From Iteration to System Failure

Characterizing the FITness of Periodic Weakly-Hard Systems

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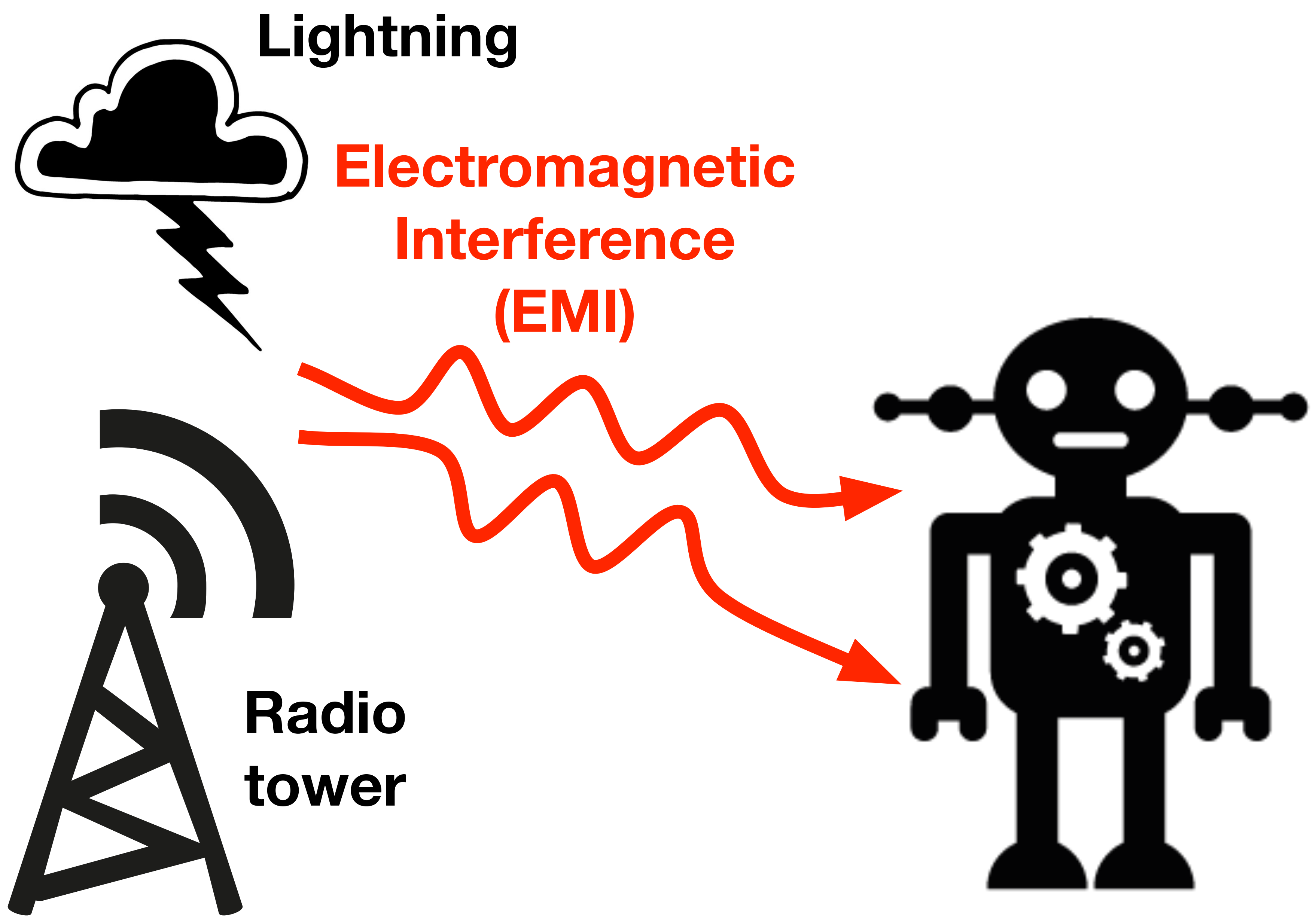
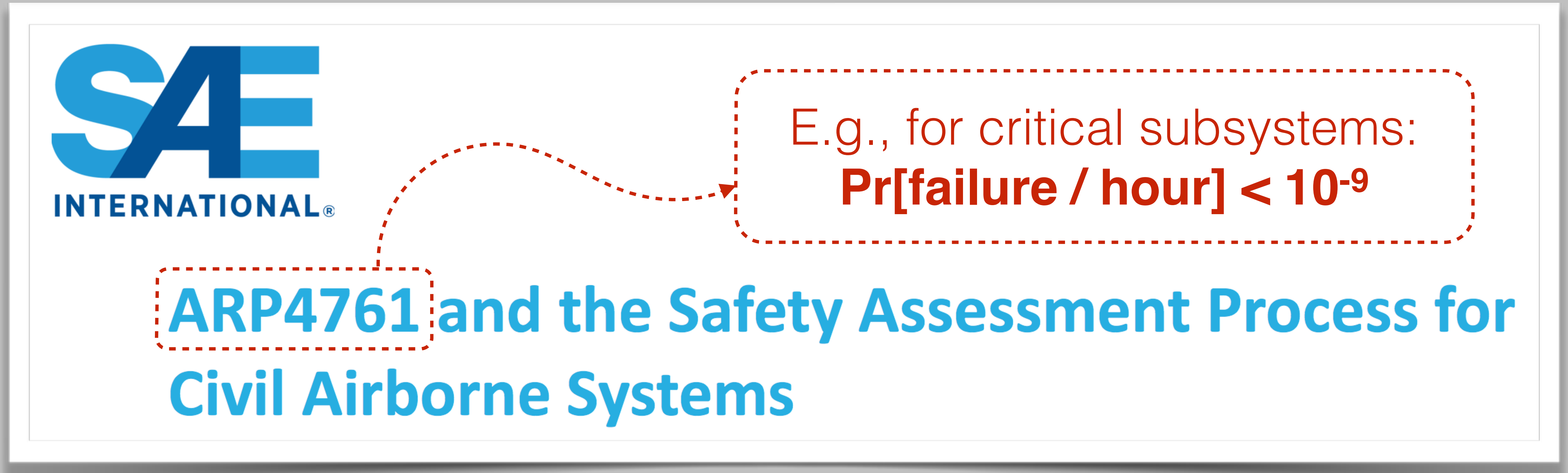
Quantitative Reliability Analysis is Essential for Safety-Critical CPS



