Qn. Set Code-1

Semester: 3rd
Programme: B.Tech
Branch: CSE, IT, CSCE

AUTUMN END SEMESTER EXAMINATION-2022

3rd Semester B.Tech

PROBABILITY AND STATISTICS

MA2011

(For 2022 (L.E), 2021 & Previous Admitted Batches)

Time: 3 Hours Full Marks: 50

Answer any SIX questions.
Question paper consists of four SECTIONS i.e. A, B, C and D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

1. Answer the following questions. $[1 \times 10]$

(a) You have three similar looking keys; only one of them fits into the lock on your door. If X is the number of trials needed to open the door find the expected number of trials E(X).

(b) Let $f(x) = ke^{-|x|},\ -\infty < x < \infty$ be the probability density function for a continuous random variable X. Determine the value of k.

(c) Let X be a continuous random variable uniformly distributed in the range $-2 \le x \le 2$, then determine $P(|X-1| > \frac{1}{2})$.

(d) Two random variables X and Y have the mean 2 and 3, respectively. If $E(aX - 2Y) = 12$, find a and then find variance $V(aX - 2Y)$.

(e) If X is a random variable such that $E(X) = 3$, $E(X^2) = 13$, using Chebyshev inequality determine the lowerbound for $P(-2 < X < 8)$.

(f) A student computed the mean and variance of a random variable as 2.4 and 3.4, respectively. Is it possible that the random variable follows the binomial distribution? Justify your answer.

(g) Determine the value of k so that the following function $p(x,y) = k|x-y|,\ x = -2,0,2; y = -2,3$ represents a joint probability mass function for two discrete random variables X and Y.

(h) If a normal distribution has mean $\mu = 30$ and standard deviation $\sigma = 5$, what is the 95^{th} percentile of the distribution? Use $\Phi(z) = 0.95$, for $z = 1.645$, $\Phi(z)$ being the cumulative distribution function for the standard normal variate z.

(i) What is the difference between a parameter and a statistic?

(j) Is confidence interval a random variable? Justify your answer.

SECTION-B

2. (a) Campus-15 of KIIT University has two lifts, operating independently. The probability that lift -1 is available when needed is 0.45 and that of lift-2 is 0.55. (i) What is the probability that a lift is available when needed? (ii) what is the probability that neither of them is available when needed? [4]

(b) If $P(A) = \frac{1}{3}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{3}$, find $P(A|B')$ and $P(A' \cup B'|B')$. [4]

3. (a) Find the maximum likelihood estimation for the mean and variance of a normal distribution. [4]

(b) In a certain assembly plant, three machines, $B1$, $B2$, and $B3$, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Suppose that a finished product is randomly selected. If it is defective what is the probability that it is made by machine B1? [4]

SECTION-C

4. (a) The 3rd semester students of KIIT university appeared the test on PS. The marks were found to be normally distributed with mean 60 and standard deviation 5. Determine the percentage of students scored (i) more than 60 marks; (ii) between 45 and 65 marks. [4]

   [Use $\Phi(0) = 0.5, \Phi(-3) = 0.0013$ and $\Phi(1) = 0.8413$].

   (b) Find the maximum likelihood estimation for the parameter p of a Binomial distribution, p being the probability of success. [4]

5. (a) Let $f(x,y) = \begin{cases} \frac{6-x-y}{8}, 0 \le x \le 2, 2 \le y \le 4 \\ 0 \quad\quad otherwise \end{cases}$ be the joint pdf for the continuous random variable X and Y. [4]

   Find $P(X + Y < 1)$ and $P(1 < Y < 3|X = 1)$.

   (b) If the joint probability mass function of X and Y is given by $f(x,y) = \frac{x+y}{30}$ $for\ x = 0,1,2,3$ and $y = 0,1,2$, find $P(X > Y)$ and $P(X + Y = 4)$. [4]

6. (a) Let $f(x,y) = \begin{cases} 24xy, 0 \le x \le 1, 0 \le y \le 1, x + y \le 1 \\ 0 \quad\quad otherwise \end{cases}$ [4]

   Find Covariance of X and Y i.e., $COV(X,Y)$.

(b) If the joint probability mass function of X and Y is given by $f(x,y) = \frac{x+y}{30}$ $for\ x = 0,1,2,3$ and $y = 0,1,2,$ find $COV(X,Y)$. [4]

SECTION-D

7. (a) Consider that the marks obtained by the students in the subject P&S follow normal distribution. If 31% of students have scored under 45 marks and that of 8% have scored over 70 marks. Determine the mean and standard deviation of the mark distribution. [4]

   [Use $\Phi(-1.405) = 0.08$ and $\Phi(-0.496) = 0.31$]

   (b) Find 90% confidence interval for the mean μ of a normal population with variance 0.25, using a sample size 100 with mean 212.3. What sample size would be needed for obtaining a 99% confidence interval of length 2? [4]

   Use the following table

γ	0.90	0.99
c	1.645	2.576

8. (a) A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Determine the expected value and the variance of the number of good components in this sample. [4]

   (b) A normal population has mean 24 and standard deviation 3. Consider a random sample of size 10 with mean $\bar{x}$. Find the region of acceptance for the null hypothesis [4]

   $\mu = \mu_0 = 24$ against the alternative hypothesis $\mu \neq \mu_0$ with 5% level of significance.

   [Use $\Phi(1.960) = 0.975, \Phi(-1.645) = 0.05$]

*****