MODULE 2

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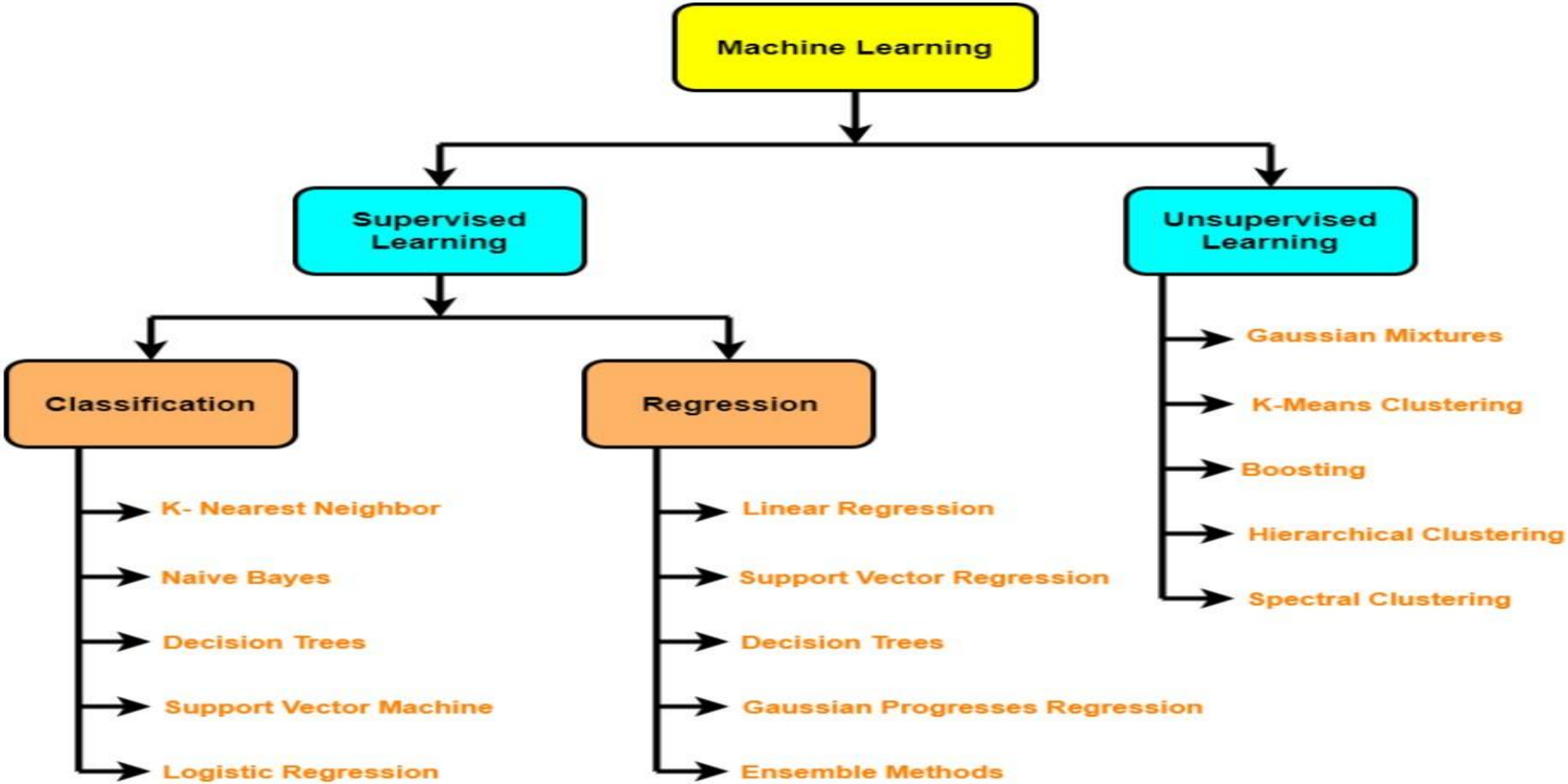