Zero risk of failures can never be achieved

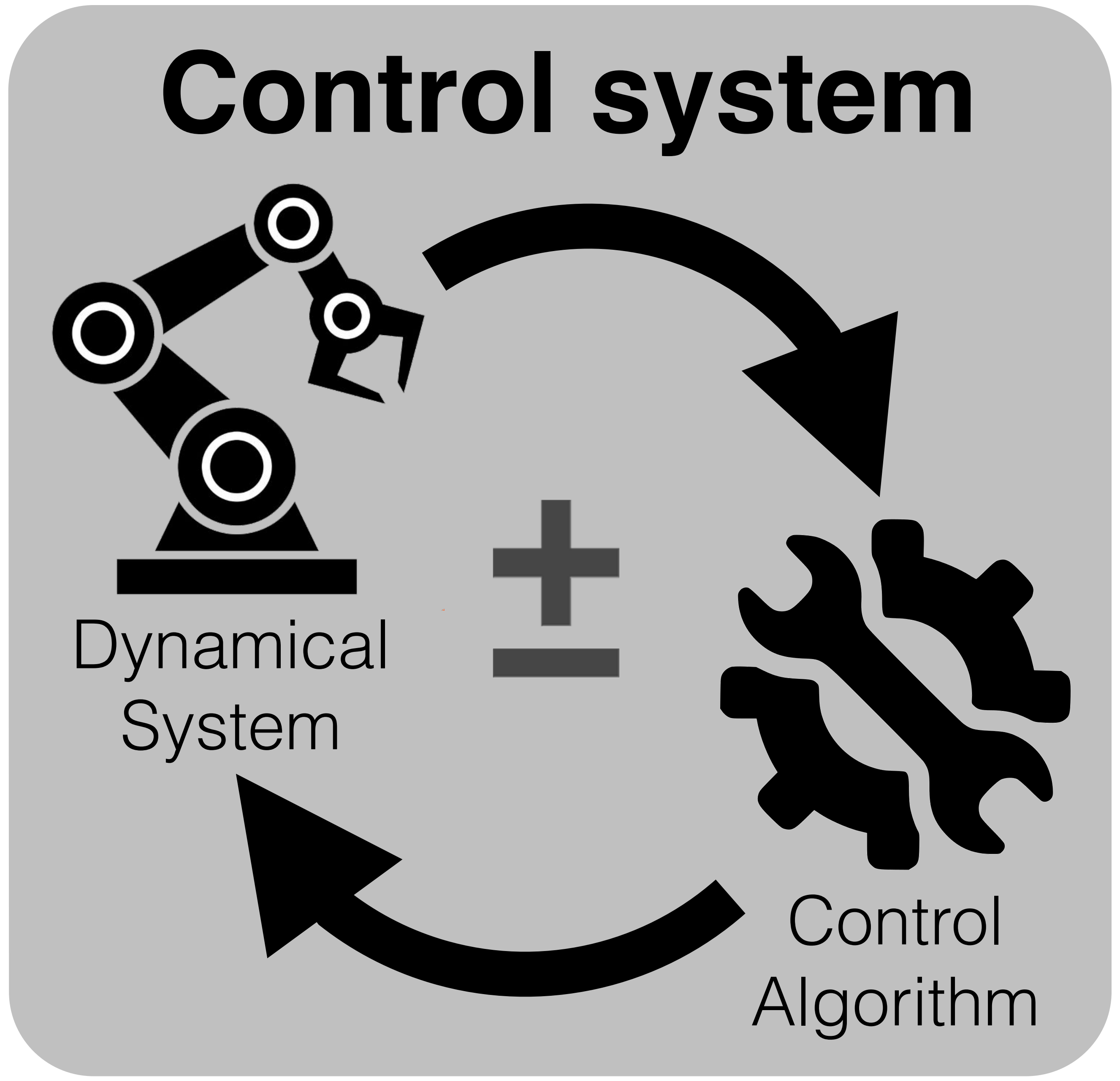
Safety certification objective: Ensure **"negligible"** failure rates


SAE INTERNATIONAL

E.g., for critical subsystems: **Pr[failure / hour] < 10^{-9}**

ARP4761 and the Safety Assessment Process for Civil Airborne Systems

How to Analyse the Reliability of **Temporally Robust Systems?**



Motivating example

- ➡ **Frequency:** 100 Hz (10 ms time period)
- ➡ **Stability requirement:** 3 out of 4 iterations execute on time
- ➡ **Schedulability analyses:** Pr[single iteration delayed] ≤ 10^{-10}

Per-iteration analyses yield pessimistic failure rates

- ➡ Computing mean time to first failed iteration ignores stability requirements
- ➡ E.g., iteration failure probability of 10^{-10} ↦ **36,000 x 10^{-9} failures / hours**

9 orders of magnitude!

Explicitly accounting for the stability requirements

- ➡ Yields more accurate failure rates
- ➡ E.g., iteration failure probability of 10^{-10} and stability requirement ↦ **1.08 x 10^{-15} failures / hours**

Not trivial anymore!

This work

How to Analyse the Reliability of **Temporally Robust Systems?**

Objectives

Generic

- ➡ Complex robustness requirements

Accurate (ideally, exact)

- ➡ Minimize pessimism in the final system reliability

Scalable

- ➡ Asymptotic requirements with large parameter values

Proposed Techniques

PMC (**P**robabilistic **M**odel **C**hecking)

- ➡ Exact, very generic, but slow

Mart (uses martingale theory)

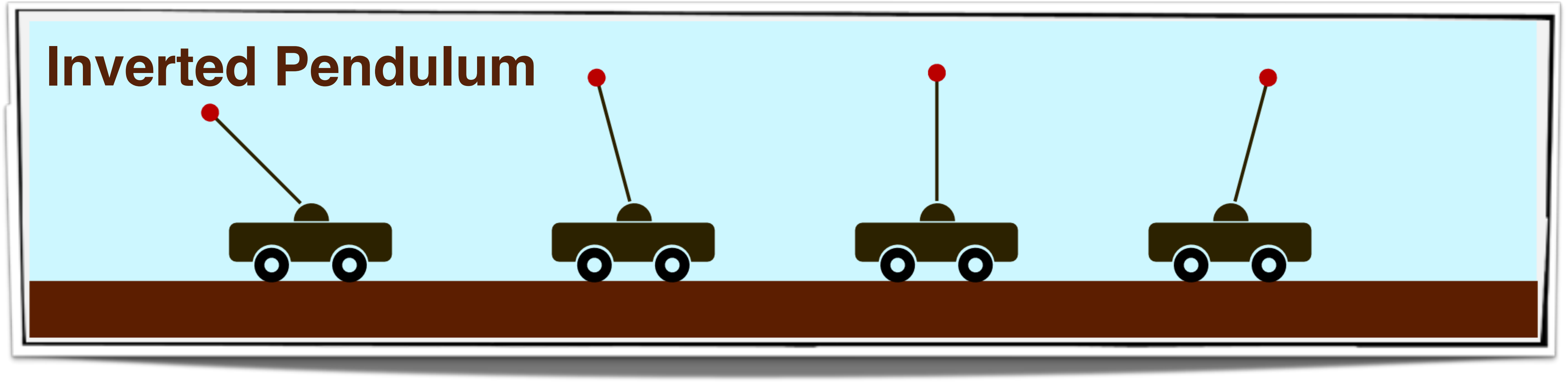
- ➡ Exact, less generic, but slightly faster

SAp (**S**ound **Ap**proximation)

- ➡ Not exact, least generic, but highly scalable

Background & System Model

Asymptotic Properties



Specification: Mass 0.5 kg, length 0.20 m, period 10 ms

Design: Current iteration is skipped ↦ Use previous iteration parameters

Asymptotically stable with **at least 76.51% successful iterations***

Doesn't specify if the system can handle a burst of skipped iterations

➡ What if the first 50 iterations are skipped? No feedback for 0.5 second

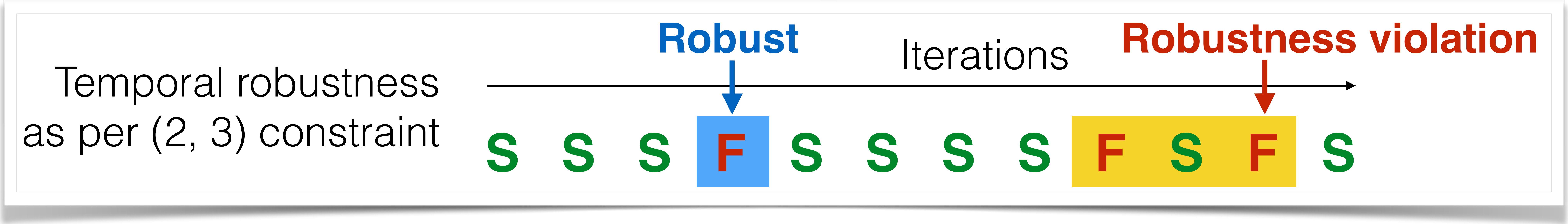
* Majumdar et al. "Performance-aware scheduler synthesis for control systems." EMSOFT, Taipei (2011)

Weakly-Hard* Constraints

Concretize asymptotic properties using finite window sizes

If each iteration is labeled either as a **S**uccess or a **F**ailure

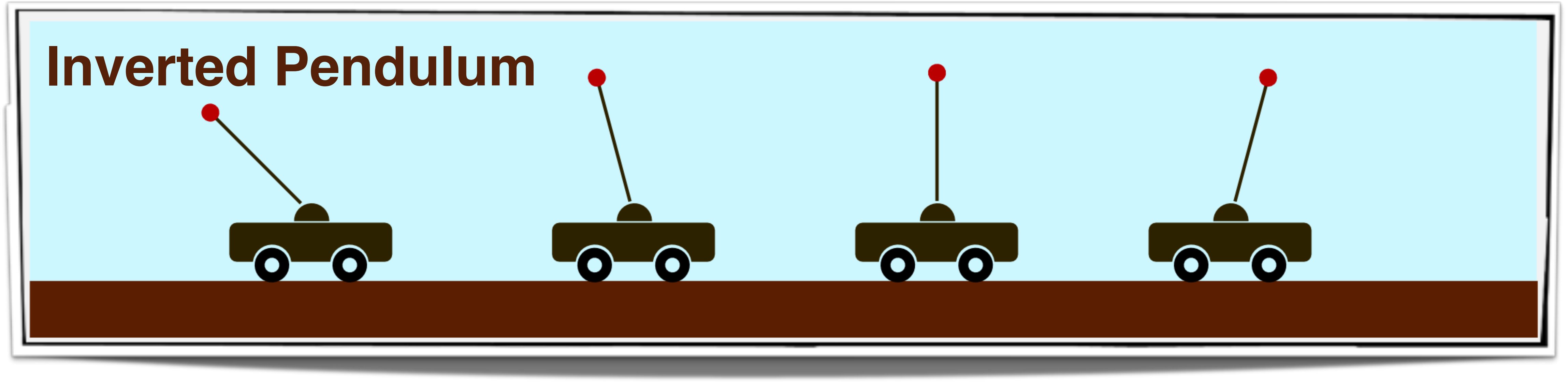
- ➡ (m, k) constraint: At least m out of every k consecutive iterations must be **S**uccessful



