T-matrix calculation procedure

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Equations are taken from:
Mishchenko, M.I., Travis, L.D., and Lacis, A.A. (2002). Scattering, absorption, and emission of light by small particles (Cambridge University Press).

T-matrix calculation procedure:

1- Evaluate surface integrals (i.e., over the surface of the particle)

$$\begin{bmatrix} J^{11}_{mnm'n'} \\ J^{12}_{mnm'n'} \\ J^{21}_{mnm'n'} \\ J^{22}_{mnm'n'} \end{bmatrix} = (-1)^m \int_S \mathrm{d}S\, \hat{\mathbf{n}} \cdot \begin{bmatrix} \mathrm{Rg}\mathbf{M}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathbf{M}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{M}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathbf{N}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{N}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathbf{M}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{N}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathbf{N}_{-mn}(k_1 r, \vartheta, \varphi) \end{bmatrix}. \quad (5.184)$$

$$\begin{bmatrix} \mathrm{Rg}J^{11}_{mnm'n'} \\ \mathrm{Rg}J^{12}_{mnm'n'} \\ \mathrm{Rg}J^{21}_{mnm'n'} \\ \mathrm{Rg}J^{22}_{mnm'n'} \end{bmatrix} = (-1)^m \int_S \mathrm{d}S\, \hat{\mathbf{n}} \cdot \begin{bmatrix} \mathrm{Rg}\mathbf{M}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathrm{Rg}\mathbf{M}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{M}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathrm{Rg}\mathbf{N}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{N}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathrm{Rg}\mathbf{M}_{-mn}(k_1 r, \vartheta, \varphi) \\ \mathrm{Rg}\mathbf{N}_{m'n'}(k_2 r, \vartheta, \varphi) \times \mathrm{Rg}\mathbf{N}_{-mn}(k_1 r, \vartheta, \varphi) \end{bmatrix}. \quad (5.190)$$

Notes:

- these are surface integrals happening on the surface of the particle. Therefore, we need to find an expression for $r(\theta, \phi)$ and $\hat{\mathbf{n}}(\theta, \phi)$, and then we perform the integral over 2D θ, ϕ plane.
- k_1 and k_2 refers to wavevector of surrounding and particle, respectively
- Calculation of vector spherical waves ($\mathbf{M}$ and **N**) is explained in: https://github.com/mhmodzoka/VectorSphericalWaves.jl

2- Calculate **Q** and Rg**Q**

$$Q^{11}_{mnm'n'} = -\mathrm{i}k_1 k_2 J^{21}_{mnm'n'} - \mathrm{i}k_1^2 J^{12}_{mnm'n'}, \quad (5.180)$$

$$Q^{12}_{mnm'n'} = -\mathrm{i}k_1 k_2 J^{11}_{mnm'n'} - \mathrm{i}k_1^2 J^{22}_{mnm'n'}, \quad (5.181)$$

$$Q^{21}_{mnm'n'} = -\mathrm{i}k_1 k_2 J^{22}_{mnm'n'} - \mathrm{i}k_1^2 J^{11}_{mnm'n'}, \quad (5.182)$$

$$Q^{22}_{mnm'n'} = -\mathrm{i}k_1 k_2 J^{12}_{mnm'n'} - \mathrm{i}k_1^2 J^{21}_{mnm'n'}, \quad (5.183)$$

$$\mathrm{Rg}Q^{11}_{mnm'n'} = -\mathrm{i}k_1 k_2 \mathrm{Rg}J^{21}_{mnm'n'} - \mathrm{i}k_1^2\, \mathrm{Rg}J^{12}_{mnm'n'}, \quad (5.186)$$

$$\mathrm{Rg}Q^{12}_{mnm'n'} = -\mathrm{i}k_1 k_2 \mathrm{Rg}J^{11}_{mnm'n'} - \mathrm{i}k_1^2\, \mathrm{Rg}J^{22}_{mnm'n'}, \quad (5.187)$$

$$\mathrm{Rg}Q^{21}_{mnm'n'} = -\mathrm{i}k_1 k_2 \mathrm{Rg}J^{22}_{mnm'n'} - \mathrm{i}k_1^2\, \mathrm{Rg}J^{11}_{mnm'n'}, \quad (5.188)$$

$$\mathrm{Rg}Q^{22}_{mnm'n'} = -\mathrm{i}k_1 k_2 \mathrm{Rg}J^{12}_{mnm'n'} - \mathrm{i}k_1^2\, \mathrm{Rg}J^{21}_{mnm'n'}, \quad (5.189)$$

$$\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \mathbf{Q} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^{11} & \mathbf{Q}^{12} \\ \mathbf{Q}^{21} & \mathbf{Q}^{22} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix}, \quad (5.179)$$

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = -\mathrm{Rg}\mathbf{Q} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} = -\begin{bmatrix} \mathrm{Rg}\mathbf{Q}^{11} & \mathrm{Rg}\mathbf{Q}^{12} \\ \mathrm{Rg}\mathbf{Q}^{21} & \mathrm{Rg}\mathbf{Q}^{22} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix}, \quad (5.185)$$

3- Calculate T-matrix

$$\mathbf{T}(P) = -(\mathrm{Rg}\mathbf{Q})\, \mathbf{Q}^{-1}. \quad (5.191)$$