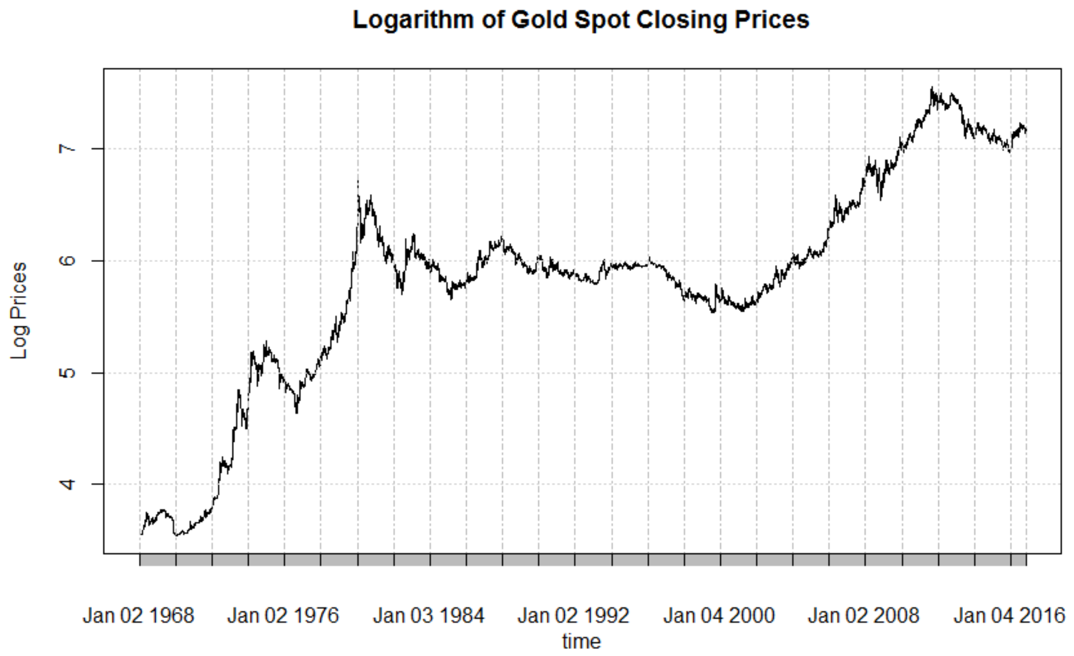
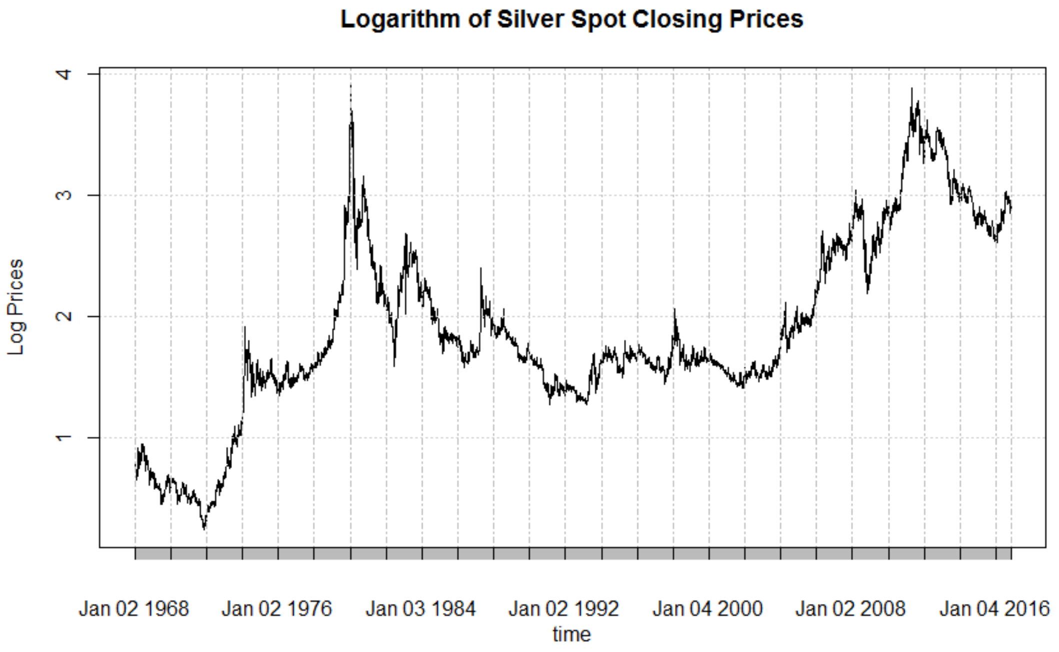
Gold Silver - Pairs Trading