Temporal robustness as per (2, 3) constraint

[*] Bernat et al. "Weakly hard real-time systems." *IEEE Transactions on Computers*, 50(4):308–321 (2001).

Temporal Robustness Criteria



Asymptotically stable with **at least 76.51% successful iterations*** ⟼ (766, 1000)

Short-range “liveness” constraints ⟼ (1, 5)

- ➡ The inverted pendulum can tolerate a small burst of skipped iterations

Robustness Criteria

Combination of two weakly-hard constraints

(766, 1000)

(1, 5)

Combination of different weakly-hard constraints

- ➡ (m, k) = Each k consecutive iterations, at least m successes needed
- ➡ ⟨m, k⟩ = Each k consecutive iterations, at least m consecutive successes needed
- ➡ $\overline{\langle m \rangle}$ = m consecutive failures should never happen

Problem Statement

Given periodic system **S**, time period **T**, iteration failure probability **P_F**, and the **temporal robustness criteria** …

Lower-bound the **Mean Time To Failure (MTTF)** of S

MTTF = Expected time to 1st temporal robustness violation

$$= \sum_{n=0}^{\infty} \left(nT \times \Pr[1^{st} \text{ violation in the } n^{th} \text{ iteration}] \right)$$

$$\propto \frac{1}{\text{failure rate} \atop \text{(e.g., failures/hour or FIT)}}$$

Assumption: **P_F** is independently and identically distributed (IID)[1, 2]

[1] Broster et al. "Timing Analysis of Real-Time Communication Under Electromagnetic Interference." Real Time Systems Journal (2005)

[2] Gujarati et al. "Quantifying the Resiliency of Fail-Operational Real-Time Networked Control Systems." ECRTS, Barcelona (2018)

Probabilistic **M**odel **C**hecking **(PMC)**

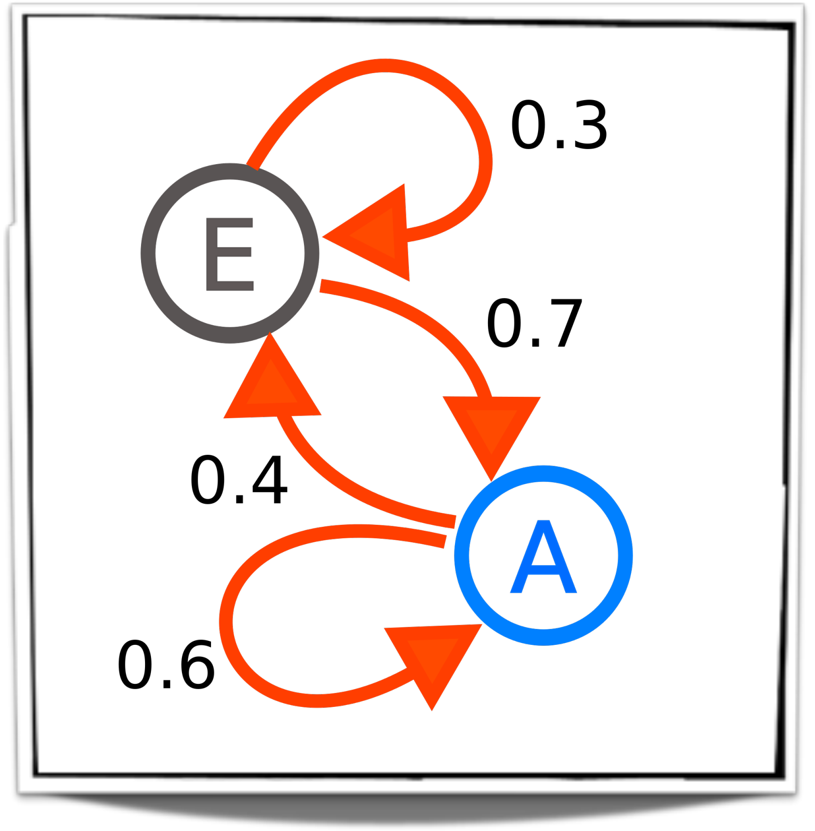
Exact, very generic, but slow

MTTF Estimation using **PMC**

Formal verification technique to model and analyze systems that exhibit **probabilistic** behaviours

Periodic system **S** with iteration failure probabilities **P_F**

System as a probabilistic model

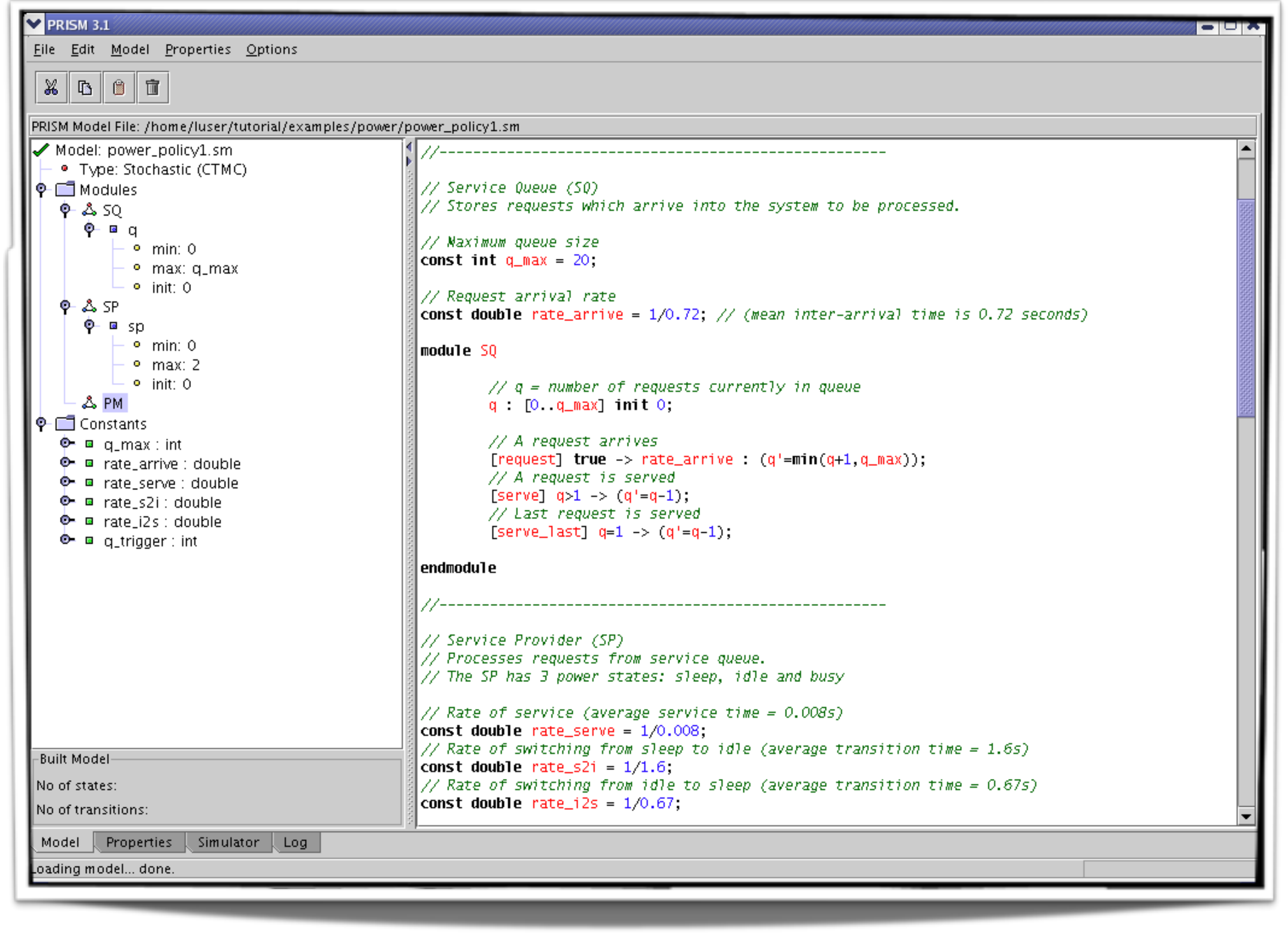


Safety properties as temporal logical specifications

```
R=? [ F safe=false ]
```

