SUBSTATION DISTRIBUTION TRANSFORMERS FAILURES AND SPARES

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Abstract: Electric utilities should have a sufficient number of spare transformers to backup substation distribution transformers to replace transformers that fail and require factory rebuild or replacement. To identify such a number, the statistical methodology was developed to analyze available failure data for different groups of transformer. That methodology enables the estimation of future numbers of failures with associated probabilities, recommends the proper number of spares, identifies the necessity and shows the means to shorten the transformer's replacement time.

Key Words: substation distribution transformers, statistical methodology, failure prediction, number of spares, replacement time.

I. INTRODUCTION

Substation distribution transformers supply power to a large number of electric utility customers. Failures of these transformers are a concern to utility management, because the long outage duration is unacceptable to customers. Usually, after the failure of a large distribution transformer, the utility personnel brings a mobile transformer to the substation, reconnects the load to the mobile transformer and makes recommendations regarding the type of repair (field repair vs. factory rebuild) and/or replacement of the failed transformer using the inventory stock. Therefore, the size of inventory stock should be adequate to replace failed transformers, which are removed from service (scrapped or requiring factory rebuild), because a complete replacement could last up to 18 months. It is undesirable to use mobile transformers as spares for extended periods of time for the following reasons:

1) Mobile transformers are usually needed for maintenance purposes.
2) Their cost is much higher than the cost of stationary transformers.
3) They have lower reliability than that of stationary transformers due to their more complex design.

This article is devoted to the establishment of a methodology to predict the number of future failures per year in different groups of substation distribution transformers. This methodology enables the prediction of the proper number of stationary spare distribution transformers required. A similar procedure could be used to establish the proper number of mobile transformers.

It should be noted that a well-established approach to solve standard inventory problems is to calculate the expected cost to a power utility of keeping any numbers of spares. Such a cost is the sum of two subcosts: one is associated with the cost of owning a spare, another is associated with the cost of nonavailability of a spare. Minimization of the expected cost regarding the number of spares produces the optimal number of spares and the minimum cost to the utility. Unavailability of distribution transformer spares can lead to the inability of the utility to supply power to its customers, which is unacceptable. Therefore, the cost of non-availability of spares could be considered as infinite.

This methodology is based on historical data regarding the number of observed failures in past years. The homogeneous Poisson process (HPP) is applied as a mathematical model of the failure process under consideration. It should be noted that:

1) There is no guarantee that the chosen model adequately represents the failure data. Therefore, the χ^2 test estimates numerically the quality of this representation.
2) There is no guarantee that all failure data is generated by the same HPP. The second of our two examples is devoted to exactly such a case.

If all failure data is naturally divided into subsets with different characteristics, a question arises about the events which lead to the observed difference. Therefore, the analysis may reveal a non-homogeneity of the failure data and the most appropriate value of the parameter of HPP must be chosen. It is assumed that more recent data has greater weight than the older data. A balanced approach to choose the proper Poisson parameter is outlined in the main body of the article.

A simple formula for the recommended number of spares which shows a dependence on the cycle time is introduced. The number of spares can be varied according to the degree of confidence desired. Most companies have the resources to implement the shortened cycle time without additional capital expenditures by modifying only their work practices.

The presented approach could be applied to a large variety of inventory problems regarding power equipment.

II. GENERAL METHODOLOGY

The whole population of substation distribution transformers is divided into groups having the same voltage but perhaps different MVA ratings in which the following holds:

a) Transformers with higher MVA rating could replace failed transformers with a lower MVA rating.
b) There is no vast difference between first cost of transformers that belong to the same group (e.g. the ratio of the first costs, $r \in (½,2)$).

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c) The sizes of the transformers in the same group should be comparable in the sense that one could be physically replaced by another one.

For any such group, the failure data could be represented in the following table:

Table 1

Year of Observation	1	2	3	...	m
Calendar Year	CY_1	CY_2	CY_3	...	CY_m
Factory Rebuild or Scrap Failures	fr_1	fr_2	fr_3	...	fr_m
Total Number of Failures	ft_1	ft_2	ft_3	...	ft_m
Exposure to Risk	T_1	T_2	T_3	...	T_m

where the total number of failures includes factory rebuild, scrap and field repairable transformers.