Machine Learning
Supervised Learning
Unsupervised Learning
Classification
Regression
K- Nearest Neighbor
Naive Bayes
Decision Trees
Support Vector Machine
Logistic Regression
Linear Regression
Support Vector Regression
Decision Trees
Gaussian Progresses Regression
Ensemble Methods
Gaussian Mixtures
K-Means Clustering
Boosting
Hierarchical Clustering
Spectral Clustering

K-Nearest Neighbor(KNN) Algorithm

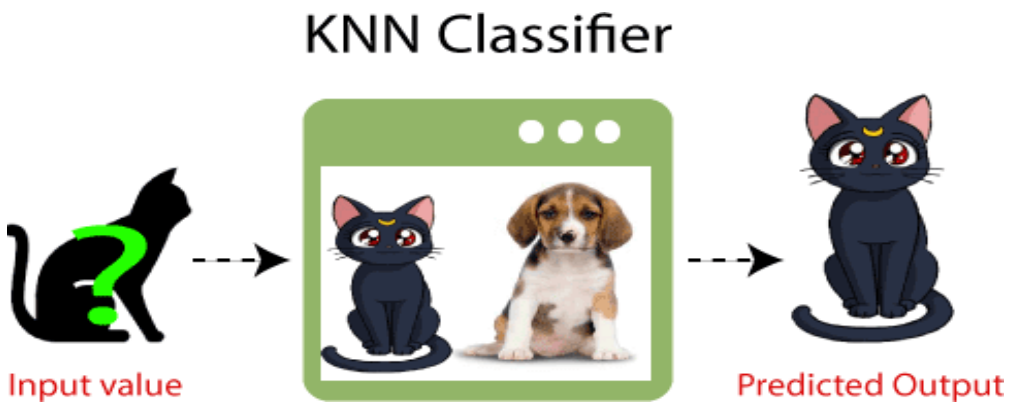
K-Nearest Neighbor(KNN) Algorithm for Machine Learning

- K-Nearest Neighbour is one of the simplest Machine Learning algorithms based on Supervised Learning technique.
- K-NN algorithm assumes the similarity between the new case/data and available cases and put the new case into the category that is most similar to the available categories.
- K-NN algorithm stores all the available data and classifies a new data point based on the similarity. This means when new data appears then it can be easily classified into a well suite category by using K- NN algorithm.
- K-NN algorithm can be used for Regression as well as for Classification but mostly it is used for the Classification problems.
- K-NN is a non-parametric algorithm, which means it does not make any assumption on underlying data.
- It is also called a lazy learner algorithm because it does not learn from the training set immediately instead it stores the dataset and at the time of classification, it performs an action on the dataset.

K-Nearest Neighbor(KNN) Algorithm for Machine Learning

KNN algorithm at the training phase just stores the dataset and when it gets new data, then it classifies that data into a category that is much similar to the new data.