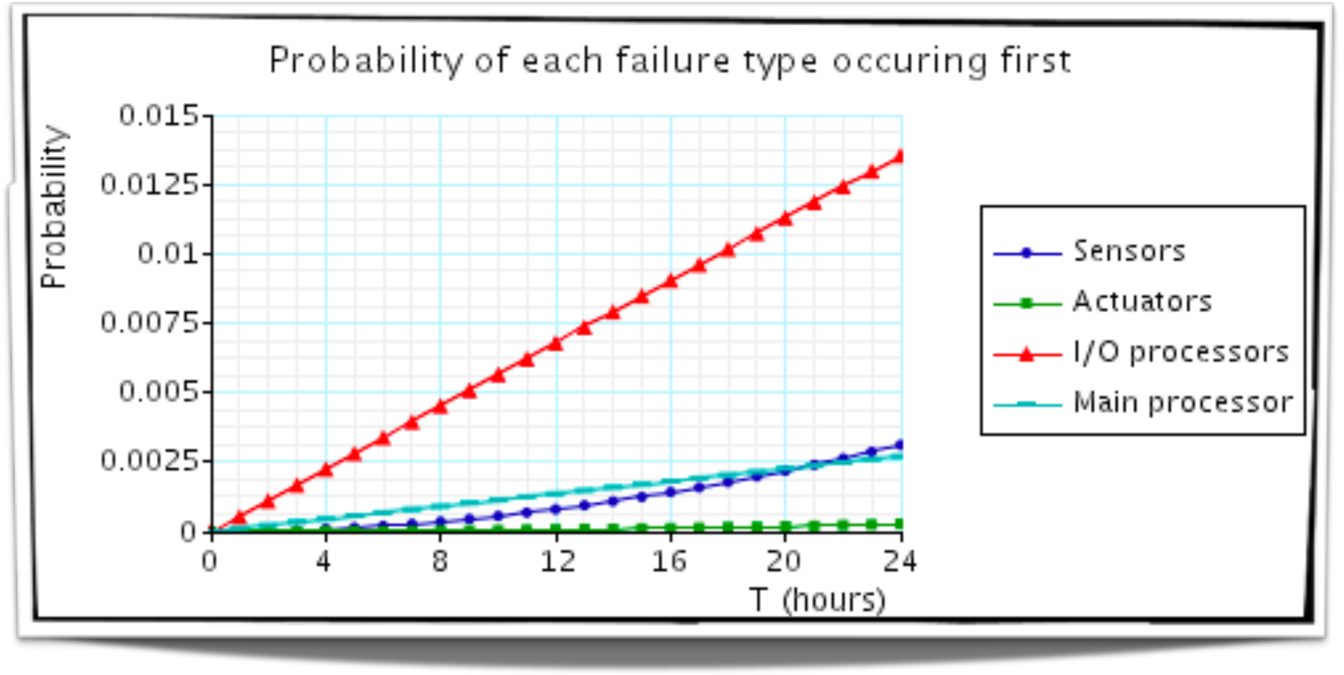
Violation of temporal robustness

Probabilistic model checker, e.g., PRISM



```
//-----------------------------------------------------

// Service Queue (SQ)
// Stores requests which arrive into the system to be processed.

// Maximum queue size
const int q_max = 20;

// Request arrival rate
const double rate_arrive = 1/0.72; // (mean inter-arrival time is 0.72 seconds)

module SQ

        // q = number of requests currently in queue
        q : [0..q_max] init 0;

        // A request arrives
        [request] true -> rate_arrive : (q'=min(q+1,q_max));
        // A request is served
        [serve] q>1 -> (q'=q-1);
        // Last request is served
        [serve_last] q=1 -> (q'=q-1);

endmodule

//-----------------------------------------------------

// Service Provider (SP)
// Processes requests from service queue.
// The SP has 3 power states: sleep, idle and busy

// Rate of service (average service time = 0.008s)
const double rate_serve = 1/0.008;
// Rate of switching from sleep to idle (average transition time = 1.6s)
const double rate_s2i = 1/1.6;
// Rate of switching from idle to sleep (average transition time = 0.67s)
const double rate_i2s = 1/0.67;
```

Quantitative results



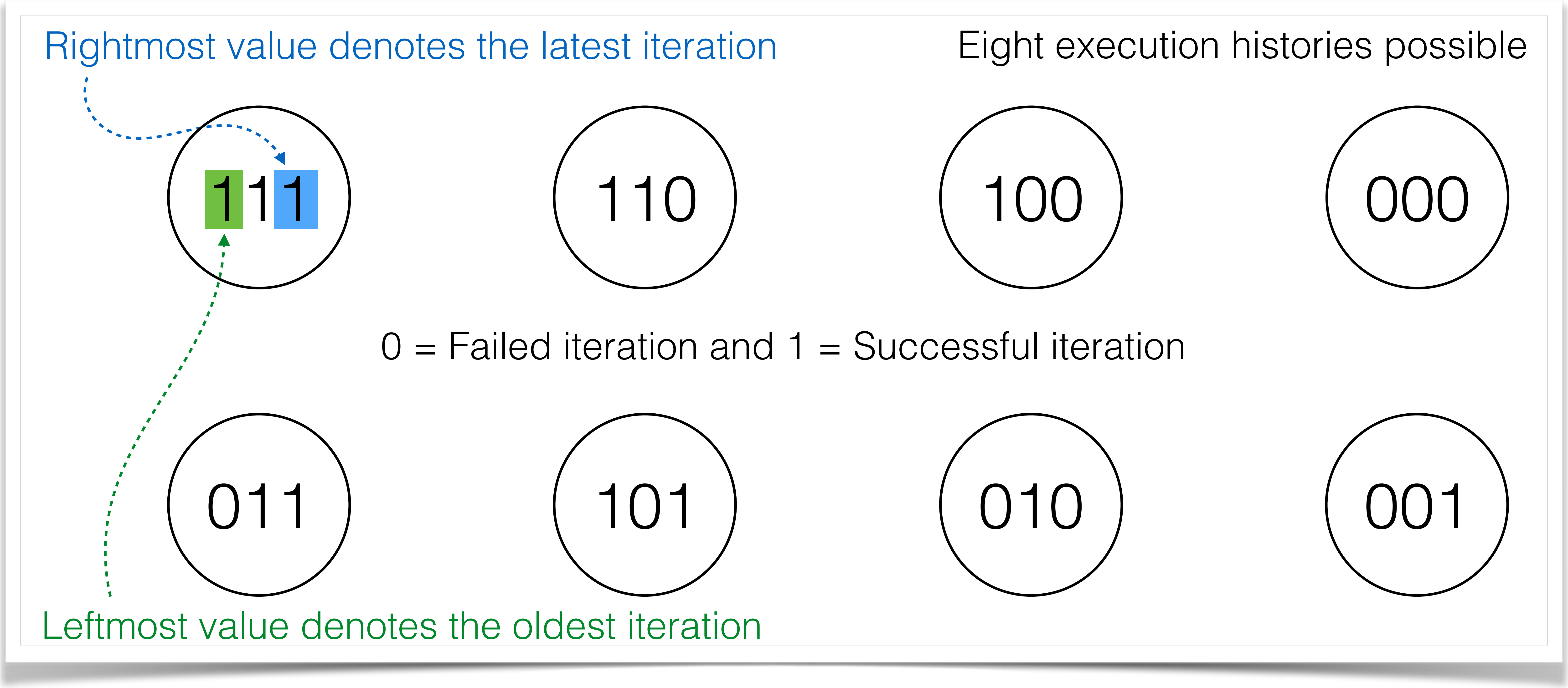
MTTF estimation using PRISM

Modeling Weakly-Hard Constraints

Key idea

Weakly-hard constraints depend on a **finite-sized history**

- ➡ E.g., (m, k) constraint depends on the k latest iterations
- ➡ Connect all possible execution histories via transition probabilities **P_F** and **$1 - P_F$**