Arpan Gupta

Cointegration of Gold and Silver

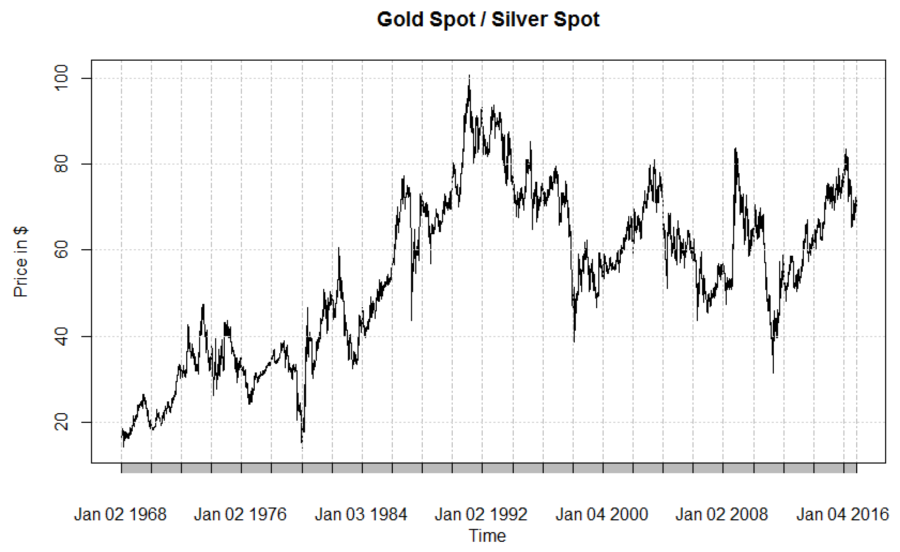
- Historical correlation of 0.91 going back to 1968





Gold Silver Ratio

- $\frac{Gold\ Spot}{Silver\ Spot}$
- Exhibits Mean-Reverting characteristics due to correlation between Gold and Silver



Thesis

- Calculate a predicted Gold/Silver Ratio based on historical data
- When the actual Gold/Silver Ratio is a certain bound away from the predicted ratio, enter a trade
  - $\frac{Actual\ Gold\ Spot}{Actual\ Silver\ Spot} > \frac{Predicted\ Gold\ Spot}{Predicted\ Silver\ Spot} + bound$
    - Short a dollar amount of Gold and Long that same dollar amount of Silver
  - $\frac{Actual\ Gold\ Spot}{Actual\ Silver\ Spot} < \frac{Predicted\ Gold\ Spot}{Predicted\ Silver\ Spot} - bound$
    - Long a dollar amount of Gold and Short that same dollar amount of Silver
- Market Neutral

Moving Average Model

- Also known as Rolling Average / Running Average
- Common way to implement Mean Reversion Strategies
- $Ratio_t = \frac{Ratio_{t-1} + Ratio_{t-2} + \cdots + Ratio_{t-n}}{n}$
- $Ratio_t \rightarrow$ Predicted Gold/Silver Ratio for day t
- $n \rightarrow$ Rolling Days

Process for Trade

- Calculate Predicted Gold/Silver Ratio using a certain number of Rolling Days for the Simple Moving Average
- If not in a trade
  - $\frac{Actual\ Gold\ Spot}{Actual\ Silver\ Spot} > \frac{Predicted\ Gold\ Spot}{Predicted\ Silver\ Spot} + bound$
    - Short a dollar amount of Gold and Long that same dollar amount of Silver
  - $\frac{Actual\ Gold\ Spot}{Actual\ Silver\ Spot} < \frac{Predicted\ Gold\ Spot}{Predicted\ Silver\ Spot} - bound$
    - Long a dollar amount of Gold and Short that same dollar amount of Silver
- If in a trade
  - $\left|\frac{Actual\ Gold\ Spot}{Actual\ Silver\ Spot} - \frac{Predicted\ Gold\ Spot}{Predicted\ Silver\ Spot}\right| < 0.1 * std\left(GS\ Ratio_{rolling\ days}\right)$
    - Exit trade

Backtesting

- Ran on Gold and Silver Spot data from 2006 – 2012 (in-Sample)
- Out of Sample – 2013 - present
- Trading iShares ETFs for Gold and Silver (IAU and SLV)
  - Holding fees of 25 bps