The number of transformer-years T_j per j^{th} year of observation may not be known due to incomplete record keeping. In such a case, numbers could be approximated using the known coefficient α of the power system growth. Therefore, if the exposure to risk (in transformer-years) during the $(m+r)^{th}$ year of observation is known and equal to T_{m+r}, then

(1) $$T_{m-j} \approx \frac{T_{m+r}}{(1+\alpha)^{r+j}}, \quad r = 0,1,\cdots$$

The goal is to establish a mathematical model which approximates the random number of failures $\{fr_j\}$ or $\{ft_j\}$. The homogeneous Poisson process (HPP) will be used to represent a random distribution of events in which each time period of equal duration has an equal chance of witnessing an event, and the occurrence of any other event. (For a more rigorous definition of HPP see [5].)

If the random number of failures f per j^{th} year of observation could be adequately represented by a Poisson process, then it holds

(2) $$Pr(n_f = f) = \frac{e^{-\lambda T_j}(\lambda T_j)^f}{f!},$$

$$f = 0,1,2,\cdots, \quad j = 1,2,\cdots$$

with the parameter of the process λ, which is the average number of failures per a transformer-year could be approximated as the Maximum Likelihood Estimator (MLE) [3]:

(3) $$\lambda = \frac{\sum_{j=1}^{m} f_j}{\sum_{j=1}^{m} T_j} = \frac{\sum_{j=1}^{m} f_j}{\frac{T_{m+r}}{(1+\alpha)^r}\sum_{j=0}^{m-1}(1+\alpha)^{-j}}$$

$$= \frac{\sum_{j=1}^{m} f_j}{T_{m+r}\frac{(1+\alpha)^m - 1}{\alpha(1+\alpha)^{m+r-1}}} \approx \frac{1+(m+r-1)\alpha}{m}\,\frac{\sum_{j=1}^{m} f_j}{T_{m+r}}$$

However, there is no assurance that all failure data in the Table 1 are generated by the same failure mechanism. In addition, the recent data is more valuable than the old data. To express this in mathematical form, the following sequence of estimators is computed:

(4) $$\lambda_s = \frac{\sum_{j=s}^{m} f_j}{\sum_{j=s}^{m} T_j}, \quad s = 1,\cdots,m$$

For s=1, $\lambda_1 = \lambda$. Now it should be decided which of the λ_j is the most appropriate value. To solve this problem (at least partially), the χ^2 test is applied. It provides a tool to compare the observed number of failures with the expected number of failures. The χ^2 test is based on computation of the following statistics:

(5) $$Q_r = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed number of failures during the i^{th} year of observation, E_i is the expected number of failures from the accepted Poisson model, and n is the number of years of observation. It is proven in statistical texts [2], that as n tends to infinity, Q_r has limiting distribution $\chi^2(r)$. A random variable of the continuous type that has the probability density function is said to have a $\chi^2(r)$ distribution with r degrees of freedom. Here

(6) $$f(x) = \frac{1}{\Gamma\left(\frac{r}{2}\right) 2^{\frac{r}{2}}} x^{\frac{r}{2}-1} e^{-\frac{x}{2}}, \quad 0 < x < \infty$$

the Euler's gamma function is defined as

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy.$$

$\chi^2(r)$ distribution is tabulated in many statistical texts. The number of degrees of freedom, r is given by

a) r = n-1, if the expected number of failures can be computed without having to estimate parameters from sample statistics. One is subtracted from n because the overall number of failures is known and therefore if n-1 number of failures is known, the remaining number can be established.

b) r = n-m-1, if the expected number of failures can be computed only by estimating m parameters from sample statistics.

Consequently, because the parameter of the process is estimated from sample data using the formula (4), the number of degrees of freedom in our case is equal to r = n-2. Because the $\chi^2(r)$ is the continuous distribution applied to discrete data, the following Yates' correction is recommended to modify the formula (5) in the case of r = 1:

(7) $$Q_1 = \sum_{i=1}^{n} \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

If positive E_i values are sufficiently small, the calculated Q_r is biased because it is larger than the theoretical $\chi^2(r)$ value it is supposed to estimate. This situation is clearly undesirable, and there is no consensus on the limits of the applicability of the χ^2 test. A thorough analysis by Cochran concludes that no expected values should be less than 1.0 and no more than 20% of the expected values should be less than 5.0. Recent analysis shows this "rule of thumb" is good.

On the other hand, it is claimed that if E_i is of the order of 2 or 3, the χ^2 test procedure is applicable. Even if $E_i = 1$ but there are sufficiently large number of classes n (e.g. n= 10) the χ^2 test produces satisfactory results [6, ch 11].

Substation distribution transformer failure data frequently do not satisfy the above mentioned limitations. One way to offset these limitations is to group failure data and corresponding exposures for several years.

