2.4.6 $\quad \phi(\rho, \varphi, z) = \rho^2 - 2\rho \cos\varphi - z^2$

Nivåyta: $\quad \phi(\vec{r}) = C' = \text{const.}$

$\rightarrow$ Alla $\vec{r}$ med samma värde av ϕ formar en yta.

här: Skriv fältet i kartesiska koordinater.

$$x = \rho \cos\varphi, \quad y = \sin\varphi, \quad z = z$$

$$\rho = \sqrt{x^2 + y^2}$$

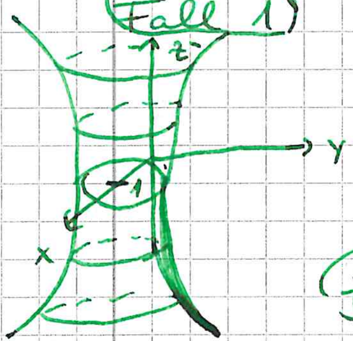
$$\rightarrow \phi(x, y, z) = x^2 + y^2 - 2x - z^2 = x^2 - 2x + \underbrace{1 - 1}_{=0} + y^2 - z^2$$

$$= (x-1)^2 + y^2 - z^2 - 1$$

definiera $\rho' = \sqrt{(x-1)^2 + y^2}$, (cylinder-) radius omkring punkten $\vec{r}_0 = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$$\rightarrow \phi(x,y,z) \rightarrow \phi(\rho', \varphi, z) = \rho'^2 - z^2 - 1 = C'$$

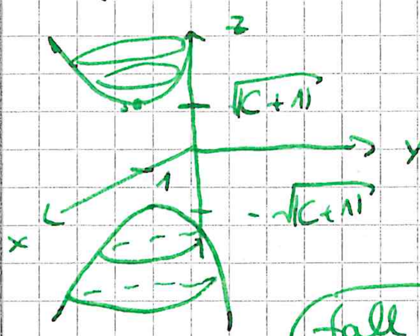
$$\Leftrightarrow \rho'^2(z) = C' + 1 + z^2$$



Fall 1) $\quad C+1 > 0 \Leftrightarrow C > -1: \quad \rho'(z) = \sqrt{C+1+z^2} > \sqrt{C+1}$

definierad $\forall z$

$\rightarrow$ enmantlad hyperboloid



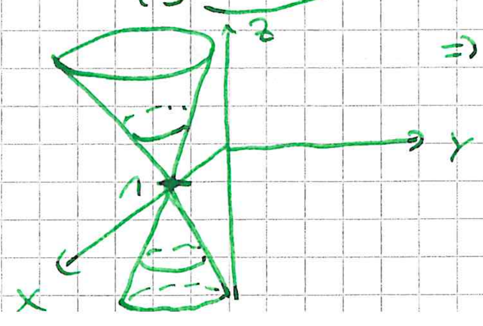
Fall 2) $\quad C'+1 < 0 \Leftrightarrow C < -1 \Rightarrow C+1 = -|C+1|$

$$\rho'(z) = \sqrt{z^2 - |C+1|}$$

bara reell för $z^2 > |C+1|$

$\Leftrightarrow z > \sqrt{|C+1|}$

$\Rightarrow$ tvåmantlad hyperboloid



Fall 3) $\quad C'+1 = 0 \Leftrightarrow C' = -1$

$$\Rightarrow \rho'(z) = \sqrt{z^2} = |z|$$

$\Rightarrow$ kon

2.4.23

$$\left.\begin{aligned} x &= \alpha\ e^{\omega}\ \cos v \\ y &= \beta\ e^{\omega}\ \sin v \\ z &= u \end{aligned}\right\} \quad \vec{r}(x,y,z) \longrightarrow \underbrace{\vec{r}(u,v,w)}_{\text{beror på } \alpha,\beta}$$

(interpretation: cylindriska koordinater med $\rho = e^{\omega}$)

Basvektorerna i det nya systemet ska vara ortogonala.

Ekv. (2.1) : $\hat{e}_i = \frac{1}{h_i}\frac{\partial \vec{r}}{\partial a_i}$ där $h_i = \left|\frac{\partial \vec{r}}{\partial a_i}\right|$

↑ vektor

$$\frac{\partial \vec{r}}{\partial u} = \begin{pmatrix}0\\0\\1\end{pmatrix} \to h_u = 1$$

$$\frac{\partial \vec{r}}{\partial v} = \begin{pmatrix}-\alpha \sin v\ e^{\omega}\\ \beta \cos v\ e^{\omega} \\ 0\end{pmatrix} \to h_v = \sqrt{\alpha^2 \sin^2 v\ e^{2\omega} + \beta^2 \cos^2 v\ e^{2\omega}}$$

$$\frac{\partial \vec{r}}{\partial \omega} = \begin{pmatrix}\alpha \cos v\ e^{\omega}\\ \beta \sin v\ e^{\omega} \\ 0\end{pmatrix} \to h_\omega = \sqrt{\alpha^2 \cos^2 v\ e^{2\omega} + \beta^2 \sin^2 v\ e^{2\omega}}$$

$\hat{e}_u \perp \hat{e}_v, \hat{e}_\omega$ automatiskt

$$\hat{e}_v \cdot \hat{e}_\omega = \frac{1}{h_v h_\omega}\left(-\alpha^2 \cos v \sin v\ e^{2\omega} + \beta^2 \sin v \cos v\ e^{2\omega}\right)$$

$$= \frac{1}{h_v h_\omega}(\cos v \sin v\ e^{2\omega})(-\alpha^2 + \beta^2) \underset{\text{ska vara}}{=} 0$$

$$\Longrightarrow \alpha^2 = \beta^2$$

$$\to h_v = e^{\omega} \underbrace{|\alpha|}_{=\sqrt{\alpha^2}} \underbrace{\sqrt{\sin^2 v + \cos^2 v}}_{=1} = e^{\omega}|\alpha|$$

$$h_\omega = \ldots = e^{\omega}|\alpha|$$

$$ds \underset{(2.6)}{=} d\vec{r}\, d\vec{r} = \sum_{i,j} h_i h_j \underbrace{\hat{e}_i \cdot \hat{e}_j}_{=0 \text{ om inte } i=j} du_i du_j = \sum_i h_i^2 du_i^2$$

$$\to ds = h_u^2 du^2 + h_v^2 dv^2 + h_\omega^2 d\omega^2$$

$$= 1 \cdot du^2 + e^{2\omega}|\alpha|^2 (dv^2 + d\omega^2)$$