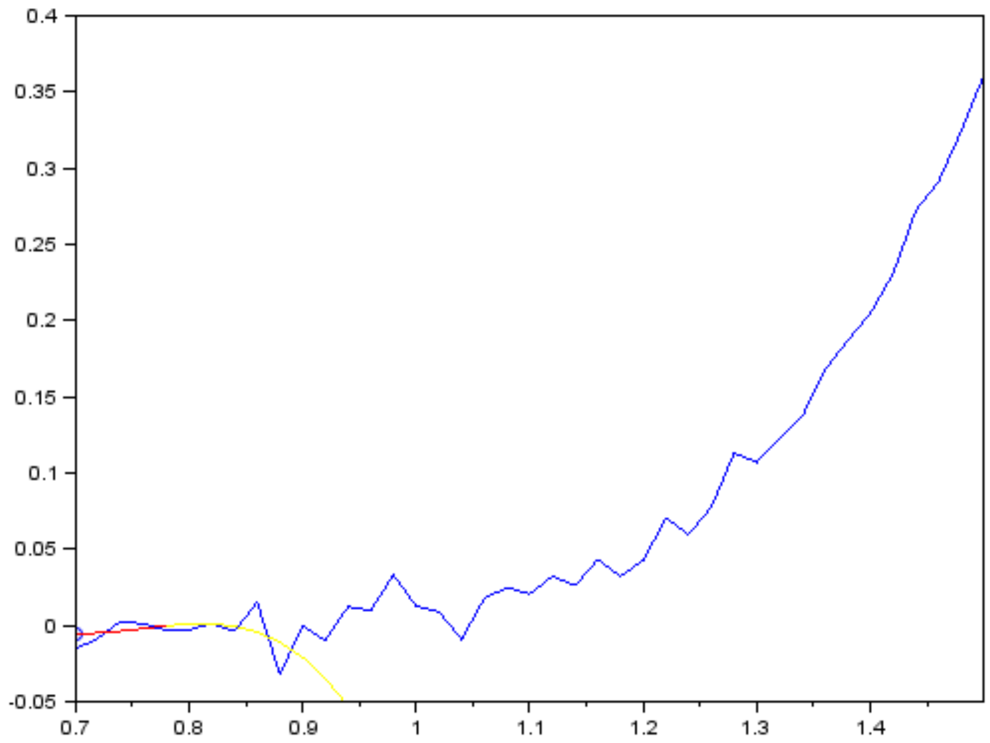
Optimization

- Two key parameters in the model are Rolling Days and the Bound
- Bound → $c * std(GSRatio_{rolling\ days})$
- Finding the best values for the parameters Rolling Days and c
- Brute Force to get all possible Sharpe Values
  - Tried all possible Rolling Days from 10 to 252
  - Tried all possible c from 0.4 to 2.0
- Large dataframe of Sharpe Values
  - Lots of Noise in Data Frame
    - Random values that come in because of noise in the data or certain idiosyncrasies that happened during that specific backtest

Savitzky Golay Filter

- Remove the noise in the backtest while keeping as much signal as possible
- Savitzky Golay Filter
  - Fits a curve to the data that will smoothen out the values
  - N degree polynomial
  - Fits the polynomial over a certain number of rolling days
  - Used a 4^{th} degree polynomial with 11 rolling days

Visualization of Savitzky Golay Filter



Picking parameters from smoothed data

- Pick the highest Sharpe (obviously)
  - Highest stable area
- How can we choose the highest stable area?
  - Look at the derivatives of the polynomials generated by the Savitzky Golay Filter
  - Look for an Area with
    - First Derivative close to 0
    - Second Derivative is negative and close to zero

Final Results

- Rolling Days -> 25
- Trading Signal -> 1.1
- Out of Sample Sharpe -> 0.69
- Downside of Model
  - All observations are weighed equally

ARIMA Model

- General model to forecast a time series
- Autoregressive Model
- Moving Average Model
- Orders of Integration

Autoregressive Model

- A representation of a series of data points where the current time's value is based on a linear combination of it's past values.
- $X_t = c + \sum_{i=1}^{p}(j_i * X_{t-i}) + \varepsilon_t$
- An AR(3) process can be modeled with this equation
  - $X_t = c + j_{i-1}X_{t-1} + j_{i-2}X_{t-2} + j_{i-3}X_{t-3} + \varepsilon_t$
- The "characteristics" of the AR model are the j terms.
  - This will be useful when we talk about Unit Root tests

Moving Average

- Completely different concept from previous Simple Moving Average Model
- Basing future predictions on the past errors of the series
- An MA(q) model can be written as
  - $X_t = \mu + \varepsilon_t + \theta_1 * \varepsilon_{t-1} + \cdots + \theta_q * \varepsilon_{t-q}$
- In order to use a MA model, the series has to be autocorrelated
  - Autocorrelation – series is correlated with itself

What is Stationarity?

