Design Project 2

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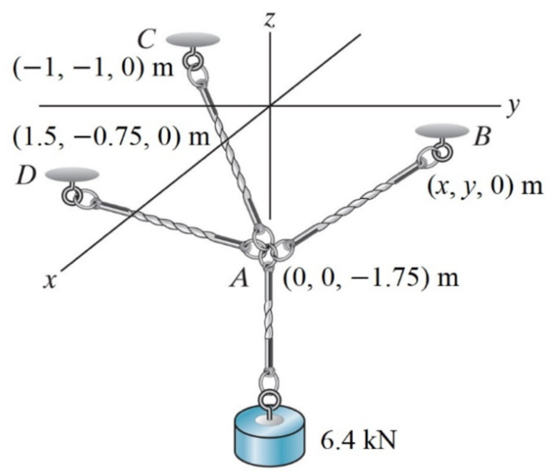
ABSTRACT

The objective of this project is to investigate the mechanics of a system of cables in relation to the position of each anchor point, as well as how those decisions might affect the cost of the system. This is done through the use of MATLAB, in which the results are calculated and plotted in a 3D graph for better visualization.

INTRODUCTION

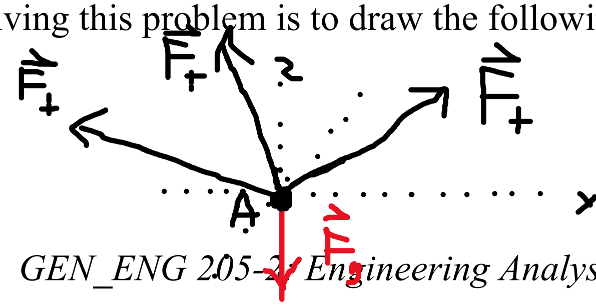
A system of four cables support a 6.4 kN weight. The suspensions created by these cables are denoted by the four points A, B, C and D, and the cables themselves can be represented as AD, AB, and AC. The positions of points A, C and D are fixed, however the x and y components of point B can be chosen from the ranges $-0.6m \leq x \leq 1.0m$ and $-0.4m \leq y \leq 1.2m$. The x and y coordinates of B have an impact on the distribution of the tension forces in the diagram, and hence the feasibility of the system.



It is also given that the cost of the cable AB can be represented by the product of the tension force on AB and its squared length. Thus, we are asked to find the effects of x and y on the cable, as well as an optimal coordinate pair for the point B.

RESULTS

The first step in solving this problem is to draw the following free body diagram with respect to A:



From the free body diagram, we know that the forces with respect to A are in equilibrium. Thus, we can say that the sum of overall tension forces and weight 6.4 kN are equal to 0.

$$\sum \boldsymbol{F} = 0 = \boldsymbol{T_{AB}} + \boldsymbol{T_{AC}} + \boldsymbol{T_{AD}} + (-6.4\ kN)$$

Each of the tension forces can be represented as the product of the magnitude of the tension force and the unit vector in that direction. Hence:

$$\sum \boldsymbol{F} = 0 = |T_{AB}|(\boldsymbol{e_{AB}}) + |T_{AC}|(\boldsymbol{e_{AC}}) + |T_{AD}|(\boldsymbol{e_{AD}}) + (-6.4\ kN)$$

In order to solve for the components of each of the vectors, and to represent the system in terms of the variables we can control, we can represent the equation above in terms of matrices. Additionally, we convert all of the units to Newtons for consistency.

$$\sum \boldsymbol{F} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = |T_{AB}| \begin{bmatrix} \frac{x}{|\boldsymbol{AB}|} \\ \frac{y}{|\boldsymbol{AB}|} \\ \frac{1.75}{|\boldsymbol{AB}|} \end{bmatrix} + |T_{AC}| \begin{bmatrix} \frac{1}{2.25} \\ \frac{1}{2.25} \\ \frac{1.75}{2.25} \end{bmatrix} + |T_{AD}| \begin{bmatrix} \frac{1.5}{(5.875)^{\frac{1}{2}}} \\ \frac{0.75}{(5.875)^{\frac{1}{2}}} \\ \frac{1.75}{(5.875)^{\frac{1}{2}}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -6400 \end{bmatrix}$$

The vector $\boldsymbol{AB}$ can be put in terms of x and y by the following expression: $\boldsymbol{AB} = \sqrt{x^2 + y^2 + 1.75^2}$. Thus, the entire expression can be rewritten as:

$$\sum \boldsymbol{F} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = |T_{AB}| \begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2 + 1.75^2}} \\ \frac{y}{\sqrt{x^2 + y^2 + 1.75^2}} \\ \frac{1.75}{\sqrt{x^2 + y^2 + 1.75^2}} \end{bmatrix} + |T_{AC}| \begin{bmatrix} \frac{1}{2.25} \\ \frac{1}{2.25} \\ \frac{1.75}{2.25} \end{bmatrix} + |T_{AD}| \begin{bmatrix} \frac{1.5}{(5.875)^{\frac{1}{2}}} \\ \frac{0.75}{(5.875)^{\frac{1}{2}}} \\ \frac{1.75}{(5.875)^{\frac{1}{2}}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -6400 \end{bmatrix}$$

