# AUTUMN MID SEMESTER EXAMINATION-2024

School of Computer Engineering
Kalinga Institute of Industrial Technology, Deemed to be University
Design and Analysis of Algorithms
[CS30001]

**Time: 1 1/2 Hours**                                          **Full Mark: 20**

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*Answer Any four questions including question No.1 which is compulsory.*
*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable and all parts of a*
*question should be answered at one place only.*

1.    Answer all the questions.                                          [ 1 Mark X 5 ]

a) Let $\Sigma$ be a sorted array of n integers. Let $T(n)$ denotes the time taken for the most efficient algorithm to determine whether there are two elements in $\Sigma$ having sum less than 100. Then what is the value of $T(n)$ in asymptotic notation? Justify your answer.

b) Work out the time complexity of the following code snippet using asymptotic notation:

```
for( int i = n;  i > 0;  i /= 2 ) {
  for( int j = 1;  j < n;  j *= 2 ) {
    for( int k = 0;  k < n;  k += 2 ) {
        ... // constant number of operations
    }
  }
}
```

Justify your answer with legitimate arguments.

c) In a graph $G = (V, E)$ with distinct costs on all edges, a subset of vertices T is selected such that $T \subseteq V$. Let $e$ be the minimum cost edge between any vertex in T and any vertex in $V - T$. Then, pick the correct statement about a minimum-cost spanning tree $\Sigma$ of G from the following:
   i.   $\Sigma$ may not include $e$ depending on the algorithm
   ii.  $\Sigma$ must not include $e$
   iii. $\Sigma$ must include $e$
   iv.  $\Sigma$ must include all the edges with cost less that $e$

d) What is the total number of merging required to generate the Huffman tree for a set of $N$ characters? Justify your answer with suitable reasoning.

e) Which data structure is commonly used to detect cycles efficiently while constructing the minimum-cost spanning tree using Kruskal's algorithm?

2. (a) There are seven (7) stock instruments available for purchase as per the following table. If you have an available fund of Rs. 1,00,000/-, create an investment strategy to maximize your overall profit. Assume that, you may purchase part of an instrument, if required.

| Instrument | Instrument Price (Rs) | Expected Profit per Instrument (Rs) | Number of instruments available |
|------------|----------------------|-------------------------------------|---------------------------------|
| A | 1000 | 150 | 6 |
| B | 4000 | 700 | 8 |
| C | 2000 | 200 | 5 |
| D | 8000 | 1500 | 4 |
| E | 4000 | 500 | 8 |
| F | 5000 | 600 | 4 |
| G | 2000 | 300 | 3 |

(b) Given $\log n$ number of sorted lists, each of which is of size $\frac{n}{\log n}$, find the total time taken to merge them into a single list. Justify your answer with proper reasoning. [ 3 + 2 = 5 Marks ]

3. (a) Consider an alternative to binary search called ternary search, which divides the input array into three equal sub-arrays and recursively searches one of these three segments. Write the algorithm for the ternary search procedure and find the key element 13 from the array A = {2, 3, 5, 6, 8, 9, 12, 13, 14} with indices from 1 to 9 using the above ternary search procedure.

(b) Solve the following recurrence using Master's Theorem: $T(n) = 2T(\sqrt{n}) + \log(n)$

[ 3 + 2 = 5 Marks ]

4. The following graph shows the cities A, B, C, D, E, F, and G represented by the vertices and the distance between them represented by the weighted edges. Find the shortest path between city A to city G using Dijkstra's algorithm. Show each step of the algorithm in detail with legitimate explanation. [ 5 Marks ]
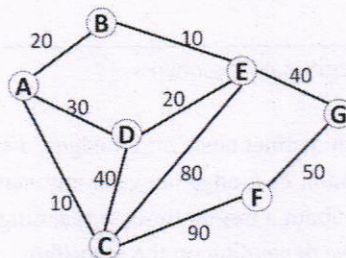


5. (a) Find a sub-sequence using the greedy approach for the following jobs with respect to their deadlines that gives a maximum profit. Note that each job takes a single unit of time for their execution.

| Job | J1 | J2 | J3 | J4 | J5 | J6 | J7 | J8 | J9 |
|----------|----|----|----|----|----|----|----|----|----|
| Profit | 15 | 20 | 30 | 18 | 18 | 10 | 23 | 1 | 25 |
| Deadline | 7 | 2 | 5 | 3 | 4 | 5 | 2 | 7 | 3 |

(b) Given the following character set with their frequencies listed side-by-side:

A: 45, B: 16, C: 12, D: 13, E: 9, F: 5

find an optimal prefix code for the character set using Huffman's algorithm. [ 3 + 2 = 5 Marks ]

*** Best of Luck ***