- The underlying statistical structure of the time series does not change with time
  - The joint probability distribution for each of the data points in the time series does not change over time
    - Strict stationarity
  - The mean and variance of the distribution of each the points does not change over time
    - Weak Stationarity (implied if strict stationarity holds)

Checking for Stationarity

- Unit Roots
  - Can calculate Unit Roots based off characteristics of AR and MA process
    - Characteristics are j in AR and θ in MA
- Augmented Dickey Fuller Test
  - $\Delta y_t = \alpha + \beta * t + \gamma * y$
  - Uses Unit Roots to give you a p-value to determine stationarity

Orders of Integration

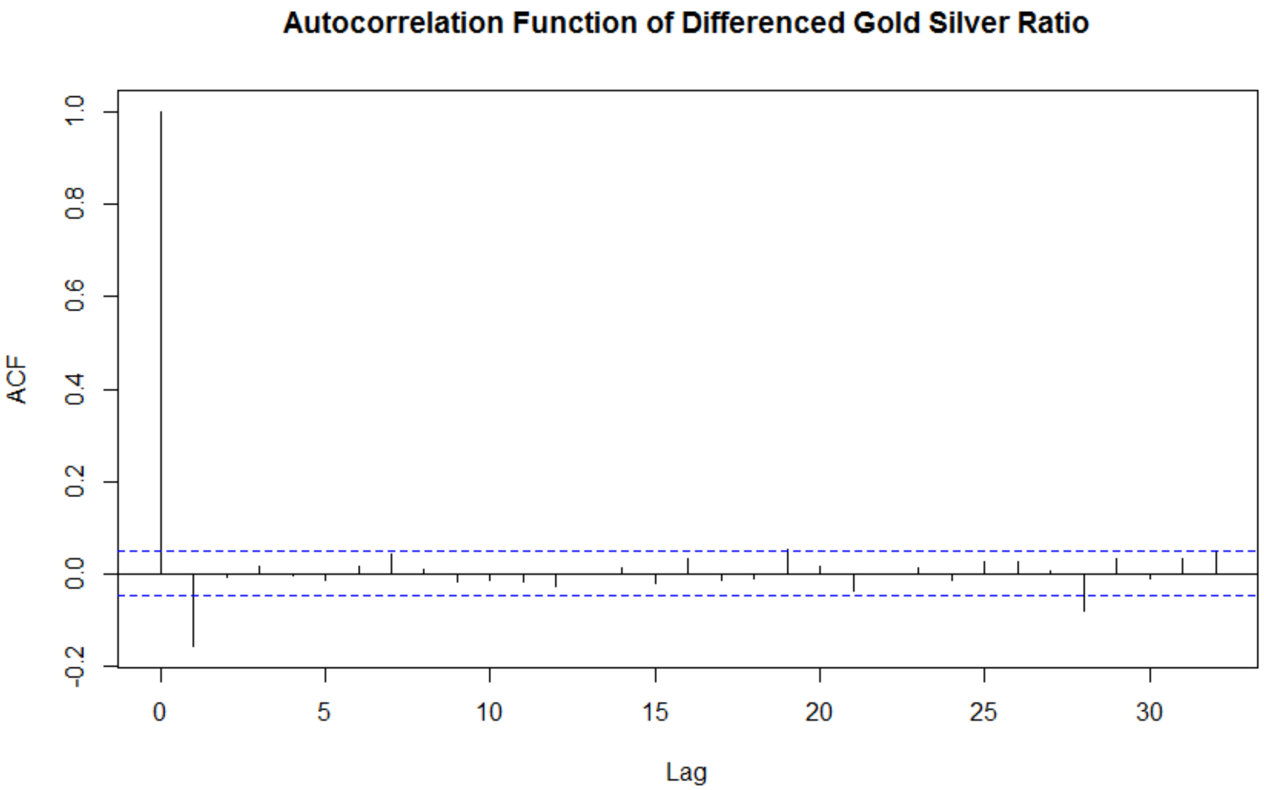
- In order to make our time series stationary, we can employ a process called differencing
- Differenced Gold/Silver Ratio
  - $Today's\ Observation\ = \frac{Gold\ Spot_{today}}{Silver\ Spot_{today}} - \frac{Gold\ Spot_{yesterday}}{Silver\ Spot_{yesterday}}$
- Instead of running the model on the Gold Silver Ratio, we’re running the model on the change in the Gold Silver Ratio.
  - We’re assuming the change in the Gold Silver Ratio is stationary

