Technical Report Writing On

Interpolation Problems

Submitted By

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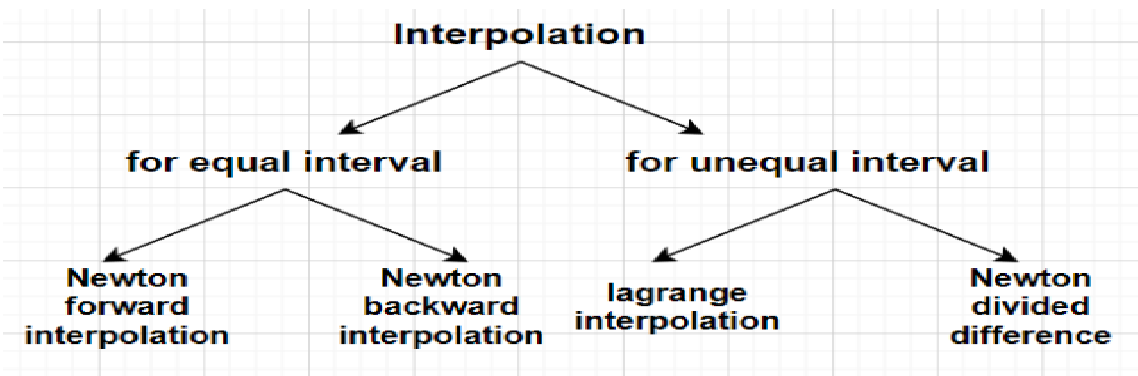
Abstract

This technical report delves into interpolation problems, exploring the significance of approximating values between known data points. Various interpolation techniques are examined, emphasizing their applications in diverse fields. The report presents a comprehensive analysis of interpolation methods, shedding light on their strengths and limitations.

Introduction

Interpolation serves as a fundamental tool in numerous scientific and engineering applications, facilitating the estimation of values between discrete data points. This report aims to provide a thorough understanding of interpolation problems, outlining the necessity of accurate predictions in scenarios where limited data is available. As technology advances, the demand for precise interpolation methods becomes increasingly crucial in fields such as finance, computer graphics, and scientific research.

Procedure and Discussion



Equal Interval Interpolation

Newton Forward Interpolation

Newton's forward interpolation is a method for approximating values between equally spaced data points. It involves constructing a forward-difference table to compute the coefficients of the interpolating polynomial. The resulting polynomial can then be used to estimate values at any desired point within the given interval.

Formula

$$p = \frac{x - x_0}{h}$$

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \cdot \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \cdot \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \cdot \Delta^4 y_0 + \ldots$$

Example

1. Find Solution using Newton's Forward Difference formula

x	f(x)
1891	46
1901	66
1911	81
1921	93
1931	101

x = 1895
Finding option 1. Value f(2)

Solution:
The value of table for x and y

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

Newton's forward difference interpolation method to find solution

Newton's forward difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		66 - 46 = 20			
1901	66		15 - 20 = -5		
		81 - 66 = 15		-3 - -5 = 2	
1911	81		12 - 15 = -3		-1 - 2 = -3
		93 - 81 = 12		-4 - -3 = -1	
1921	93		8 - 12 = -4		
		101 - 93 = 8			
1931	101				

The value of x at you want to find the $f(x)$: $x = 1895$

$h = x_1 - x_0 = 1901 - 1891 = 10$

$$p = \frac{x - x_0}{h} = \frac{1895 - 1891}{10} = 0.4$$

Newton's forward difference interpolation formula is

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!} \cdot \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \cdot \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \cdot \Delta^4 y_0$$

$$y(1895) = 46 + 0.4 \times 20 + \frac{0.4(0.4-1)}{2} \times -5 + \frac{0.4(0.4-1)(0.4-2)}{6} \times 2 + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24} \times -3$$

$$y(1895) = 46 + 8 + 0.6 + 0.128 + 0.1248$$

$$y(1895) = 54.8528$$

Solution of newton's forward interpolation method $y(1895) = 54.8528$

Advantages

Simplicity: Newton Forward Interpolation is relatively straightforward to implement and understand, making it accessible for learners and practitioners.