However, it is not a very simple task. The major problem here is the loss of information when failure data and exposures are combined. Indeed, in the limiting case of combining all failure data and all exposures, there appear two numbers: the number of all failures through the period of observation f_Σ and the overall exposure (in transformer-years) through the same period T_Σ. In this case it is hard to check whether the HPP with the parameter $\hat{\lambda} = f_\Sigma / T_\Sigma$ adequately represents the failure process; essential failure information is gone. On the other hand, it is hard to use the original failure information due to the above mentioned χ^2 test limitation. Therefore, the failure data should be combined to satisfy the limitation needed to successfully apply the χ^2 test and, at the same time, preserve statistical information as much as possible. Suggested optimal procedure is placed into Appendix B.

The statistic Q_r (5) shows the correlation between actual number of failures and expected number of failures, calculation of which depends only upon the estimation of the Poisson parameter λ. Indeed, the expected number of failures with exposure to risk T could be approximated by

$$E\,[n_f] \approx \hat{\lambda}_j T \tag{8}$$

The larger value of Q_r indicates the greater disagreement between the data and hypothesis about the parameter λ. The probability $Pr(\chi^2 \geq Q_r)$ serves as such a measure of disagreement. If this probability is small, then a disagreement between the data and expected numbers of failures, calculated using (8), is large. Estimation of $Pr(\chi^2 \geq Q_r)$ useful for numerical evaluations is given in Appendix A.

The following step is to choose a proper value of $\hat{\lambda}$ among all $\hat{\lambda}_j$, j=1, ..., m. The chosen value should:

a) represent a sufficiently large subset of failure data and therefore not be influenced by strong random fluctuations;
b) be reasonably conservative because of relatively low precision of statistical data.

After the choice of $\hat{\lambda}$ it is possible to estimate the number of future failures with associated probabilities. To achieve this, the following cumulative probabilities are tabulated,

$$Pr(n_f \leq N) = e^{-\hat{\lambda} T} \sum_{j=0}^{N} \frac{(\hat{\lambda} T)^j}{j!} \tag{9}$$

and after that the smallest value of N such that

$$Pr(n_f \leq N) \geq \beta, \tag{10}$$

or, equivalently,

$$Pr(n_f > N) \leq 1 - \beta, \tag{11}$$

for sufficiently large $\beta \in (0,1)$ is chosen. The value β could be interpreted as a measure of confidence. Indeed, for values of β close to one, $Pr(n_f>N)$ is very small and we are confident that with rare exceptions the number of failures n_f does not exceed N. Interpreting β as frequency, it is possible to estimate the rate of occurrence of the above mentioned "rare exceptions".

Consider now the examples in which presented methodology is applied to real life situations.

III. EXAMPLES

Example 1. Consider the group of 69/12 kV transformers, nominally rated 10.6 MVA or smaller. The group consists of 136 substation distribution transformers with load tap changers (LTC). Table 1.1, which contains all failure data looks as follows:

Table 1.1

Year of Observation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Calendar Year	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
Factory Repairable or Scrap Failures	1	2	0	1	2	2	0	0	0	1	1	0	1	2	0	0	2
Exposure to Risk	111.7	112.9	114.2	115.4	116.7	118.0	119.3	120.6	121.9	123.3	124.6	126.0	127.4	128.8	130.2	131.6	133.1

The following Table 2 consists of estimators { $\hat{\lambda}_j$ } calculated by formulas (4):

Table 2 MLEs of the Parameter λ

Year of Start of Observation	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
$\hat{\lambda}$	.0072	.0071	.0065	.0069	.0068	.0060	.0050	.0055	.0061	.0068	.0067	.0064	.0077	.0076	.0051	.0076	.0150

In Table 2, $\bar{\lambda}_1 = 0.0072$ was calculated using all failure data from Table 1.1, and e.g. $\bar{\lambda}_3 = 0.0065$ was calculated using failure data from 1978 to 1992.

Applying the χ^2 test, the new group of the failure data is established using the approach of Appendix B. Assuming L=3, the following is true: $\bar{\lambda}_1 = 0.0072$, $L / \bar{\lambda}_1 = 416.67$, $m_1 = 4$; $\bar{\lambda}_5 = 0.0068$, $L / \bar{\lambda}_5 = 441.18$, $m_2 = 8$; $\bar{\lambda}_9 = 0.0061$, $L / \bar{\lambda}_9 = 491.80$, $m_3 = 12$; $\bar{\lambda}_{13} = 0.0077$, $L / \bar{\lambda}_{13} = 389.61$, $m_4 = n = 17$. Consequently, the new grouping of the failure data is established in the following table.