Rightmost value denotes the latest iteration
Eight execution histories possible
111
110
100
000
0 = Failed iteration and 1 = Successful iteration
011
101
010
001
Leftmost value denotes the oldest iteration

Example:
(2, 3) constraints

Modeling Weakly-Hard Constraints

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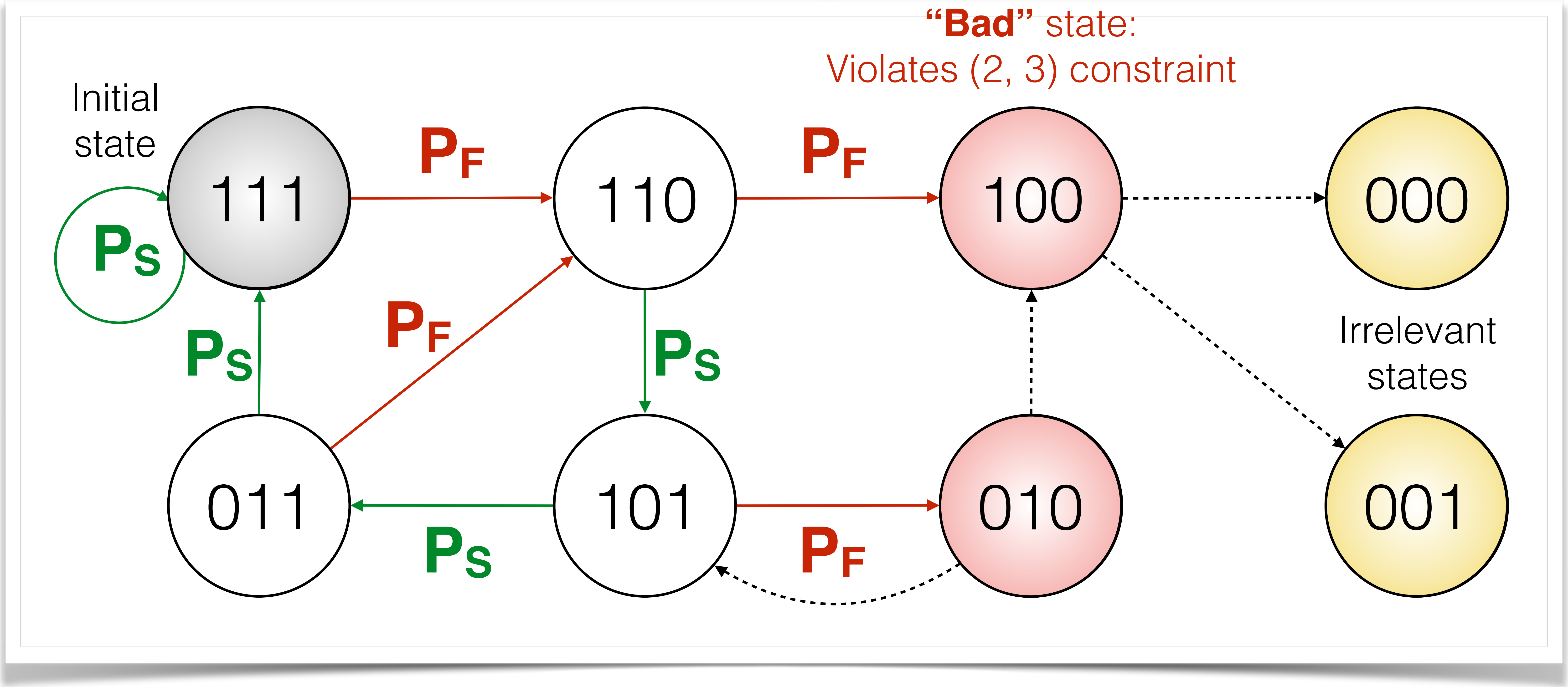

Example:
(2, 3) constraints

Modeling Weakly-Hard Constraints

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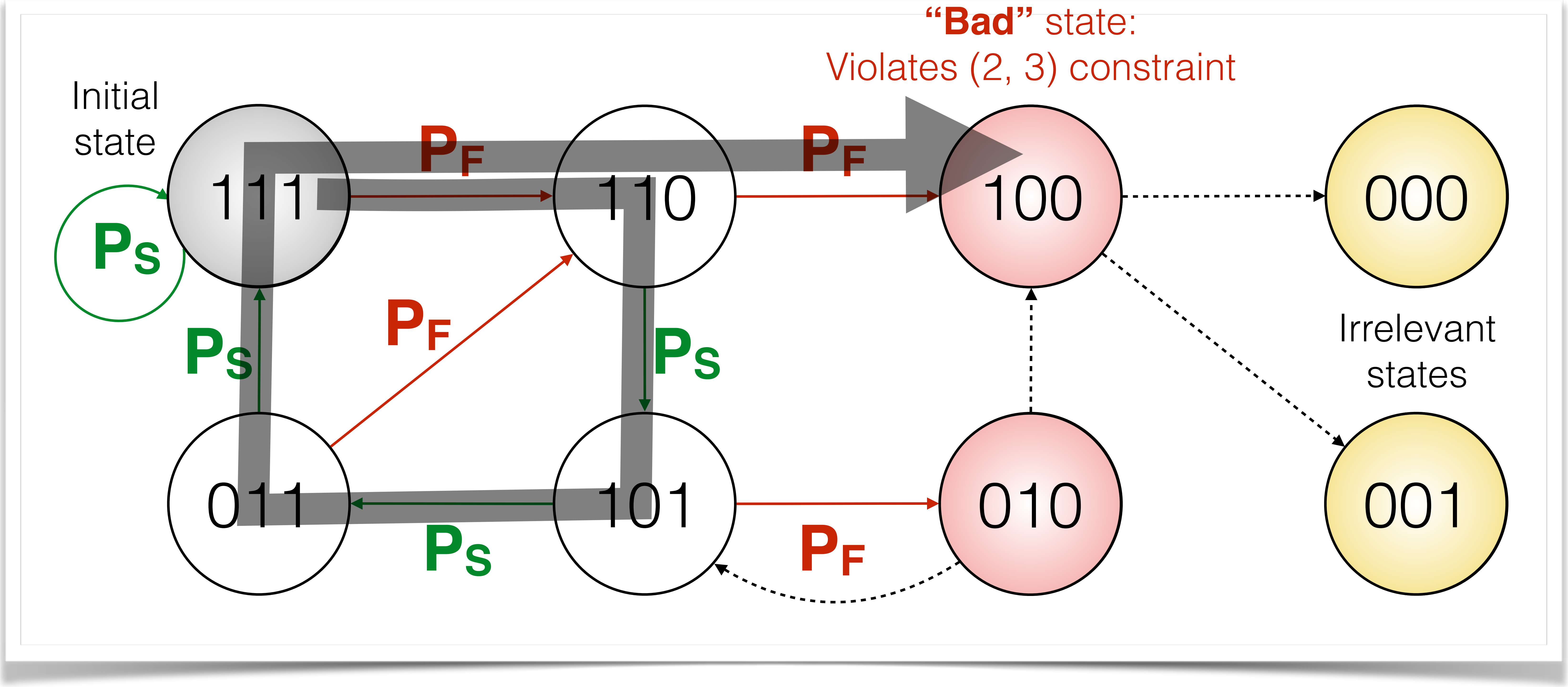
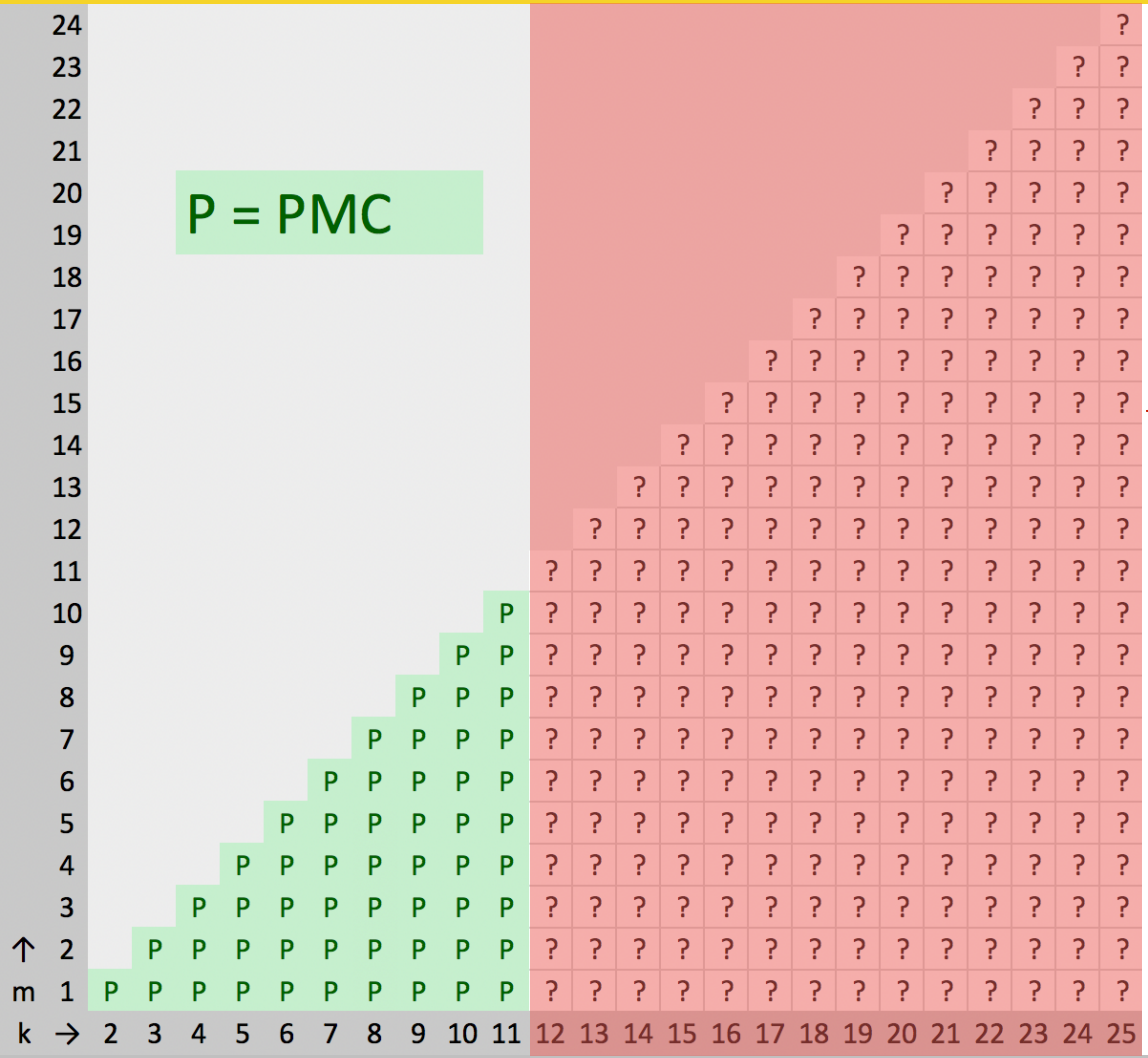


