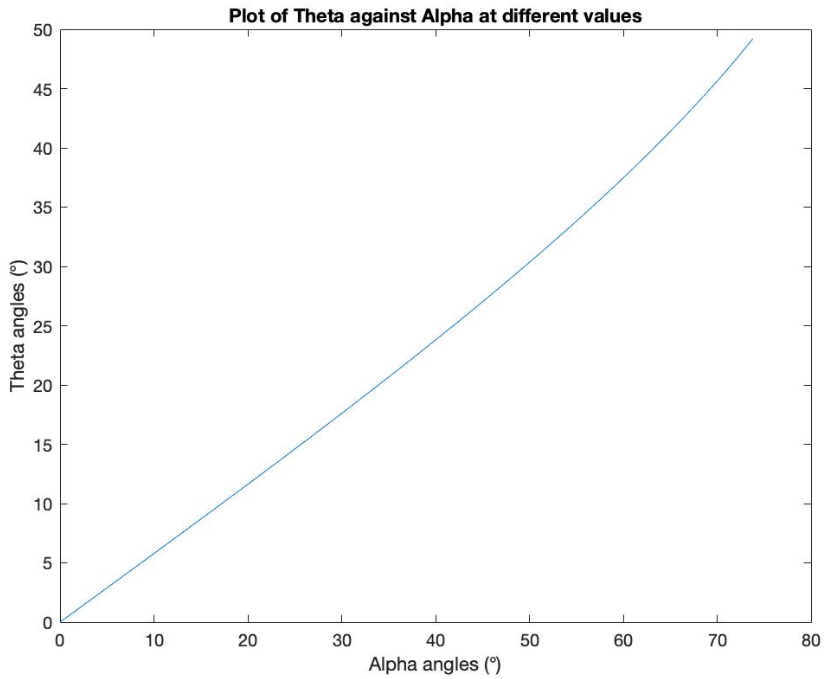
$$\cos(\theta)^3 = \cos(\alpha)$$

And hence, the critical angle θ_c is equal to:

$$\theta_c = \arccos(\cos(\alpha)^{1/3})$$

ANALYSIS AND DISCUSSION

Question 4



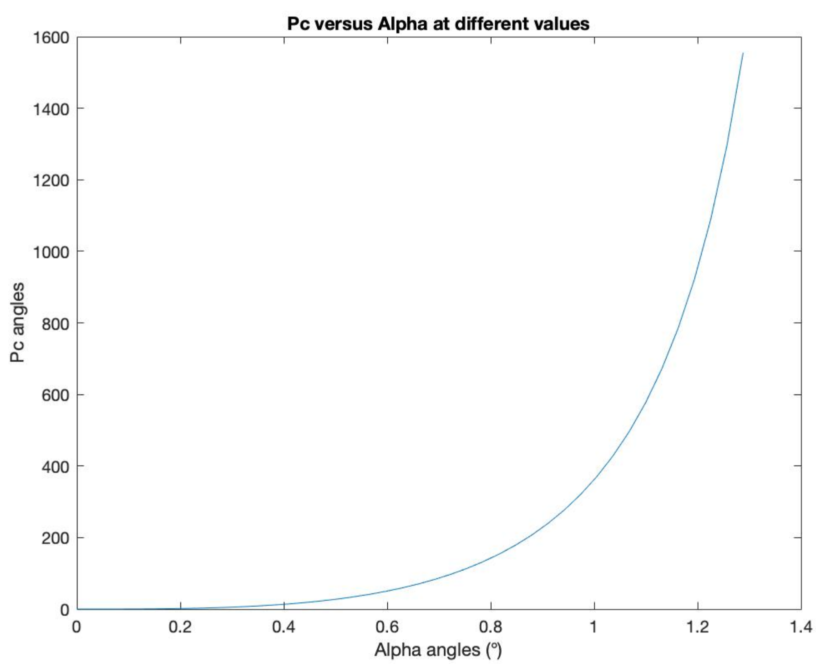


Figure 2

Figure 2 depicts the graphs of P_c against α at the given range values and the graph of θ_c against α at the given range values. These graphs show the dependency relationship between both P_c and θ_c with regards to the parameter α. It can be seen that P_c has an exponential relationship with α while θ_c has a relatively linear relationship.

