Importance of Computation In Physics

Jadavpur University

Contents

Chapter 1

Nonlinear Algebric Equation

1.1 Introduction

An expression is a number, a variable, or a combination of numbers and variables and operation symbols. An equation is made up of two expressions connected by an equal sign. For example, 3x - 2 is an expression. Also 5 + x is an expression. But an equation means that two different expressions are connected to each other by an equal to sign in between. For example, 3x - 2 = 5 + x is an equation.
Based on the degree and variable in the equations, they are classified as linear and nonlinear equations.

An equation in which the maximum degree of a term is one is called a linear equation. A linear equation forms a straight line on the graph.
An equation in which the maximum degree of a term is 2 or more than two is called a nonlinear equation. A nonlinear equation forms a curve on the graph.
In this chapter we will discuss about how to solve Nonlinear Equations in Computer.

1.2 The Bisection Method

In this section we consider a popular way of finding root named **Bisection Method**. This process suggest the way of finding a **root**, or a solution, of an equation of the form $f(x) = 0$, for a given function f.

Let, f is a continuous function defined on the interval $[a, b]$, with f (a) and f(b) of opposite sign. A number p exists in (a, b) with $f(p) = 0$.

Let, $a_1 = a$ and $b_1 = b$, and p_1 be the midpoint of $[a, b]$ then,

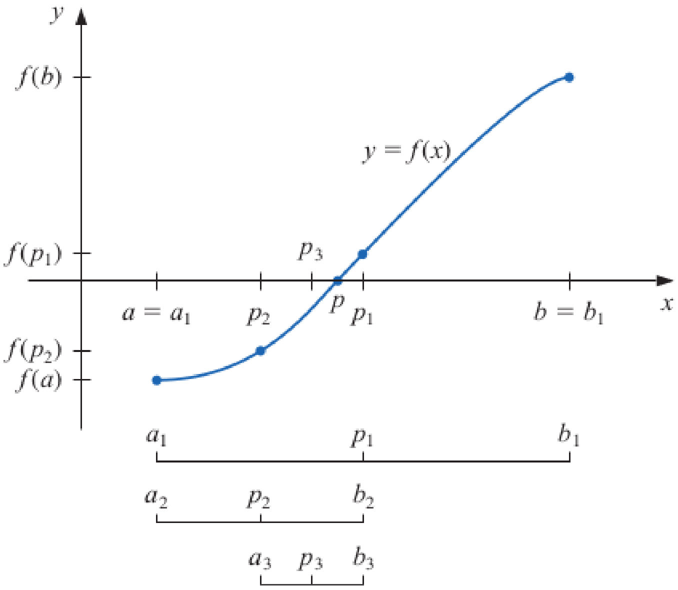
$$p_1 = \frac{a_1 + b_1}{2}$$

If $f(p_1) = 0$, then Root or $p = p_1$, and we are done.
If $f(p_1) \neq 0$, then $f(p_1)$ is positive or negetive,

- If $f(p_1)$ and $f(a_1)$ have the same sign, then $p \in (p_1, b_1)$. Then set $a_1 = p_1$ and b_1 keep same.

- If $f(p_1)$ and $f(a_1)$ have the opposite sign, then $p \in (a_1, p_1)$. Then set $b_1 = p_1$ and a_1 keep same.

Then reapply the process to the new interval $[a_1, b_1]$.



Lets write the code in FORTRAN...

```
subroutine nonlnr_eqn(n,interval,a1,b1,p1,root)
integer ::n
real ::a1, b1, p1, root
1  b1 =a1 + interval  ! interval value depends on problem
   n = 0          ! Variable to get relief from Infinte Loop
   if ( (f(a1) * f(b1)) > 0.0) then
      a1 = b1
      goto 1
   else
2     p1 = ((a1 + b1) / 2)
      n = n + 1
      if(n == 100)then
         write(*,*)"Infinite loop..."
         stop
      endif
      if(abs(f(p1)) < 0.001)then
         root = p1  ! This is the Solution
         goto 3
      else
         if(f(p1) < 0)then
            a1 = p1
            goto 2
         else
            b1 = p1
            goto 2
         endif
      endif
   endif
3  continue
end subroutine
```

Here ”f” is a function, like,

```
real function f(x)
implicit none
real::x
f = (x**2 - 2) ! Expression of function
end function
```

In this way we can find any nonlinear equations roots by changing the expression of function only.