Example:
(2, 3) constraints

$$\text{MTTF} = \underbrace{\left(\begin{array}{c}\text{Expected \# steps}\\ \text{to a “}\textbf{bad}\text{” state}\end{array}\right) \times T}_{\text{PRISM}}$$

Does PMC **Scale** with k?



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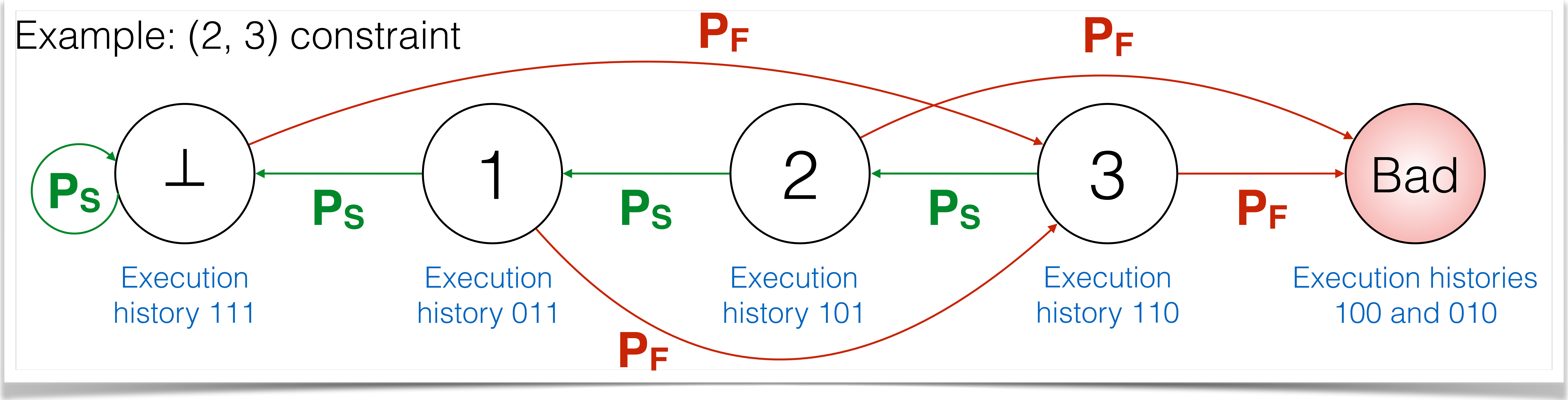
m	1	P	P	P	P	P	P	P	P	P	P	?	?	?	?	?	?	?	?	?	?	?	?	?	?
k	→	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

P = PMC

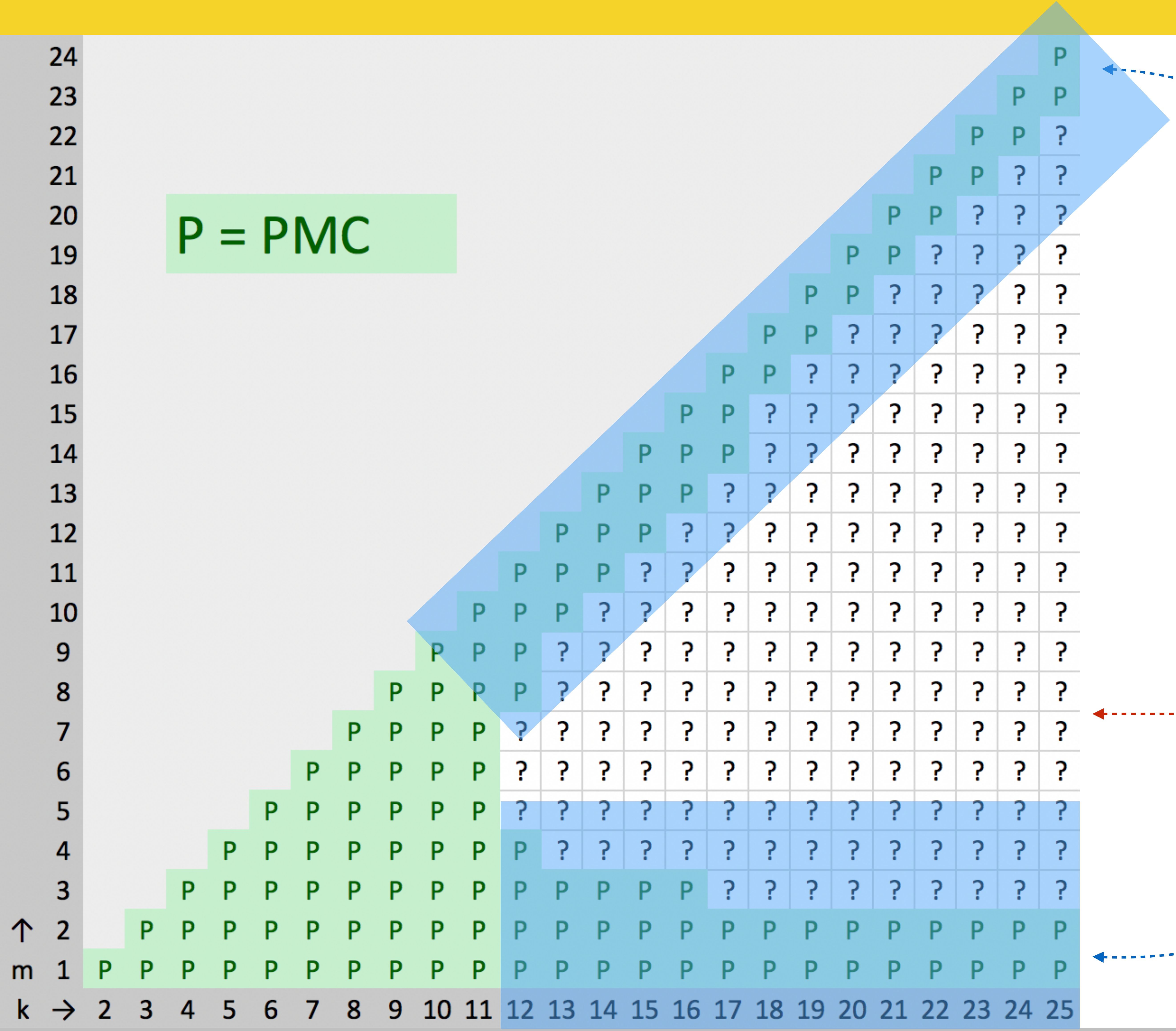
PMC times out after 1 hour for each k > 11