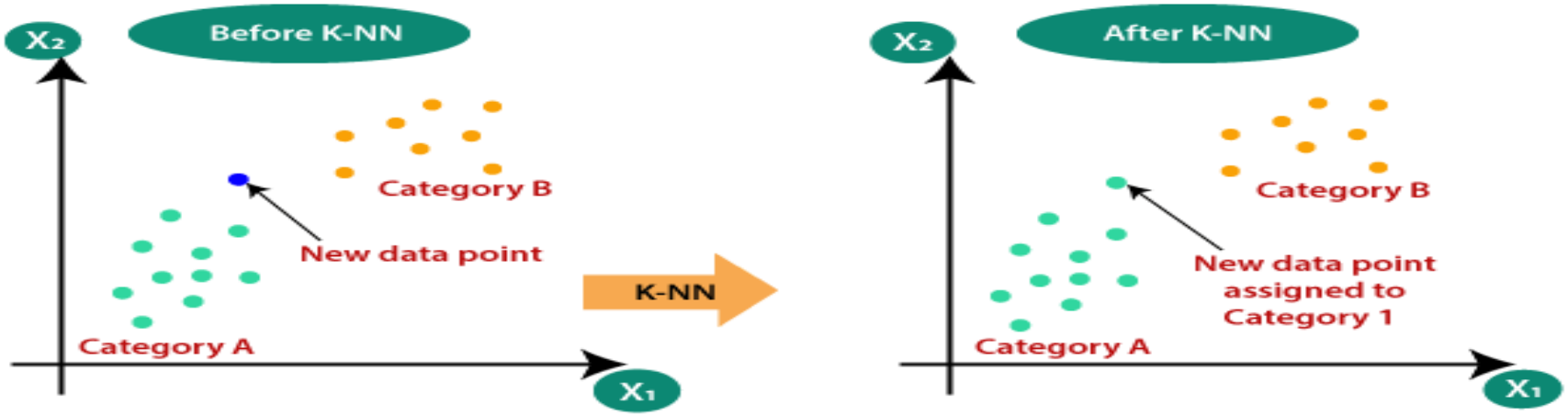
Example: Suppose, we have an image of a creature that looks similar to cat and dog, but we want to know either it is a cat or dog. So for this identification, we can use the KNN algorithm, as it works on a similarity measure. Our KNN model will find the similar features of the new data set to the cats and dogs images and based on the most similar features it will put it in either cat or dog category.



K-Nearest Neighbor(KNN) Algorithm for Machine Learning

Why do we need a K-NN Algorithm?

Suppose there are two categories, i.e., Category A and Category B, and we have a new data point x1, so this data point will lie in which of these categories. To solve this type of problem, we need a K-NN algorithm. With the help of K-NN, we can easily identify the category or class of a particular dataset. Consider the below diagram:



K-Nearest Neighbor(KNN) Algorithm for Machine Learning

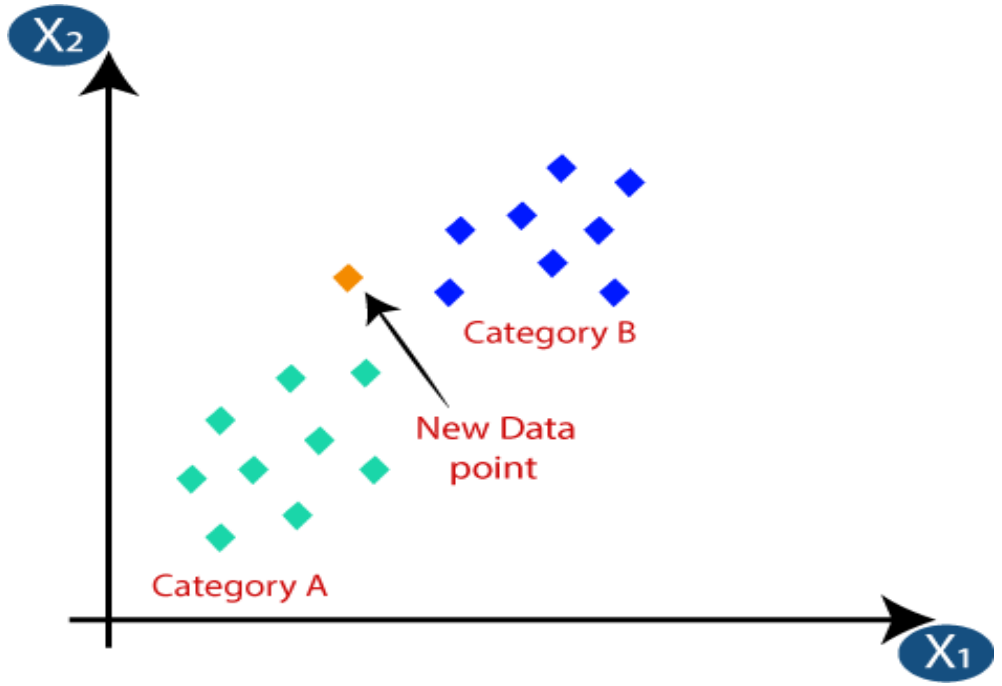
How does K-NN work?

