Kalinga Institute of Industrial Technology

Deemed to be University, Bhubaneswar

Mid Semester Examination-SPRING-2020

Probability & Statistics

MA-2011

Semester- 4th

Time: 1 hour 30 minutes **Full Marks: 20**

Answer four questions including question number 1

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. Answer all parts. $[1 \times 5 = 5]$

(i) A die is thrown 4 times. Find the probability of getting at least one "six".

(ii) For any events A and B with $P(B) > 0$, show that $P(A \mid B) + P(A' \mid B) = 1$.

(iii) Find the suitable value of k, for which the function $f(x) = \frac{k\mu^x}{x!}$, $(\mu > 0)$, $x = 0,1,2,3,\ldots$, represents a probability mass function.

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(v) Find $E(2X + 5)$, given that mean of X is 6.

(vi) Show that $Var(aX + b) = a^2 Var(X)$, where a, b are constants where $Var(X)$ denotes variance of random variable X.

2. (a) Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products. [3]

   i. What is the probability that a randomly selected adult regularly consumes both coffee and soda?

   ii. What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

   (b) A boiler has five identical relief valves. The probability that any particular valve will open on demand is 0.95. Assuming independent operation of the valves, calculate P (at least one valve opens) and P (at least one valve fails to open). [2]

3. (a) The probability density function of a random variable X is [1+2]

$$f(x) = \{k(x-1)(2-x) \ for\ 1 \le x \le 2;\ 0\ elsewhere$$

(i) Determine the value of the constant k,

(ii) Find using distribution function of $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$.

(b) For any events A, B, C, show that $P(A \cap B \cap C) = P(A)P(B \mid A)P(C \mid AB)$ [2]

4. (a) Find the mean and variance of the binomial (n, p) distribution. [3]

(b) Show that $Var(X) = E(X(X-1)) - E(X)(E(X)-1)$. [2]

5. (a) A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is given in the accompanying table. [2]

x	0	1	2	3	4	5	6
$p(x)$	0.10	0.15	0.20	0.25	0.20	0.06	0.04

Calculate the probability of each of the following events.

(i) {between two and four lines , inclusive, are not in use}

(ii) {at least four lines are not in use}

(b) A chemical supply company currently has in stock 100 lb of a certain chemical, which it sells to customers in 5-lb batches. Let X = the number of batches ordered by a randomly chosen customer, and suppose that X has pmf [3]

x	1	2	3	4
$p(x)$	0.2	0.4	0.3	0.1

Compute $E(X)$ and $V(X)$. Then compute the expected number of pounds left after the next customer's order is shipped and the variance of the number of pounds left

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