Table 3

Years of Observation	1-4	5-8	9-12	13-17
Number of Failures	4	4	2	5
Exposure to Risk	454.19	474.51	495.73	650.97

In the following Table 4, expected numbers of failures E_i are calculated using equation (8).

Table 4 Expected Number of Failures

$\bar{\lambda}_1 = 0.0072$	3.27	3.42	3.57	4.69
$\bar{\lambda}_5 = 0.0068$		3.23	3.37	4.43
$\bar{\lambda}_{13} = 0.0077$	3.50	3.65	3.82	5.01

Here $\bar{\lambda}_1$ and $\bar{\lambda}_5$ are chosen from Table 2 in accordance with the grouping of the data in Table 3. One might see that the choice of $\bar{\lambda}_j$ in Table 3 depends upon the grouping of the failure data. Actually any $\bar{\lambda}_j$ from Table 2 may be used in Table 4. Only results from statistical test could justify inclusion of any number into Table 4 to compute expectations and establish the quality of included estimator. To illustrate such a point of view, the most conservative estimate $\bar{\lambda}_{13}$ was included into this table. The value of $\bar{\lambda}_{17} = 0.0150$ was not included in Table 4 because it reflects only the last year's failure data, and does not characterize an essential subset of the data.

Computing statistics Q_{n-2} for all estimators $\bar{\lambda}_j$ in Table 4, the following could be observed:

a) For $\bar{\lambda}_1 = 0.0072$, $Q_2(\bar{\lambda}_1) = 0.97$. Linearly interpolating the following entries from the statistical table: $\Pr(\chi^2 \geq 0.713) = 0.7$ and $\Pr(\chi^2 \geq 1.386) = 0.5$ the probability $\Pr(\chi^2 \geq 0.97) = 0.62$ is obtained.

b) For $\bar{\lambda}_5 = 0.0068$, $Q_1(\bar{\lambda}_5) = 0.25$. Using similar procedure the following probability $\Pr(\chi^2 \geq 0.25) = 0.63$ is obtained.

c) For $\bar{\lambda}_{13} = 0.0077$, $Q_2(\bar{\lambda}_{13}) = {}_2Q(\bar{\lambda}_1)$. Therefore, estimators $\bar{\lambda}_1$ and $\bar{\lambda}_{13}$ are equivalent from the viewpoint of representation of all failure data. Applying $\bar{\lambda}_{13}$ only to the exposure to risk from the 5th to the 17th years of observation, $Q_1(\bar{\lambda}_{13}) = 0.51$ is computed. This already demonstrates the superiority of the estimator $\bar{\lambda}_5$ over $\bar{\lambda}_{13}$ in representation of the failure data in years 5-17 of observation.

The question: which of the estimators $\bar{\lambda}_1$, $\bar{\lambda}_5$ or $\bar{\lambda}_{13}$ should be selected for computation of probabilities for the future number of failures. Fortunately, the results of computations are stable relative to small variations of estimators $\bar{\lambda}_j$. To demonstrate this, the following table of calculated exposures for the future 5 years will be used.

Table 5 Calculated Exposures

Year	1995	1996	1997	1998	1999
Exposure	137.50	139.01	140.54	142.08	143.65
	E_1	E_2	E_3	E_4	E_5

In the following Table 6 the function (9) is tabulated for $\bar{\lambda}_1$, $\bar{\lambda}_5$, $\bar{\lambda}_{13}$ and all exposures from Table 5. Such a tabulation enables the establishment of the future number of failures with associated probabilities.

Table 6 Probability Limits $\beta = \Pr(n_f \leq N)$

	N	1	2	3	4	5
$\bar{\lambda}_1 = 0.0072$	E_1	73.81%	92.08%	98.14%	99.64%	99.94%
	E_2	73.40%	91.88%	98.07%	99.63%	99.94%
	E_3	73.00%	91.68%	98.00%	99.61%	99.94%
	E_4	72.59%	91.47%	97.93%	99.59%	99.93%
	E_5	72.17%	91.25%	97.86%	99.57%	99.93%
$\bar{\lambda}_5 = 0.0068$	E_1	76.04%	93.16%	98.49%	99.73%	99.96%
	E_2	75.66%	92.98%	98.43%	99.71%	99.96%
	E_3	75.28%	92.80%	98.37%	99.70%	99.95%
	E_4	74.90%	92.62%	98.31%	99.69%	99.95%
	E_5	74.51%	92.43%	98.25%	99.67%	99.95%
$\bar{\lambda}_{13} = 0.0077$	E_1	71.51%	90.91%	97.74%	99.54%	99.92%
	E_2	71.09%	90.68%	97.66%	99.52%	99.92%
	E_3	70.66%	90.45%	97.57%	99.50%	99.91%
	E_4	70.22%	90.22%	97.49%	99.47%	99.91%
	E_5	69.78%	89.97%	97.40%	99.42%	99.90%