1.3 Some examples on Bisection Method

PROBLEM 01: Find the value of $\sqrt{2}$

Let, $x = \sqrt{2}$ then we can write, $x^2 = 2$ and $x^2 - 2 = 0$.
This is a nonlinear equation in the form of $f(x) = 0$. So we can find the value of x by putting the left hand side expression in the function of the code. The value of x is the value of $\sqrt{2}$.
Lets set $a_1 = 0$ and interval value is 0.1
Main program -

```
program main
implicit none
real  :: a1,b1,p1,n,interval
real  :: f
real  :: root
   interval = 0.1
   a1 = 0
call nonlnr_eqn(n,interval,a1,b1,p1,root)
write(*,*)'Root=',root
end program main
```

Function -

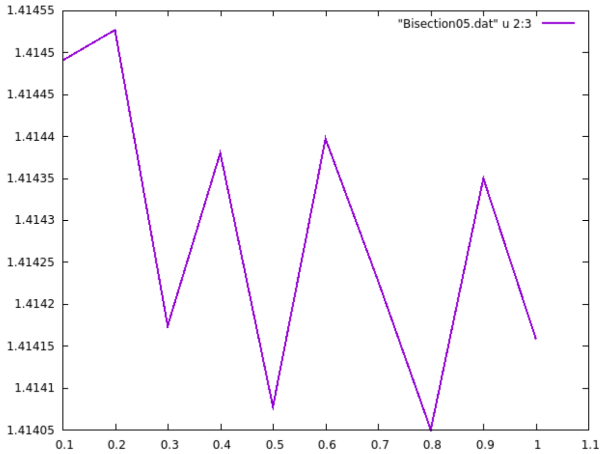
```
real function f(x)
implicit none
real::x
f = (x**2 - 2) ! Expression of function
end function
```

Subroutine of Bisection Method -

```
subroutine nonlnr_eqn(n,interval,a1,b1,p1,root)
integer ::n
real  ::a1, b1, p1, root
1  b1 =a1 + interval  ! interval value depends on problem
   n = 0          ! Variable to get relief from Infinte Loop
   if ( (f(a1) * f(b1)) > 0.0) then
      a1 = b1
      goto 1
   else
2     p1 = ((a1 + b1) / 2)
      n = n + 1
      if(n == 100)then
         write(*,*)"Infinite loop..."
         stop
      endif
      if(abs(f(p1)) < 0.001)then
         root = p1  ! This is the Solution
         goto 3
      else
         if(f(p1) < 0)then
            a1 = p1
            goto 2
         else
            b1 = p1
            goto 2
         endif
      endif
   endif
3  continue
end subroutine
```

From here we get the value of $\sqrt{2}$ is = 1.41448975
Being Numarical Solution, the roots are aproximate, it depends on many things like value of interval, considerable decimal digits of root, etc. Dependency on interval value shown in a table.

No.	Interval	Value of $\sqrt{2}$
1 .	0.1	1.41448975
2 .	0.2	1.41452622
3 .	0.3	1.41417384
4 .	0.4	1.41437984
5 .	0.5	1.41407776
6 .	0.6	1.41439664
7 .	0.7	1.41422737
8 .	0.8	1.41405022
9 .	0.9	1.41434956
10 .	1.0	1.41415787



PROBLEM 02: Show that $f(x) = x^3 + 4x^2 - 10$ has a root in $[1, 2]$, and use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-4} .

PROBLEM 03: Find the value of x where, $\tan(x) = x$.

PROBLEM 04:

Chapter 2

Ordinary Differential Equation

Chapter 3

Partial Differential Equation

Chapter 4

Chapter 5

Chapter 6

Chapter 7

Chapter 8

Chapter 9