Optimizing for the **Common Case k - m ≪ k**

Store **positions of all failed iterations**, instead of the entire history



Does the Optimized PMC **Scale** with k?



P = PMC

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3			P	P	P	P	P	P	P	P	P	P	P	P	P	?	?	?	?	?	?	?	?	?
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m 1	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
k →	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

PMC scales for large k if m « k or k - m « k

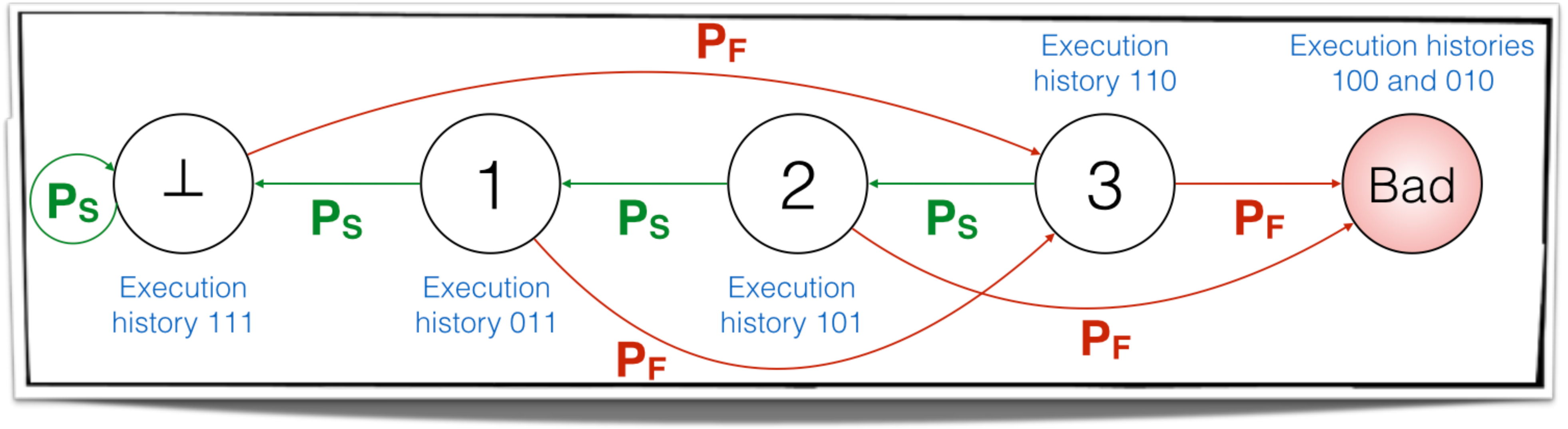
Scalability still a problem for the general case

The Martingale Approach **(Mart)**

Exact, less generic, but slightly faster

Exact Model Checking Slows Down PRISM

Markov model



System of linear equations

Model building

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ -1 & 110 & 10 & 110 \\ -1 & 10 & 1090/9 & 10 \\ -1 & 0 & 100/9 & 1000/9 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Model solving

MTTF (expected reward)

Probabilistic model checking (PRISM under the hood)

For error-free computation

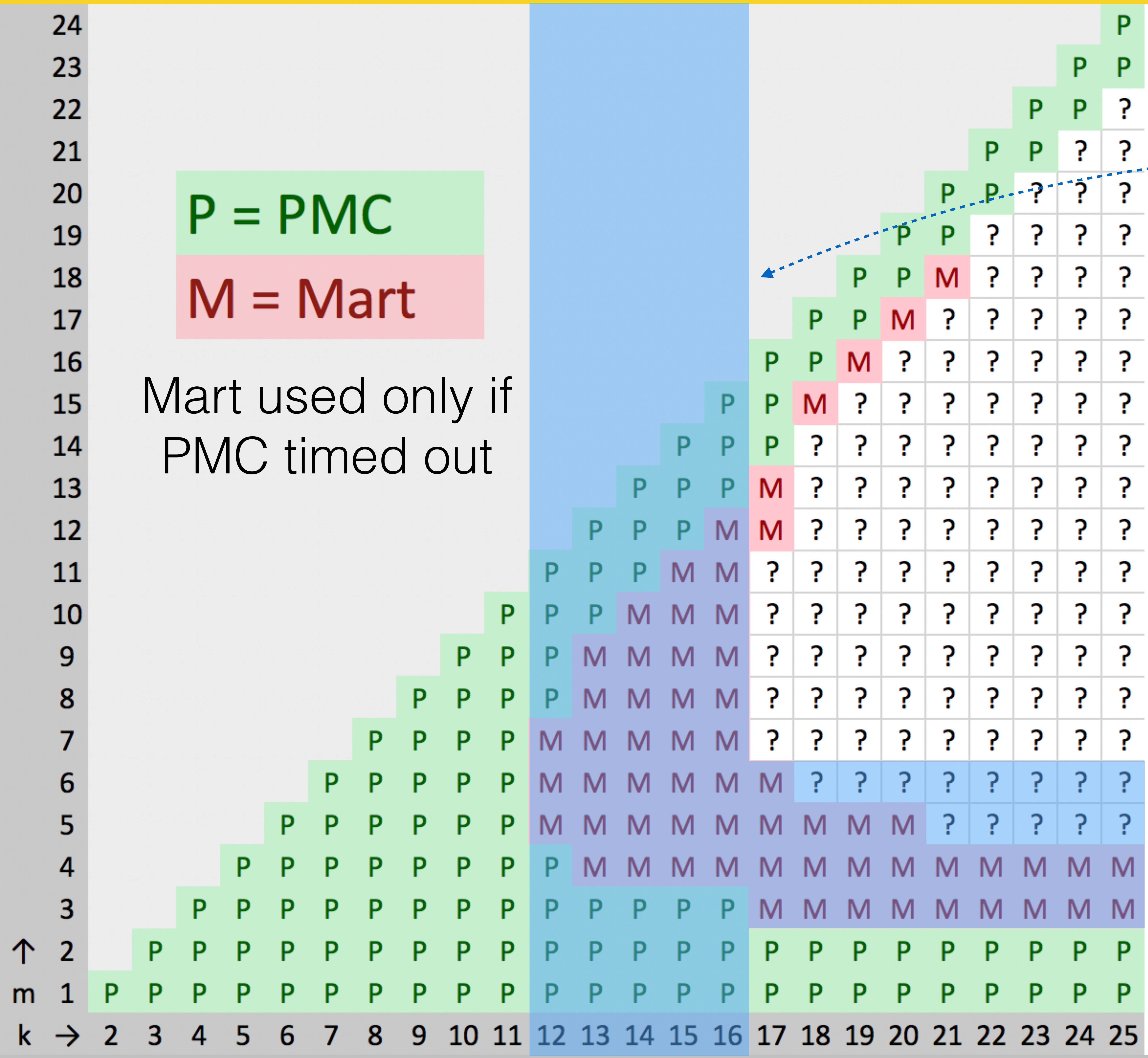
PRISM must be configured with **exact model checking** (i.e., no floating points)

Using **martingale theory***

- ➡ Linear equations obtained directly
- ➡ Bypass PRISM, use highly-scalable BLAS/ LAPACK libraries, with very high precision

* Li. “A Martingale Approach to the Study of Occurrence of Sequence Patterns in Repeated Experiments.” The Annals of Probability 8.6 (1980):1171–1176.