Gold/Silver Ratio Dickey Fuller Test

- After running the Adjusted Dickey Fuller Test on the Gold Silver Ratio, we get a Dickey-Fuller number of -2.2629 and a p-value of 0.4627.
  - This is obviously not optimal
- We can difference the series once and run the Adjusted Dickey Fuller Test again
  - This time we get a Dickey Fuller number of -47.718 and a p-value that is less than .01
  - We can assume the series is stationary

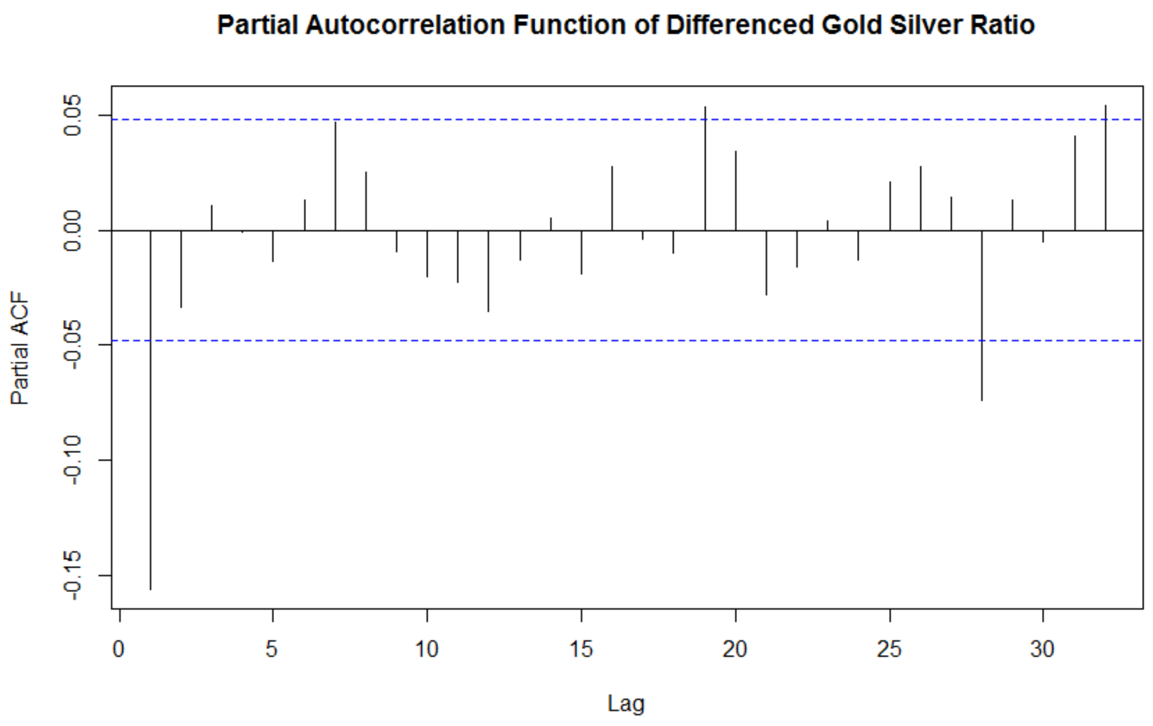
Autocorrelation Function

- The correlation of a time series with it’s own lagged values



Partial Autocorrelation Function

- Gives you the Partial Correlation of a time series with it’s own lagged values



Akaike Information Criterion

- $N * \ln\left(\frac{SS_{errors}}{N}\right) + 2\mathrm{k}$
  - N -> Number of Observations
  - K -> Number of parameters
- Akaike Information Criterion
  - Want the smallest value
- Comparing AIC values for different models
  - ARIMA(1,1,0), ARIMA(1,1,1), ARIMA(0,1,1)

Choosing the best model

- From the ACF and PACF we have
  - ARIMA(1,1,1)
  - ARIMA(1,1,0)
    - Same as AR(1)
  - ARIMA(0,1,1)
    - Same as MA(1)
- We can use the Akaike Information Criterion to pick the best model
- ARIMA(1,1,1) has the lowest Akaike Information Criterion