

# A Cost-Reliability Model with Spares in Electric Power Systems

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## Abstract:

The 3 - component systems are termed triple systems. The three - phase electric power system can be considered as a triple system since it includes within its different sections, three similar components of certain items. These items may be transformers, rectifiers, thyristors, protection equipment... etc. In order to achieve a certain reliability level for this system, it is also necessary to include spares in the system for each item and this will bring about an increase in the cost. This paper presents the design issue for the optimal number of spare units in a triple section including faults due to common - cause failure. Two main problems are tackled in this paper; the first is how to minimize the average total system cost, subject to a certain reliability level, for different values of failure and repair rates; the second is how to maximize the system reliability with imperfect coverage and common cause failure included. Application and a numerical example illustrates the results which are compared with the results obtained from other reference for a different triple system.

## 1 - Introduction :

It is generally known that electric power systems cannot achieve their intended reliability without using redundancy. Typical examples are the station transformer configuration, generating

station auxiliaries, transmission and distribution network, substation configurations and high voltage direct current systems (HVDC). Several techniques have been developed to achieve fault tolerance using sparing redundancy in different systems [1-6], but in [7] the reliability and cost effectiveness of triple - modular redundancy with spares (TMRWS) systems have been analyzed when they are subjected to common - cause failure and fault coverage. The TMRWS system consists of a triple modular redundancy (TMR) system with a set of spare units. The TMR functions properly as long as any two units are operational; when one of the three active unit fails, it is replaced by a spare and the basic TMR operation continues.

In this paper, a different system is analyzed. This system consists also of three units in an electric power system but it can function properly only as long as all the units are operational. This system can be a generating transformer substation [8] that consists of two identical three-phase transformer banks, each bank consisting of three identical single phase transformers. Each of these banks is considered to have failed totally and must be removed from service if any one of the three single-phase transformers in the bank fails. The bank can be returned to service when the failed transformer has been repaired or replaced. The techniques used in [7] are

applied with some modifications due to the difference between the two systems.

The problem of determining the optimal number of spares that minimizes the system cost at different failure and repair rates is considered. If the reliability corresponding to the optimal configuration is too low to be accepted, then the optimization criterion is to minimize the cost subject to a minimum designed reliability level.

The paper also considers the optimum number of spare units that maximizes the reliability of the system if the concept of fault coverage and its effect on reliability model of a repairable system is taken into consideration as well as the common cause failure.

## Notation

$S$	number of spares units
$S^*$	optimum number of spares units that provides minimum cost
$S_{op}$	optimum number of spares units that provides maximum reliability
$x$	cost of each unit
$y$	cost of system failure
$p$	unit reliability ( $q = 1 - p$ )
$R(s)$	reliability of the system
$\lambda$	failure rate of each unit
$u$	repair rate
$C(s)$	average system cost
$P_c$	Pr {fault detection and switching mechanism operate correctly   failure of one unit}
$P_f$	Common cause failure reliability
$R_c(s)$	System reliability including fault coverage and common cause failure
binf	binomial function
gilb	greatest integer lower bound

## 2 - Cost Optimization Model :

Consider a triple component section in an electric power system, this section includes spare units i.e.  $(s+3)$  section. The units are 2-state : good or failed and the failures of the components are statistically independent. The system is considered to be failed totally and must be removed from service if any one of the three component fails. The reliability of this system  $R(s)$  is given by :-

$$R(s) = \text{binf}(s; q; s+3) \quad (1)$$

$R(s)$  is an increasing function of the number of spare units. It is required to get an optimal solution which gives a balance between the reliability and the cost of the system  $C(s)$  as the number of spares increases. The average system cost  $C(s)$  is the cost of all components in the system plus the cost incurred when the system has failed and is given by :-

$$C(s) = x(s+3) + y[1 - R(s)] \quad (2)$$

It is generally known that the probability of finding a two-state unit on outage at a time  $t$ , given that it was operating successfully at  $t = 0$  is :-

$$q = \frac{\lambda}{\lambda + u} - \frac{\lambda}{\lambda + u} e^{-(\lambda + u)t} \quad (3)$$

The minimum cost for different failure rates ( $\lambda$ ) and repair rates ( $u$ ) can be obtained in this section. Also the number of spares that will provide this minimum cost is also obtained as will be shown. The three equations (1), (2) and (3) are all combined for determining  $s^*$  which is the  $s$  that minimizes  $C(s)$  under different conditions as follows :-

By using the identity [7] :-

$$\text{binf}(s+1; q; s+4) = \text{binf}(s; q; s+3) + (s+3) q^{s+1} p^3 \quad (4)$$

$$\text{Let } f(s) = (s+3) q^{s+1} p^3 \quad (5)$$

$$\therefore \Delta f(s) = f(s+1) - f(s)$$

$$\therefore \Delta f(s) = (s+4) q^{s+2} p^3 - (s+3) q^{s+1} p^3$$

$$\therefore \Delta f(s) \geq 0 \text{ iff :-} \\ (s+4) q \geq s+3$$

from which :

$$s \leq \text{gilb} (1/p - 4) \equiv S_0$$

Likewise,  $\Delta f(s) \leq 0$  iff:  $S \geq S_0$ .