Rearranging and simplifying the expression, we get that:

$$\begin{bmatrix} \frac{x}{\sqrt{x^2 + y^2 + 1.75^2}} & \frac{1}{2.25} & \frac{1.5}{(5.875)^{\frac{1}{2}}} \\ \frac{y}{\sqrt{x^2 + y^2 + 1.75^2}} & \frac{1}{2.25} & \frac{0.75}{(5.875)^{\frac{1}{2}}} \\ \frac{1.75}{\sqrt{x^2 + y^2 + 1.75^2}} & \frac{1.75}{2.25} & \frac{1.75}{(5.875)^{\frac{1}{2}}} \end{bmatrix} \begin{bmatrix} |T_{AB}| \\ |T_{AC}| \\ |T_{AD}| \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6400 \end{bmatrix}$$

By using the backslash operator in MATLAB, we can solve this system of equation for the tension force and plot the results. Furthermore, the cost of the beam can be derived from the resultant tension force, and thus the optimal point for B can be found.

ANALYSIS AND DISCUSSION

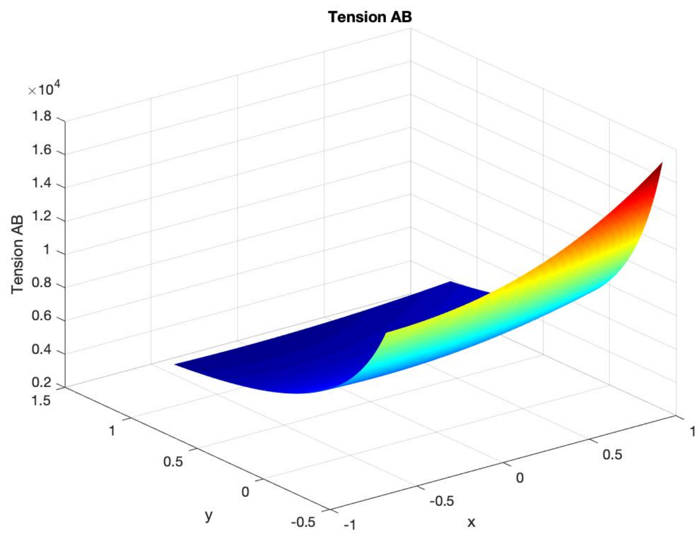


Figure 1

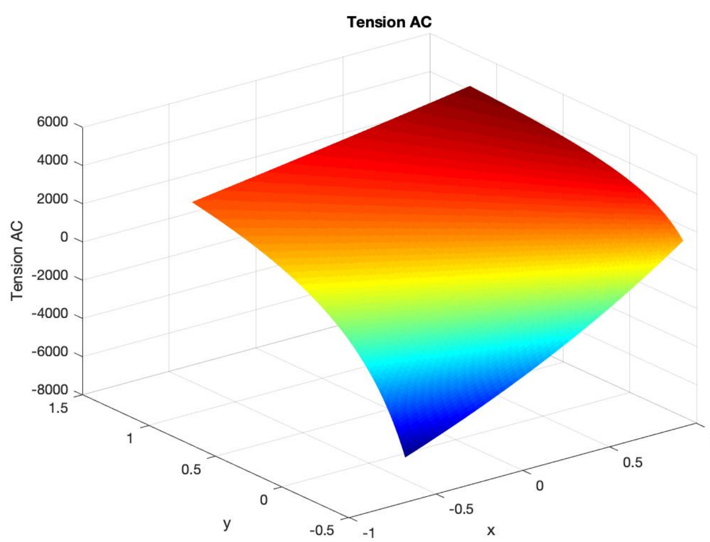


Figure 2

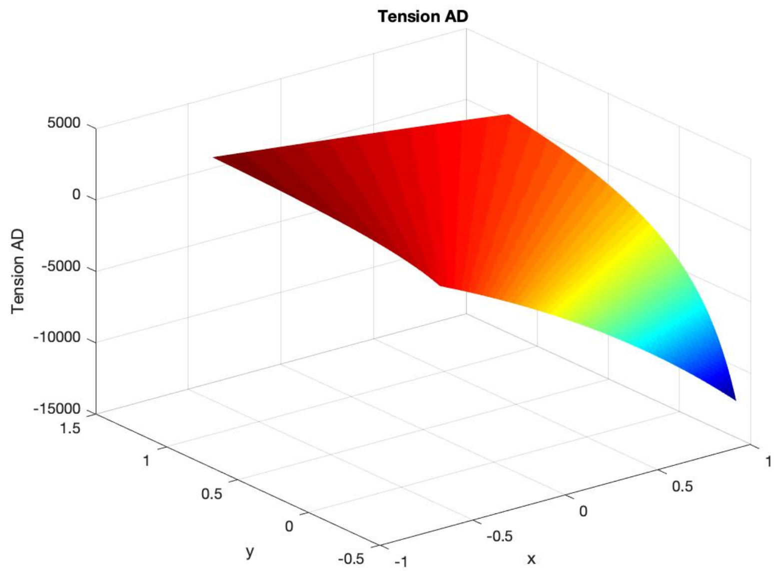


Figure 3

Figure 1, figure 2 and *figure 3* represent the tension forces for the vectors AD, AB, and AC. The magnitude of the tension forces is plotted against the different values of x and y for point B, and the results can be seen above. As can be seen, some values of x and y are unfeasible as they create negative tensions. The rate at which the tension changes depends on each of the variables x and y.