Efficiency: It is computationally efficient when dealing with equally spaced data points. Memory Usage: Requires less memory as compared to Lagrange Interpolation, especially when dealing with a large number of data points.

Disadvantages

Limited Applicability: Primarily suitable for equally spaced data, limiting its application in scenarios with unevenly spaced data points.

Sensitivity to Data Arrangement: The method's accuracy can be affected by the arrangement of data points, particularly if they are subject to fluctuations or outliers.

Newton Backward Interpolation

Similar to forward interpolation, Newton's backward interpolation is employed when dealing with equally spaced data points. The main difference lies in the direction of the difference table, which starts from the last data point. The backward interpolation method is particularly useful when the values are known in reverse chronological order.

Formula

$$p = \frac{x - x_n}{h}$$

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!} \cdot \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \cdot \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \cdot \nabla^4 y_n + \ldots$$

Example

1. Find Solution using Newton's Backward Difference formula

x	f(x)
1891	46
1901	66
1911	81
1921	93
1931	101

x = 1925

Solution:
The value of table for x and y

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

Newton's backward difference interpolation method to find solution

Newton's backward difference table is

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

The value of x at you want to find the $f(x) : x = 1925$

$h = x_1 - x_0 = 1901 - 1891 = 10$

$$p = \frac{x - x_n}{h} = \frac{1925 - 1931}{10} = -0.6$$

Newton's backward difference interpolation formula is

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!} \cdot \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \cdot \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \cdot \nabla^4 y_n$$

$$y(1925) = 101 + (-0.6) \times 8 + \frac{-0.6(-0.6+1)}{2} \times -4 + \frac{-0.6(-0.6+1)(-0.6+2)}{6} \times -1 + \frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{24} \times -3$$

$y(1925) = 101 - 4.8 + 0.48 + 0.056 + 0.1008$

$y(1925) = 96.8368$

Solution of newton's backward interpolation method $y(1925) = 96.8368$

Advantages

Applicability to Reverse Data: Well-suited for scenarios where data points are arranged in reverse chronological order.

Simplicity: Like Newton Forward Interpolation, it is relatively simple to implement.

Disadvantages

Limited Applicability: Similar to Newton Forward, it is primarily applicable to equally spaced data points in a reverse chronological order.

Sensitivity to Data Arrangement: Accuracy may be affected by the arrangement of data points.

Unequal Interval Interpolation

Lagrange Interpolation

Lagrange interpolation is a method applicable to unequally spaced data points. It involves constructing a polynomial of the least degree that passes through all the given data points. The Lagrange interpolating polynomial provides a flexible way to estimate values within the range of the data set, and its expression is directly influenced by the given data points.

Formula

$$y(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} \times y_2 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} \times y_n$$

Example

1. Find Solution using Lagrange's Interpolation formula

x	f(x)
300	2.4771
304	2.4829
305	2.4843
307	2.4871

x = 301

Solution:

The value of table for x and y

x	300	304	305	307
y	2.4771	2.4829	2.4843	2.4871

Lagrange's Interpolating Polynomial

The value of x at you want to find $P_n(x) : x = 301$

Lagrange's formula is

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$y(301) = \frac{(301-304)(301-305)(301-307)}{(300-304)(300-305)(300-307)} \times 2.4771 + \frac{(301-300)(301-305)(301-307)}{(304-300)(304-305)(304-307)} \times 2.4829 + \frac{(301-300)(301-304)(301-307)}{(305-300)(305-304)(305-307)} \times 2.4843 + \frac{(301-300)(301-304)(301-305)}{(307-300)(307-304)(307-305)} \times 2.4871$$