Mart Scales Better than PMC



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	3			P	P	P	P	P	P	P	P	P	P	P	P	P	M	M	M	M	M	M	M	M	M
↑	2		P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
m	1	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
k	→	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

P = PMC

M = Mart

Mart used only if PMC timed out

Mart helps scale up exact MTTF estimation to **k = 16**

Also, Mart implicitly benefits from small values of m

Scalability still a problem for the general case

Sound **Ap**proximation **(SAp)**

Not exact, least generic, but highly scalable

Sound Approximation (SAp) for Single (m, k) Constraint

MTTF = Expected time to 1st temporal robustness violation

$$= \sum_{n=0}^{\infty} \left(nT \times \overbrace{\Pr[1^{st} \text{ violation in the } n^{th} \text{ iteration}]}^{f(n)} \right)$$

Approximation accuracy

- ➡ Accuracy of $f_{LB}(n)$ (reliability modeling literature*)
- ➡ The choice of $n_0, n_1, n_2, \ldots, n_D$ (heuristics based on $f_{LB}(n)$'s shape)

$\mathbf{MTTF_{LB}}$

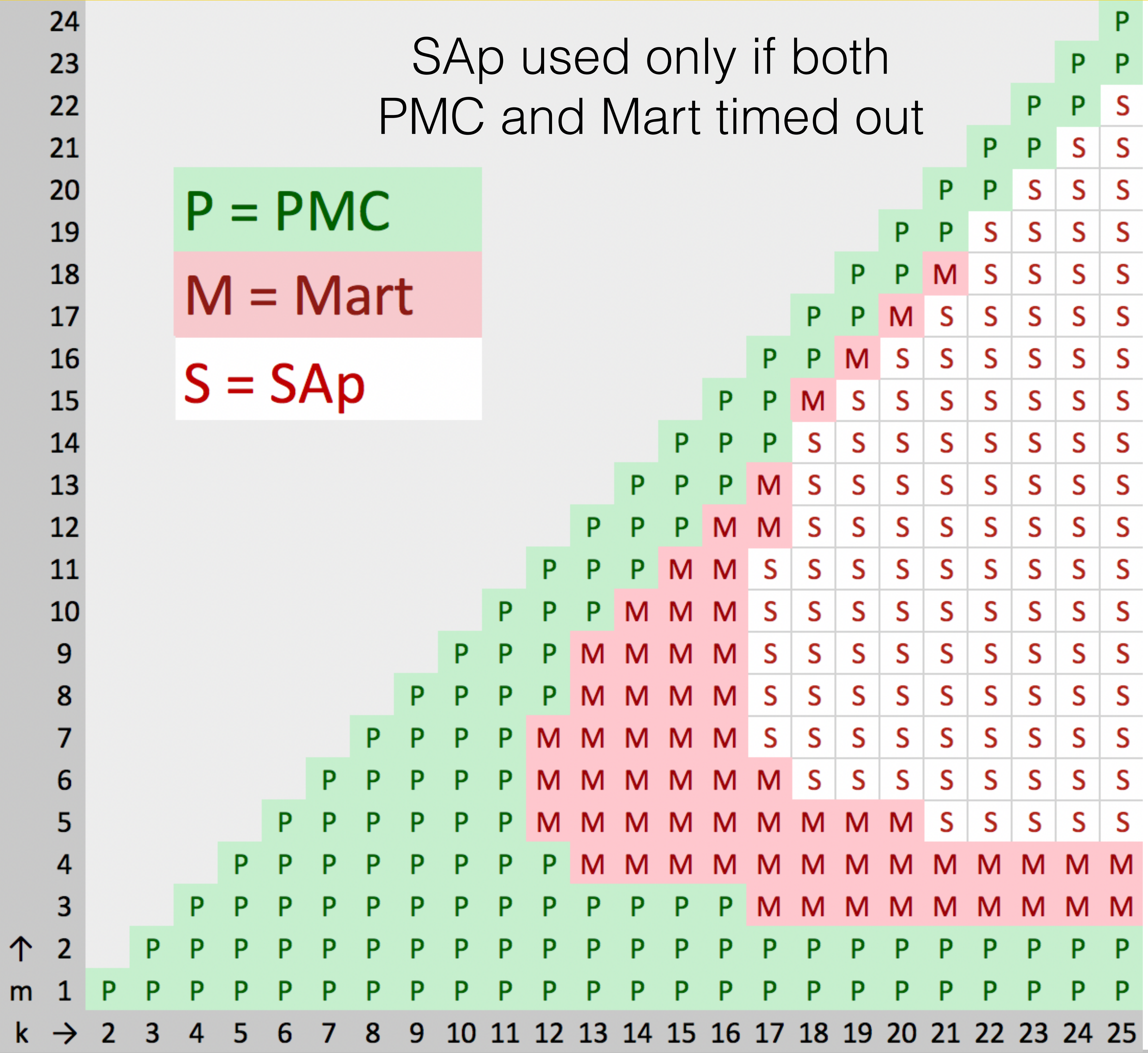
① Obtain $\mathbf{f_{LB}(n) \leq f(n)}$ that can be quickly computed for large n

② Compute $\mathbf{f_{LB}(n_0), f_{LB}(n_1), \ldots, f_{LB}(n_D)}$

③ **Numerically integrate** over subintervals $(n_0, n_1], \ldots, (n_{D-1}, n_D]$

* Sfakianakis et al.. "Reliability of a consecutive k-out-of-r-from-n: F system." IEEE Transactions on Reliability 41.3 (1992): 442-447.

SAp is Scalable to Very Large Window Sizes



SAp used only if both PMC and Mart timed out

P = PMC

M = Mart

S = SAp

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SAp comfortably scales for windows of size k = 1000

How **Accurate** is SAp?

All errors are positive (SAp is proven to under-approximate the exact MTTF)

	11											81.45
	10		Error > 50 %								79.25	69.20
	9		25 % < Error ≤ 50 %							76.61	65.85	58.25
	8		Error ≤ 25 %						73.38	62.01	54.73	47.79
	7							69.36	57.75	50.31	44.30	39.06
	6						64.29	52.99	44.91	40.11	35.35	31.43
	5					57.84	47.34	39.12	34.28	31.22	28.13	25.06
	4				49.62	39.96	33.44	28.17	24.82	22.98	21.64	20.28
	3			39.29	30.21	25.73	22.68	20.09	17.80	15.93	14.55	13.63
↑	2		25.91	19.44	15.77	13.60	12.30	11.50	10.98	10.62	10.35	10.13
m	1	05.76	05.76	05.76	05.76	05.76	05.76	05.76	05.76	05.76	05.76	05.76
k	→	2	3	4	5	6	7	8	9	10	11	12

SAp is reasonably accurate

Example: If $\text{MTTF}_{\text{exact}} = 10^9$ hours,
100% error ➡ $\text{MTTF}_{\text{SAp}} = 0.5 \times 10^9$ hours

Relative errors significant even for small k

- ➡ Exact analysis needed when feasible

Summary

Approach	Accuracy	Expressiveness	Scalability
PMC	Exact	General system, any weakly-hard constraint	Poor (k ≤ 11)
Mart	Exact	IID systems, any weakly-hard constraint	Poor (k ≤ 16)
SAp	Sound approx. (≤ 100%)	IID systems, single (m, k) constraint	Good (k ≤ 1000)

Future work: Make **SAp more expressive**

- ➡ Handle other / multiple weakly-hard constraints
- ➡ Beyond IID iteration failure probabilities

More in the paper!

- ➡ PRISM code and Mart example
- ➡ PMC / Mart for $\langle m, k \rangle$ and $\overline{\langle m \rangle}$ constraints
- ➡ SAp details and soundness proofs
- ➡ More extensive evaluation of PRISM