It is clear that for the greatest value of $\bar{\lambda}_j$ the probability limit $\Pr(n_f \leq N)$ is smallest. Then, taking this $\bar{\lambda}_j$, the most conservative result is obtained. For example, observing the part of Table 6 for $\bar{\lambda}_{13}$, one can see that with probability not less than 97.4% transformers of the considered group will not fail more frequently than 3 failures per year during the next 5 years. These are the transformers which fail in the factory rebuild and scrap modes. Considering all failures including field repairable ones and using the same methodology, one might predict yearly number of all failures. This is needed to help establish the quantity of mobile transformers, which are used for immediate replacement of any failed transformer.

If the probability of having not more than 2 failures per year equal to 90.91% is satisfactory then, at least in 1995, only 2 spares could be recommended to offset these failures.

Example 2. Consider the group of 69/12 kV transformers, nominally rated 10.6 MVA or smaller without LTC. This group consists of 240 transformers. Table 1.2 with the failure data looks as follows:

Table 1.2

Year of Obser-vation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Calendar Year	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
Factory Repairable or Scrap Failures	3	4	4	5	4	3	4	5	1	4	0	3	2	1	1	0	2
Exposure to Risk	197.1	199.3	201.5	203.7	205.9	208.2	210.5	212.8	215.13	217.5	219.9	222.3	224.8	227.2	229.7	232.3	234.2

Table 7 MLEs of the Parameter λ

Year of Start of Observation	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
$\hat{\lambda}$	.0126	.0124	.0119	.0114	.0105	.0098	.0094	.0085	.0069	.0072	.0057	.0066	.0052	.0043	.0043	.0043	.0085

Observation of Table 7 shows a significant systematic variation between $\hat{\lambda}_i$. Between 1976 and 1984, the number of failures monotonically decrease. After that they stabilize. Inquiry on this interesting failure activity reveals that a large part of these transformers were acquired from the same vendor. The transformers had a serious defect in their design, which resulted in high failure activity (common cause failures). These transformers were redesigned and rebuilt. Clearly, newly acquired transformers of this group were free of this defect. Consequently, it should not be expected that the unique value of $\hat{\lambda}$ would adequately represent all failure data from Table 1.2.

Applying Appendix B for L=4, the following chain of computations comes out: L=4, $\hat{\lambda}_1$ = 0.0126, L / $\hat{\lambda}_1$ = 317.46, $m_1 = 2$; $\hat{\lambda}_3$ = 0.0119, L / $\hat{\lambda}_3$ = 336.13, $m_2 = 4$; $\hat{\lambda}_5$ = 0.0105, L / $\hat{\lambda}_5$ = 380.95, $m_3 = 6$; $\hat{\lambda}_7$ = 0.0094, L / $\hat{\lambda}_7$ = 425.53, m_4 = 9; $\hat{\lambda}_{10}$ = 0.0072, L / $\hat{\lambda}_{10}$ = 555.56, $m_5 = 12$; $\hat{\lambda}_{13}$ = 0.0052, L / $\hat{\lambda}_{13}$ = 769.23, m_6 = 17. These computations lead to the establishment of the following:

Table 8

Years of Obser-vation	1-2	3-4	5-6	7-9	10-12	13-17
Number of Failures	7	9	7	10	7	6
Exposure to Risk	396.37	405.14	414.10	638.39	659.7	11148.8

Now, the table of expected numbers of failures looks as follows:

Table 9 Expected Number of Failures

$\hat{\lambda}_1$ = 0.0126	4.99	5.10	5.22	8.04	8.31	14.47
$\hat{\lambda}_3$ = 0.0119		4.82	4.93	7.60	7.85	13.67
$\hat{\lambda}_5$ = 0.0105			4.35	6.70	6.93	12.06
$\hat{\lambda}_7$ = 0.0094				6.00	6.02	10.80

The last row corresponds to the χ^2 test with 1 degree of freedom. Therefore, the $\hat{\lambda}_{10}$ and $\hat{\lambda}_{13}$ are not included into Table 9. Just mere observation of Tables 8 and 9 reveals the vast discrepancy between the actual number of failures and the expected number of failures. For example, the corrected formula (7) produces $Q_1(\hat{\lambda}_7) = 3.77$ or $\Pr(\chi^2 \geq 3.77) \approx 0.05$. Application of the χ^2 test to other $\hat{\lambda}_j$ does not exhibit a better result. That shows that large $\hat{\lambda}_j$, j = 1, 3, 5, 7 can not generate failure data observed since 1984. Therefore, it makes sense to analyze the data starting from 1984. Applying Appendix B with L = 3 the following table is obtained.