Since  $p$  is often high, then for any given  $p$  ( $1/4 < p < 1$ )  $S_0 < 0$ ; or  $S \geq 0$ , then  $\Delta f(s) \leq 0$  for all  $S$ .  $f(s)$  is decreasing in  $S$  for all  $S$ . Also, the equation of variation of cost as the number of spares is varied can be put as follows :-

$$\Delta C(s) = C(s+1) - C(s)$$

but from (2);

$$\Delta C(s) = x - y [R(s+1) - R(s)]$$

From (4), (5)

$$\therefore \Delta C(s) = x - y f(s)$$

$$\therefore \Delta C(s) \leq 0 \quad \text{if } x \leq y f(s) \\ \text{or } f(s) \geq x/y$$

$$\text{also } \Delta C(s) \geq 0 \quad \text{if } x \geq y f(s) \\ \text{or } f(s) \leq x/y$$

From this it can be shown that the minimum cost  $C(s)$  takes place if the number of spares is  $S'$  where :-

$$\Delta C(s') = C(s'+1) - C(s') > 0$$

which entails  $f(s') < x/y$

$$\text{or } s' = \inf \{ S : f(s) < x/y \}$$

The previous explanation presents an optimal solution that minimizes  $C(s)$ .

Practically, for a certain values of  $p$ ,  $x$ ,  $y$ , the optimization configuration is minimizing  $C(s)$  subject to a minimum designed reliability  $Z$ . If  $S'$  is not kept within the acceptance designed reliability level,  $Z : R(S') \geq Z$ ; then some modifications are needed to obtain the optimum number of spares in the system.

### 3 - Application Example :

Consider a transformer substation which is to connect 100MW generating system to a transmission network[8]. This substation consists of a single bank contains three single phase transformers. The bank is considered to have failed totally and must be removed from service if any one of these transformers fails. The following data are available :-

Failure rate  $\lambda = 1.5, 1, 0.5$ , and  $0.1$  failure/year.

Repair rate  $u = 4, 8, 12$  repairs/year.

(These being equivalent to repair times of about 3 months, 1.5 month and 1 month).

The results are shown in table (1) which indicates the variation of  $R(s)$ ,  $C(s)$  with  $S$ ,  $\lambda$  and  $u$  if  $C = 10$  price units and  $x/y = 0.01$ , the number of spares ( $S'$ ) that provides the minimum cost is also shown. Figs.(1a, 1b, 1c and 1d) show the relation between the cost and failure rate ( $\lambda$ ) for a different number of spares and different repair rates ( $u$ ). The results can be summarized as shown in fig. (1)

### 4 - Fault Coverage and Common Cause Failure :

In some cases the fault detection and switching mechanism might not be able to identify that a unit is faulty, remove it and replace it with a fault free one, this is termed as imperfect coverage. Also the simultaneous failure of all units is another aspect which must be considered in the assessment.

The reliability of the system including both effects is given by (6) as follows:-

$$R_c(s) = P_f \sum_{r=3}^{s+3} b \inf(r, p, s+3) P_c^{s+3-r} \quad (6)$$

Now, we would like to obtain the optimal value of  $S$  such that the system reliability  $R_c(s)$  is maximized.

Table (1) shows a comparison between reliability results obtained From ref [7] for TMRWS and results obtained for a generating transformer substation system.

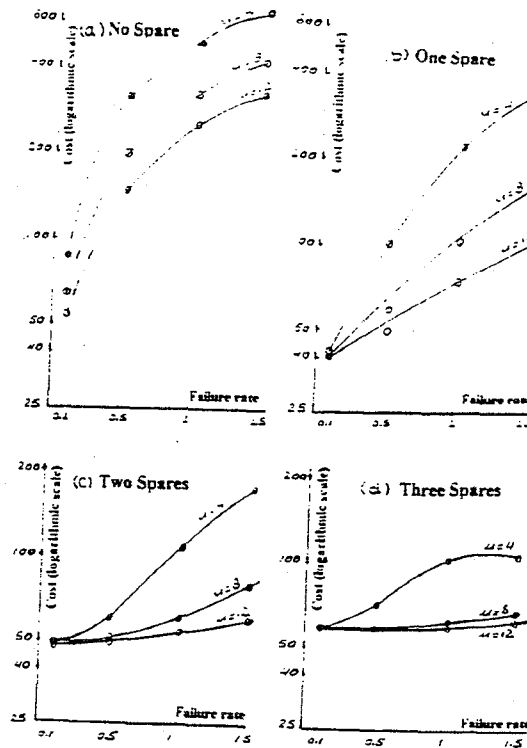


Fig. 1a, 1b, 1c and 1d show the relation between the cost and failure rate (a) for a different number of spares and different repair rates (μ).

**Table (1)**  
Comparison of Reliability results  
for  $P = 0.9928$ ,  $P_c = 0.99$ ,  $P_f = 0.9994$

No. of spares	Results from ref [7] for TMRS	Results for a generating transformer substation
0	0.999032	0.977912
1	0.999110	0.998801
2	0.999040	0.999032
3	0.998968	0.998961
4	0.998824	0.998882

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