Method-2
Let $f(x)=2x^3-2x-5$

Here $2x^3-2x-5=0$

$\therefore 2x^3=2x+5$

$\therefore x^3=2x+52$

$\therefore x=\sqrt[3]{(2x+5)/2}$

$\therefore \phi(x)=\sqrt[3]{(2x+5)/2}$

Here $f(1.5)=-1.25<0$ and $f(1.8)=3.064>0$

$\therefore$ Root lies between $1.5$ and $1.8$

$x_0=1.5+1.82=1.65$

$x_1=\phi(x_0)=\phi(1.65)=1.607$

$x_2=\phi(x_1)=\phi(1.607)=1.6014$

$x_3=\phi(x_2)=\phi(1.6014)=1.6007$

$x_4=\phi(x_3)=\phi(1.6007)=1.6006$

Approximate root of the equation $2x^3-2x-5$ using Iteration method is $1.6006$ (After 4 iterations)

| $n$ | $x_0$ | $x_1=\phi(x_0)$ | Update | Difference $x_1-x_0$ |
|---|---|---|---|---|
| 2 | 1.65 | 1.607 | $x_0=x_1$ | 0.043 |
| 3 | 1.607 | 1.6014 | $x_0=x_1$ | 0.0056 |
| 4 | 1.6014 | 1.6007 | $x_0=x_1$ | 0.0007 |
| 5 | 1.6007 | 1.6006 | $x_0=x_1$ | 0.0001 |

Find the positive root of $x^3 - 2x - 8 = 0$ by method of successive substitution correct upto two places of decimal.

**Solution**

$$f(x) = x^3 - 2x - 8$$

To find the approximate location of the root (+ ive) we try to evaluate the function values at different x and tabulate as follows :

| **x** | 0 | 1 | 2 | 3 | x > 3 |
|---|---|---|---|---|---|
| **f(x)** | – 8 | – 9 | – 4 | 13 | + ive |
| **Sign of f(x)** | – | – | – | + | + |

The root lies between 2 and 3. Let us choose the initial approximation as $x_0 = 2.5$.

Let us express f(x) = 0 as $x = \phi(x)$ in the following forms and check whether $|\phi'(\alpha)| < 1$ for
x = 2.5.

(i) $x = x^3 - x - 8$

(ii) $x = \frac{1}{2}(x^3 - 8)$

(iii) $x = (2x + 8)^{\frac{1}{3}}$

We see that in cases (i) and (ii) $|\phi'(x)| > 1$, hence we should discard these representations. As the third case satisfies the condition, $|\phi'(x)| = \left| \frac{1}{3(2x+8)^{\frac{2}{3}}} \right| < 1$ for x = 2.5 we have the iteration scheme as,

$$x_{n+1} = (2x_n + 8)^{\frac{1}{3}}$$

Starting from $x_0 = 2.5$, we get the successive iterates as shown in the table below :

| **n** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **$x_n$** | 2.5 | 2.35 | 2.33 | 2.33 |

## Find a root of an equation $f(x)=x^2+2x-8$ between 1 and 4, using Bisection method

## Solution:

Here $x^2+2x-8=0$

Let $f(x)=x^2+2x-8$

Here

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | -5 | 0 | 7 | 16 |

Here $f(2)=0$

Root of the equation $x2+2x-8$ is $2$

Bisection method solve the equ.x2+2x–8=0 in the interval [1,4].Use4iterations.
x^2+2*x-8

| Step | x0 | x1 | x2 | f(x2) |
|---|---|---|---|---|
| 1 | 1.000000 | 4.000000 | 2.500000 | 3.250000 |
| 2 | 1.000000 | 2.500000 | 1.750000 | -1.437500 |
| 3 | 1.750000 | 2.500000 | 2.125000 | 0.765625 |
| 4 | 1.750000 | 2.125000 | 1.937500 | -0.371094 |