Table 10

Years of Observation	9-10	11-13	14-17
Number of Failures	5	5	4
Exposure to Risk	432.63	666.95	697.67

The table of expected number of failures is:

Table 11

λ_9 = 0.0069	2.99	4.60	4.81
λ_{10} = 0.0072	3.11	4.80	5.02
λ_{17} = 0.0085	3.68	5.67	5.93

It was written earlier that it is lawful to test any λ_j for the goodness of fit to the failure data. Here two conservative estimates λ_{10} and λ_{17} are added.

Applying the corrected version of the χ^2 test (7) to the data of Tables 10 and 11, the results could be obtained after the linear interpolation of the χ^2 statistical table entries:

a) λ_9 = 0.0069, $Q_1(\lambda_9) = 0.77$, $\Pr(\chi^2 \geq 0.77) = 0.398$;
b) λ_{10} = 0.0072, $Q_1(\lambda_{10}) = 0.69$, $\Pr(\chi^2 \geq 0.69) = 0.424$;
c) λ_{17} = 0.0085, $Q_1(\lambda_{17}) = 0.53$, $\Pr(\chi^2 \geq 0.53) = 0.476$.

That shows that λ_{17} provides the best fit for the failure data since 1984.

The table of calculated exposures is needed to compute the table of probability limits.

Table 12 Calculated Exposures

Year	1995	1996	1997	1998	1999
Exposure	242.64	245.61	248.01	250.74	253.49
	E_1	E_2	E_3	E_4	E_5

Table 13 Probability Limits $\beta = \Pr(n_f \leq N)$

	N	1	2	3	4
λ_9 =0.0069	E_1	49.99%	76.28%	91.00%	97.17%
	E_2	49.41%	75.80%	90.72%	97.06%
	E_3	48.83%	75.30%	90.44%	96.94%
	E_4	48.25%	74.80%	90.16%	96.81%
	E_5	47.67%	74.30%	89.86%	96.68%
λ_{10} =0.0072	E_1	47.96%	74.55%	90.01%	96.75%
	E_2	47.38%	74.04%	89.71%	96.62%
	E_3	46.80%	73.52%	89.40%	96.48%
	E_4	46.21%	73.00%	89.09%	96.34%
	E_5	45.63%	72.47%	88.77%	96.20%
λ_{17} =0.0085	E_1	38.82%	65.86%	84.49%	94.11%
	E_2	38.23%	65.25%	84.06%	93.89%
	E_3	37.64%	64.63%	83.63%	93.66%
	E_4	37.05%	64.00%	83.18%	93.43%
	E_5	36.46%	63.37%	82.73%	93.18%

In this case, four spares are adequate with probability not smaller than 93.18% what is valid for each of the future 5 years.

Both examples demonstrate significant stability of result relatively to substantial variations of value of λ_j.

IV. LTC VS. NON-LTC FAILURE ACTIVITY

It is commonly believed that transformers with LTC are less reliable than similar transformers without LTC. The core of the problem is not failure of LTC, because these failures are usually field repairable. The importance of the problem of increasing failure activity of LTC transformers lies in higher frequency of the winding failures for LTC transformers in comparison with Non-LTC ones. This higher frequency of winding failures for LTC transformers can be attributed to the more complicated designs of this winding, greater possibility of unbalanced forces and exposure of tapped turns to short circuit forces produced by misoperation of the tap changing mechanism.

Despite the plausibility of engineering explanations, this paper relies on numerical estimations of mathematical statistics. Therefore, available failure data for similar groups of substation distribution transformers, which differ only by LTC installation, is considered on a quantitative basis for analysis only in an attempt to prove the hypothesis of higher winding failure activity for LTC transformers.

Unfortunately available data does not allow a rigorous proof of the above mentioned hypothesis. However, comparison of Tables 2 and 7 starting 1988 and 1992 shows that corresponding values of parameter λ are greater for LTC transformers. As written previously, greater entries in Table 7 in the first years of observation could be explained by common cause failures in the Non-LTC group of transformers under consideration.