Question 5

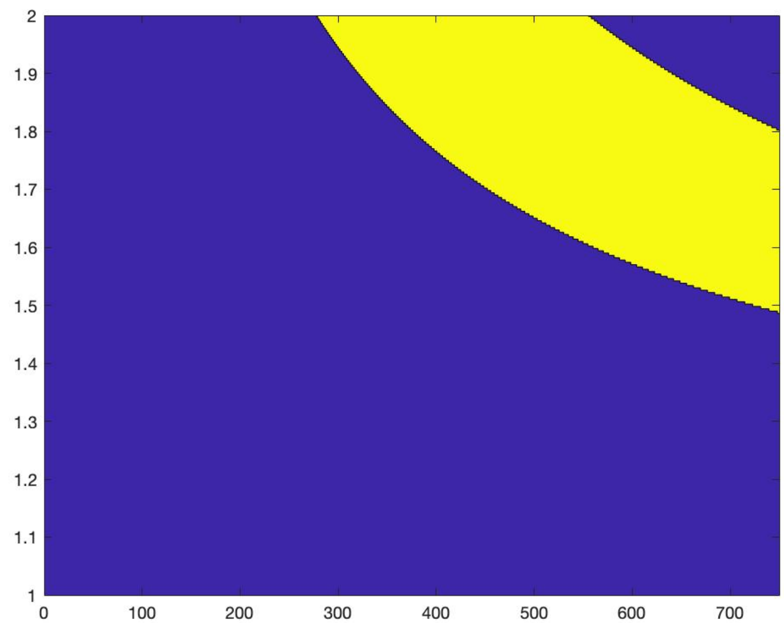


Figure 3

Figure 3 represents the feasible ranges of designs using the design specifications. The yellow strip represents the feasible ranges for the different parameters, while purple represents the unreasonable ranges. This graph can give some insight into what kind of a relationship each of the parameters have on one another, which can be helpful to someone wanting to manufacture similar apparatuses.

CONCLUSION

The project highlights application of spring systems and force diagrams, and how engineers can model such systems using computational techniques. Specific emphasis is put on variable dependencies and design specifications, which is very important when trying to model systems involving multiple variables.

APPENDIX

MATLAB Code

```
%Question 4

%Initialize the Variables
k = 500;
L = 1;
a = 0:pi/100:(5*pi/12);
theta = acos((cos(a)).^(1/3));
Pc = 2*-1*k*L.*sin(theta).*((1./cos(theta))-(1./cos(a)));

%Plots
figure
plot(a*180/pi,theta*180/pi);
title("Plot of Theta against Alpha at different values");
xlabel(['Alpha angles (' 176 ')']);
ylabel(['Theta angles (' 176 ')']);

figure
plot(a,Pc)
title("Pc versus Alpha at different values");
xlabel(['Alpha angles (' 176 ')']);
ylabel(['Pc angles']);

%Question 5

%Initialize the Variables
k = [0:0.004:750];
L = 1;
L0 = [1:0.004:2];
m = zeros(length(k),length(L0));
limit = zeros(length(k), length(L0));

%Solve
for ii = 1:length(k)
    for jj = 1:length(L0)
        limit(ii,jj) = k(ii)*2*L*((sin(acos((L./L0(jj)).^(1/3)))/(L./L0(jj)))
- tan(acos(((L./L0(jj)).^(1/3)))));
        if limit(ii,jj) >= 250 && limit(ii,jj) <= 500
            m(ii,jj) = 1;
        end
    end
end

%Plot
contourf(k,L0,m',1);
```