Mathematics I (BSM 101)

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BSM 101

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February 6, 2023

Outlines

1. Introduction to Integration
2. Geometrical meaning of Indefinite Integration
3. Basic Rules of Integration with Examples
4. Integration by substitution with Examples
5. Classwork Problem Set

Introduction to Antiderivative

A function $F(x)$ is said to be an **antiderivative** of $f(x)$ if

$$F'(x) = f(x)$$

for every x in the domain of $f(x)$. The process of finding antiderivatives is called antidifferentiation or indefinite integration.

Note: Sometimes we write the equation

$$F'(x) = f(x) \quad \text{as} \quad \frac{dF}{dx} = f(x)$$

Example

A function has more than one antiderivative.
For example, one antiderivative of the function $f(x) = 3x^2$ is $F(x) = x^3$, since

$$F'(x) = 3x^2 = f(x)$$

but so are $x^3 + 12$ and $x^3 - 5$ and $x^3 + \pi$, since

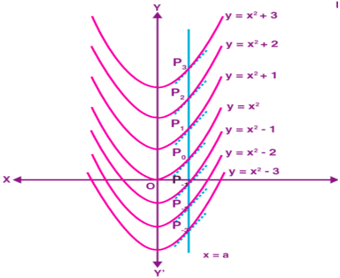
$$\frac{d}{dx}\left(x^3 + 12\right) = 3x^2 \quad \frac{d}{dx}\left(x^3 - 5\right) = 3x^2 \quad \frac{d}{dx}\left(x^3 + \pi\right) = 3x^2$$

Antiderivative

If $F(x)$ is one antiderivative of the continuous function $f(x)$, then all such antiderivatives may be characterized by $F(x) + C$ for constant C.
The family of all antiderivatives of $f(x)$ is written

$$\int f(x)dx = F(x) + C$$

and is called the indefinite integral of $f(x)$.
The integral is "indefinite" because it involves a constant C that can take on any value.



So, $\int f(x) = F(x) + C$, with different value of C, that will represent the different member of the family of curve as shown in the above figure. And slope of tangent at each curve at the same point are parallel. i.e derivatives of all the family of function are equal.

Basic Rules of Integration

Algebraic Rules for Indefinite Integration

- The constant multiple rule: $\int kf(x)dx = k\int f(x)dx$ for constant k
- The sum rule: $\int[f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$
- The difference rule: $\int[f(x)-g(x)]dx = \int f(x)dx - \int g(x)dx$

Rules for Integrating Common Functions:

- The constant rule: $\int kdx = kx + C$ for constant k
- The power rule: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for all $n \neq -1$
- The logarithmic rule: $\int \frac{1}{x}dx = \ln|x| + C$ for all $x \neq 0$
- The exponential rule: $\int e^{kx}dx = \frac{1}{k}e^{kx} + C$ for constant $k \neq 0$

Example/Classwork

1. Integrate: a. $\int \left(x^2 - 2x + 5\right) dx$ b. $\int \left(3e^{-5t} + \sqrt{t}\right) dt$

Solution:

$$\int \left(x^2 - 2x + 5\right) dx = \int x^2 dx - \int 2x dx + \int 5 dx$$
$$= \frac{x^3}{3} - x^2 + 5x + C$$

$$\int \left(3e^{-5t} + \sqrt{t}\right) dt = \int \left(3e^{-5t} + t^{1/2}\right) dt$$
$$= 3\left(\frac{1}{-5}e^{-5t}\right) + \frac{1}{3/2}t^{3/2} + C$$
$$= -\frac{3}{5}e^{-5t} + \frac{2}{3}t^{3/2} + C$$

Find the following integrals:

a. $\int \left(2x^5 + 8x^3 - 3x^2 + 5\right) dx$ b. $\int \left(\frac{x^3 + 2x - 7}{x}\right) dx$

Substitution Rule of Integration

Using Substitution to Integrate $\int f(x)dx$

Step 1. Choose a substitution $u = u(x)$ that "simplifies" the integrand $f(x)$.

Step 2. Express the entire integral in terms of u and $du = u'(x)dx$. This means that all terms involving x and dx must be transformed to terms involving u and du.

Step 3. When step 2 is complete, the given integral should have the form

$$\int f(x)dx = \int g(u)du$$

If possible, evaluate this transformed integral by finding an antiderivative $G(u)$ for $g(u)$.

Step 4. Replace u by $u(x)$ in $G(u)$ to obtain an antiderivative $G(u(x))$ for $f(x)$, so that

$$\int f(x)dx = G(u(x)) + C$$

Example 1

Integrate: $\int \sqrt{2x+7}dx$

Solution:

We choose $u = 2x + 7$ and obtain

$$du = 2dx \quad \text{so that} \quad dx = \frac{1}{2}du$$

Then the integral becomes

$$\begin{aligned}\int \sqrt{2x+7}dx &= \int \sqrt{u}\left(\frac{1}{2}du\right)\\ &= \frac{1}{2}\int u^{1/2}du\\ &= \frac{1}{2}\frac{u^{3/2}}{3/2} + C = \frac{1}{3}u^{3/2} + C\\ &= \frac{1}{3}(2x+7)^{3/2} + C\end{aligned}$$

Example: a. $\int 8x\left(4x^2-3\right)^5 dx$ b. $\int e^{5x+2}dx$ c. $\int \frac{(lnx)^2}{x}dx$

Example 2

Integrate: $\int x^3 e^{x^4+2} dx$

Solution: If the integrand of an integral contains an exponential function, it is often useful to substitute for the exponent. In this case, we choose

$$u = x^4 + 2 \quad \text{so that } du = 4x^3 dx$$

and

$$\begin{aligned} \int x^3 e^{x^4+2} dx &= \int e^{x^4+2} \left(x^3 dx\right) \\ &= \int e^u \left(\frac{1}{4} du\right) \quad \text{since } du = 4x^3 dx \\ &= \frac{1}{4} e^u + C \\ &= \frac{1}{4} e^{x^4+2} + C \end{aligned}$$

Example 3

Solve: $\int \frac{1}{1+e^{-x}} dx$

Solution: You may try to substitute $w = 1 + e^{-x}$. However, this is a dead end because $dw = -e^{-x}dx$ but there is no e^{-x} term in the numerator of the integrand. Instead, note that

$$\begin{aligned} \frac{1}{1+e^{-x}} &= \frac{1}{1+\frac{1}{e^x}} = \frac{1}{\frac{e^x+1}{e^x}} \\ &= \frac{e^x}{e^x+1} \end{aligned}$$

Now, if you substitute $u = e^x + 1$ with $du = e^x dx$ into the given integral, you get

$$\begin{aligned} \int \frac{1}{1+e^{-x}} dx = \int \frac{e^x}{e^x+1} dx = \int \frac{1}{e^x+1} \left(e^x dx\right) &= \int \frac{1}{u} du \\ &= \ln|u| + C \\ &= \ln|e^x+1| + C \quad \text{substitute } e^x+1 \text{ for } u \end{aligned}$$

Classwork

Integrate using substitution rule:

- $\int xe^{x^2} dx$
- $\int 2xe^{x^2-1} dx$
- $\int t\left(t^2+1\right)^5 dt$
- $\int 3t\sqrt{t^2+8}dt$
- $\int x^2\left(x^3+1\right)^{3/4} dx$
- $\int x^5 e^{1-x^6} dx$
- $\int \frac{2y^4}{y^5+1} dy$

Thank You