Two other similar groups of LTC and Non-LTC transformers exist for comparison. The first group consists of 86 34.5/12 kV LTC transformers rated 10.6 MVA or smaller. The second group consists of 98 34.5/12 kV Non-LTC transformers rated 10.6 MVA or smaller.

Compare MLE values of parameter λ_j for the groups in the following:

Table 14 MLE of the Parameter λ for Two 34.5/12 kV Groups

Year of Start of Observation	76	77	78	79	80	81	82
λ (LTC)	.0069	.0056	.0060	.0046	.0039	.0042	.0046
λ (Non-LTC)	.0040	.0042	.0045	.0048	.0043	.0037	.0020

83	84	85	86	87	88	89	90	91	92
.0050	.0041	.0046	.0035	.0041	.0019	.0060	.0040	.0060	.0000
.0022	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

The comparison identifies LTC transformers as more prone to winding failures than Non-LTC transformers. Here all factory rebuild failures are considered to be winding failures — the assumption, which is sufficiently close to truth.

V. From the Number of Failures to the Number of Spares

The required number of spares depends on the mean replenishment (cycle) time, which goes on from a transformer failure to the return of the repaired/newly purchased transformer into operational state. The mean cycle time is equal to

$$\tau = t_e + t_r + t_T$$

where t_e is the time which is required by electric utility personnel to evaluate the state of a failed transformer, to perform administrative activities as financing of repair or purchase of a new spare, approvals, etc., to make an arrangement with a repair facility to rebuild the failed transformer, if necessary, to organize a transportation to and from the repair facility; t_r is the time of actual rebuild or purchase of a transformer; t_T is a transportation time.

It is usually difficult to shorten the times t_r and t_T without additional expenditures. Contrary to this the time t_e, which usually is not smaller than 20% of the mean cycle time τ, could be shortened by improvement of the organization of work virtually without additional expenditures. Such an improvement could be achieved by clear identification of the job assignments for the transformer rebuild procedure. Strict job schedule should be developed and make clear to all involved parties. Auxiliary and preliminary information on all stages of repair/replacement and necessary data should be collected beforehand. It is very important that funds for rebuilds/replacements be available and secured for that purpose only; unnecessary and redundant approvals be eliminated, clerical work be shortened, etc. The optimal scheduling methodology as e.g. probablistic PERT technique [4] could be recommended.

Let us estimate the number of spares for the group of similar operating transformers.

During a cycle of the duration τ, which starts as a failure of a distribution transformer in a factory repairable or scrap mode, the following inequality

$$(s-1) - \lambda m\tau + 1 \geq 0$$

should hold to prevent lack of spares (in average). Indeed, there were s spares before the beginning of the cycle. One failure has occurred immediately;

$$E[n_f(\tau)] = \lambda m\tau$$

failures are expected during cycle duration τ amoung the group of m transformers; and repaired or newly acquired transformer is to be returned to operation at the end of the cycle. From this inequality the following minimum spare requirement holds:

$$s \approx \{\ \lambda m\tau\} \qquad (12)$$

where {x} is the smallest integer greater than or equal to x. If $\tau \ll 1$ then with high probability the number of required spares is equal to zero because the mobile is available. It is possible to develop a more rigorous approach to the problem of establishment of optimal inventory for large distribution transformers taking into account stochastic variation of the cycle time τ. Such a development might require use of queuing theory .

VI. APPENDIX A. Estimation of $Pr(\chi^2 \geq Q_r)$

The probability

$$Pr(\chi^2 \geq Q_r) = [2^{\frac{r}{2}}\Gamma(\frac{r}{2})]^{-1}\int_{Q_r}^{\infty} x^{\frac{r}{2}-1} e^{-\frac{x}{2}} dx$$

where r is a positive integer number and $Q_r \geq 1$, could be estimated from statistical tables. This was done in the main text. However, for developing of a software package and for the achievement of any desired accuracy, the following approaches could be used.

For r > 30 it holds the following useful approximation:

$$Pr(\chi^2 \geq Q_r) \approx \frac{1}{\sqrt{2\pi}}\int_{x_1}^{\infty} e^{-\frac{t^2}{2}} dt,$$

where

$$x_1 = \left[(\frac{Q_r}{r})^{\frac{1}{3}} - (1 - \frac{2}{9r})\right] \Big/ \sqrt{\frac{2}{9r}}$$

These expressions could be employed for numerical computations.