$$y(301) = \frac{(-3)(-4)(-6)}{(-4)(-5)(-7)} \times 2.4771 + \frac{(1)(-4)(-6)}{(4)(-1)(-3)} \times 2.4829 + \frac{(1)(-3)(-6)}{(5)(1)(-2)} \times 2.4843 + \frac{(1)(-3)(-4)}{(7)(3)(2)} \times 2.4871$$

$$y(301) = \frac{-72}{-140} \times 2.4771 + \frac{24}{12} \times 2.4829 + \frac{18}{-10} \times 2.4843 + \frac{12}{42} \times 2.4871$$

$$y(301) = 2.4786$$

Solution of the polynomial at point 301 is $y(301) = 2.4786$

Advantages

General Applicability: Suitable for both equally and unequally spaced data points, offering more flexibility.

Global Polynomial: Constructs a single polynomial that passes through all data points, providing a global representation of the data.

Disadvantages

Computational Complexity: The method can become computationally expensive, especially as the number of data points increases.
Run-Time Overhead: Lagrange polynomials involve multiple computations, potentially leading to higher run-time overhead compared to Newton methods.

Newton Divided Difference Interpolation

Newton's divided difference interpolation is another approach suitable for unequal intervals. This method involves constructing a divided difference table, which aids in determining the coefficients of the interpolating polynomial. Newton's divided difference formula is particularly efficient for computing interpolating polynomials when the data points are irregularly spaced.

Formula

$$y(x) = y_0 + \left(x - x_0\right)f\left[x_0, x_1\right] + \left(x - x_0\right)\left(x - x_1\right)f\left[x_0, x_1, x_2\right] + \ldots$$

Example

1. Find Solution using Newton's Divided Difference Interpolation formula

x	f(x)
300	2.4771
304	2.4829
305	2.4843
307	2.4871

x = 301

Solution:
The value of table for x and y

x	300	304	305	307
y	2.4771	2.4829	2.4843	2.4871

Numerical divided differences method to find solution

Newton's divided difference table is

x	y	1^{st} order	2^{nd} order
300	2.4771		
		$\frac{2.4829 - 2.4771}{304 - 300} = 0.0014$	
304	2.4829		$\frac{0.0014 - 0.0014}{305 - 300} = 0$
		$\frac{2.4843 - 2.4829}{305 - 304} = 0.0014$	
305	2.4843		$\frac{0.0014 - 0.0014}{307 - 304} = 0$
		$\frac{2.4871 - 2.4843}{307 - 305} = 0.0014$	
307	2.4871		

The value of x at you want to find the $f(x) : x = 301$

Newton's divided difference interpolation formula is

$$f(x) = y_0 + \left(x - x_0\right)f\left[x_0, x_1\right] + \left(x - x_0\right)\left(x - x_1\right)f\left[x_0, x_1, x_2\right]$$

$$y(301) = 2.4771 + (301 - 300) \times 0.0014 + (301 - 300)(301 - 304) \times 0$$

$$y(301) = 2.4771 + (1) \times 0.0014 + (1)(-3) \times 0$$

$$y(301) = 2.4771 + 0.0014 + 0$$

$$y(301) = 2.4785$$

Solution of divided difference interpolation method $y(301) = 2.4785$

Advantages:

Efficiency: Efficient for unequally spaced data points, especially when constructing a divided difference table.

Local Polynomial: Like Lagrange, constructs a polynomial that passes through all data points, offering a local representation.

Disadvantages:

Computational Complexity: Similar to Lagrange, the method can become computationally expensive with an increasing number of data points.

Memory Usage: May require more memory compared to Newton methods, especially for larger datasets.

Conclusion:

In conclusion, this report underscores the significance of interpolation in bridging data gaps, enabling more accurate predictions and analyses. Understanding the strengths and limitations of various interpolation methods is vital for informed decision-making in scientific and engineering endeavors. As technology evolves, continuous refinement and innovation in interpolation techniques will be crucial for addressing emerging challenges.

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