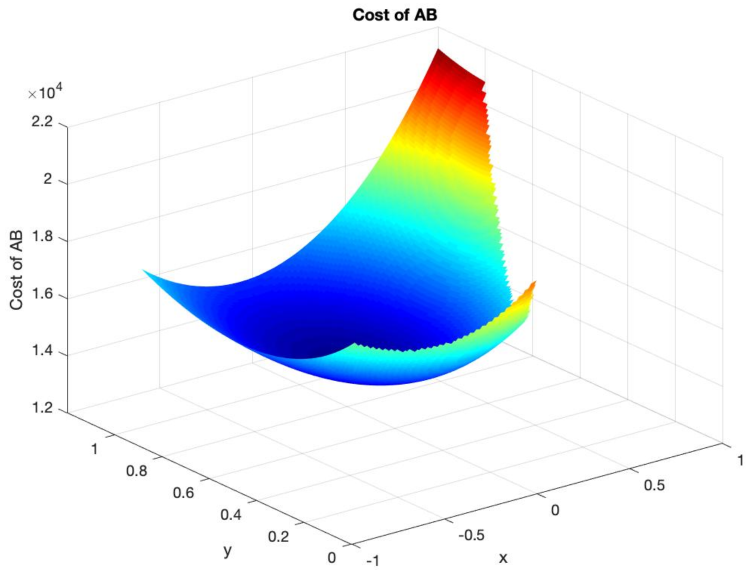


Figure 4

From figure 4, the optimal point for B as well as the cost can be deduced. The optimal point for B is defined by the coordinate pair (-0.0700, 0.7300, 0.0000). This point minimizes the cost of the system while ensuring that only tension forces are acting on the cables. The proportional cost of the system is $2124.2.

CONCLUSION

This project highlights applications of cable systems and how engineers can model mechanical systems using linear algebra and computational techniques. Specific emphasis is put on thinking about the cost-benefit relationship within components of mechanical systems, as this can come up quite frequently in industry.

APPENDIX

<u>MATLAB Code</u>

```
x=-0.6:.01:1.0;
y=-0.4:.01:1.2;
Tension1=zeros(length(x),length(y));
Tension2=zeros(length(x),length(y));
Tension3=zeros(length(x),length(y));
TotalForce=[0; 0; 6400];

for ii=1:length(x)
    for jj=1:length(y)
       Matrix = [3*((94)^.5)/47 -4/9 x(ii)/sqrt((x(ii))^2+(y(jj))^2+1.75^2);
        -3/sqrt(94) -4/9 y(jj)/sqrt((x(ii))^2+(y(jj))^2+1.75^2);
        7/sqrt(94) 7/9 1.75/sqrt((x(ii))^2+(y(jj))^2+1.75^2)];
```

```
        Final = Matrix\TotalForce;

        Tension1(ii,jj)=Final(1,1);
        Tension3(ii,jj)=Final(2,1);
        Tension2(ii,jj)=Final(3,1);
    end
end


figure
surf(x,y,Tension1','edgecolor','none');
colormap(jet)
title('Tension AD')
xlabel('x')
ylabel('y')
zlabel('Tension AD')


figure
surf(x,y,Tension3','edgecolor','none');
colormap(jet)
title('Tension AC')
xlabel('x')
ylabel('y')
zlabel('Tension AC')


figure
surf(x,y,Tension2','edgecolor','none');
colormap(jet)
title('Tension AB')
xlabel('x')
ylabel('y')
zlabel('Tension AB')


mininum = 100000000000000000;
NetCost=zeros(length(x),length(y));
Xpos=0;
Ypos=0;

for ii=1:length(x)
    for jj = 1:(length(y))
        % (NetCost(ii,jj)<mininum) &&
         if( Tension1(ii,jj)>0 && Tension3(ii,jj)>0 && Tension2(ii,jj)>0)
            NetCost(ii,jj)=Tension2(ii,jj)*((x(ii)^2+y(jj)^2+1.75^2));
            mininum=NetCost(ii,jj);
            Xpos=x(ii);
            Ypos=y(jj);
         else
             NetCost(ii,jj)= NaN;
         end

        end
```

```
    end

% end


figure
surf(x,y,NetCost','edgecolor','none');
colormap(jet)
title('Cost of AB')
xlabel('x')
ylabel('y')
zlabel('Cost of AB')
fprintf(Xpos+" "+Ypos);
fprintf(" " + min(min(NetCost)));
```