It is not hard to compute numerically $Pr(\chi^2 \geq Q_r)$ using theory of gamma function. Indeed, it holds

$$2^{\frac{r}{2}}\Gamma(\frac{r}{2}) = \begin{cases} 2^n(n-1)!\,, & if\ r=2n \\ 2\sqrt{2\pi}\prod_{j=0}^{n-1}\frac{n+j}{2}, & if\ r=2n+1 \end{cases}$$

The integral in $Pr(\chi^2 \geq Q_r)$ could be estimated as follows after integrating by parts:

$$I = \int_{Q_r}^{\infty} x^{\frac{r}{2}-1} e^{-\frac{x}{2}} dx = 2e^{-\frac{Q_r}{2}} Q_r^{\frac{r}{2}-1} \sum_{k=0}^{m-1}\prod_{j=1}^{k}(r-2j)$$
$$+ \prod_{j=1}^{m-1}(r-2j)\int_{Q_r}^{\infty} x^{\frac{r}{2}-m} e^{-\frac{x}{2}} dx,$$

where

$$m = [\frac{r}{2}] + 1$$

and [x] is the greatest integer less than or equal to x.

In this case r/2 < m. Splitting the integral I into the sum of two parts

$$\int_{Q_r}^{R} + \int_{R}^{\infty}$$

it is easy to estimate the second one and to establish a proper value of R to achieve a desired accuracy ϵ:

$$\int_{R}^{\infty} x^{\frac{r}{2}-m} e^{-\frac{x}{2}} dx < \int_{R}^{\infty} e^{-\frac{x}{2}} dx = 2e^{-\frac{R}{2}} < \epsilon,$$

Then, R should be chosen to satisfy the following inequality:

$$R > \left[2 \ln \frac{2}{\varepsilon}\right]$$

VII. APPENDIX B. OPTIMAL PROCEDURE TO COMBINE FAILURE DATA

Referring to the failure data in the Table 1, one can assume the data is grouped in such a manner that the limitation of the applicability of the χ^2 test is satisfied. This means that there exists the integer numbers $m_1, m_2, \ldots, m_r$ such that:

(B1)
$$\lambda_1 \sum_{j=1}^{m_1} T_j \geq L, \quad \lambda_1 \sum_{j=m_1+1}^{m_2} T_j \geq L, \quad \ldots \quad \lambda_1 \sum_{j=m_r+1}^{m_n} T_j \geq L$$
$$\lambda_{m_1+1} \sum_{j=m_1+1}^{m_2} T \geq L, \quad \ldots \quad \lambda_{m_1+1} \sum_{j=m_r+1}^{m_n} T_j \geq L$$
$$\ldots \quad \lambda_{m_r+1} \sum_{j=m_r+1}^{n} T_j \geq L$$

Here L is the low bound for expectations E_i required for the applicability of the χ^2 test. It is assumed here that L = 3, or 4 as was outlined earlier. Our goal is to establish the smallest numbers $m_1, m_2, \ldots, m_r$ (and the greatest number r) such that the inequalities (B1) are valid.

To achieve that, imagine that inequalities (B1) are split into vertical columns. Considering the first one, the following inequality appears

(B2)
$$\sum_{j=1}^{m_1} T_j \geq \frac{L}{\lambda_1}$$

from which the smallest m_1 is established. Second column

(B3)
$$\sum_{j=m_1+1}^{m_2} T_j \geq \frac{L}{\lambda_1} \; and$$
$$\sum_{j=m_1+1}^{m_2} T_j \geq \frac{L}{\lambda_{m_1+1}} \; or$$
$$\sum_{j=m_1+1}^{m_2} T_j \geq \max\left(\frac{L}{\lambda_1}, \frac{L}{\lambda_{m_1+1}}\right)$$

producesand because m_1 is already known, it is easy to establish a smallest value of m_2 which satisfies (B3). Such a procedure could be repeated up to establishment of the smallest value of m_r. If the last inequality in (B1) is not satisfied then the last group of data should be aggregated with the adjacent one.

VIII. CONCLUSIONS

1) Flexible statistical methodology for analysis and prediction of failure performance of substation distribution transformers is developed. This methodology does not require a complete homogeneity of the failure data, and is based on the χ^2 statistic for goodnes of fit.
2) Developed methodology is applied to establishment of the adequate number of spares for substation distribution transformers. Suggested number of spares is associated with the computed probabilities on the future number of failures.
3) Developed approach particularly fits the analysis of actual engineering failure data. It exhibits a good stability of results regarding the variance of Poisson parameter (rate).

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XI. BIOGRAPHIES

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