The K-NN working can be explained on the basis of the below algorithm:

- **Step-1:** Select the number K of the neighbors
- **Step-2:** Calculate the Euclidean distance of **K number of neighbors**
- **Step-3:** Take the K nearest neighbors as per the calculated Euclidean distance.
- **Step-4:** Among these k neighbors, count the number of the data points in each category.
- **Step-5:** Assign the new data points to that category for which the number of the neighbor is maximum.
- **Step-6:** Our model is ready.

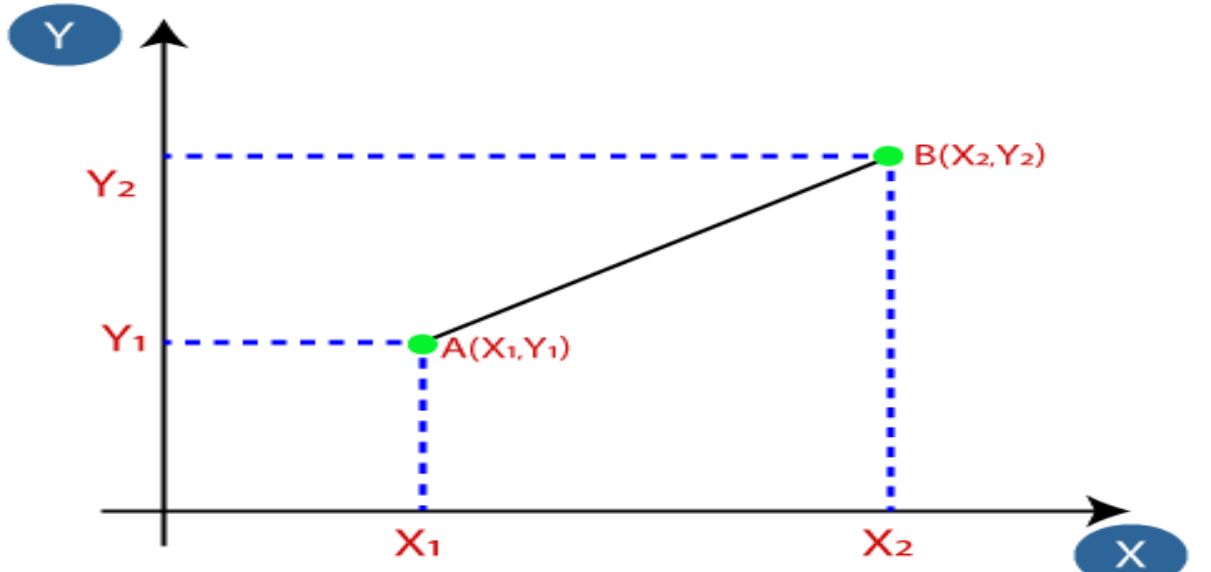
K-Nearest Neighbor(KNN) Algorithm for Machine Learning

Suppose we have a new data point and we need to put it in the required category. Consider the below image:



K-Nearest Neighbor(KNN) Algorithm for Machine Learning

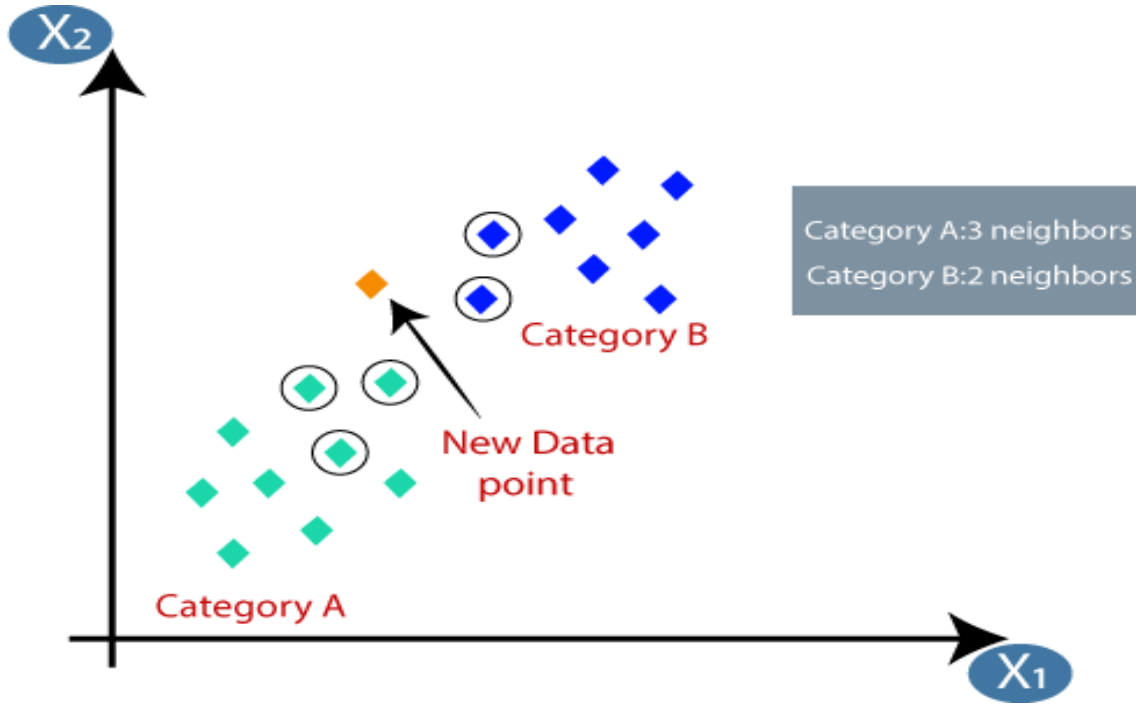
- Firstly, we will choose the number of neighbors, so we will choose the k=5.
- Next, we will calculate the **Euclidean distance** between the data points. The Euclidean distance is the distance between two points, which we have already studied in geometry. It can be calculated as:



Euclidean Distance between A_1 and $B_2 = \sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}$

K-Nearest Neighbor(KNN) Algorithm for Machine Learning

- By calculating the Euclidean distance we got the nearest neighbors, as three nearest neighbors in category A and two nearest neighbors in category B. Consider the below image:



- As we can see the 3 nearest neighbors are from category A, hence this new data point must belong to category A.

K-Nearest Neighbor(KNN) Algorithm for Machine Learning

- There is no particular way to determine the best value for "K", so we need to try some values to find the best out of them. The most preferred value for K is 5.
- A very low value for K such as K=1 or K=2, can be noisy and lead to the effects of outliers in the model.
- Large values for K are good, but it may find some difficulties. Advantages of KNN

Algorithm:

- It is simple to implement.
- It is robust to the noisy training data
- It can be more effective if the training data is large.

Disadvantages of KNN Algorithm:

- Always needs to determine the value of K which may be complex some time.
- The computation cost is high because of calculating the distance between the data points for all the training samples.

KNN Algorithm e.g.

Name	Age	Gender	sport
Ajay	32	M	Football
Mark	40	M	Neither
sara	16	F	Cricket
Zaira	34	F	Cricket
Sachin	55	M	Neither
Rahul	40	M	Cricket
Pooja	20	F	Neither
Smith	15	M	Cricket
Laxmi	55	F	Football
Michael	15	M	Football

KNN Algorithm e.g.

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Ajay	32	M	Football
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Rahul	40	M	Cricket
Pooja	20	F	Neither
Smith	15	M	Cricket
Laxmi	55	F	Football
Michael	15	M	Football
Angelina	5	F	?

male = 0

Female = 1

The distance equation is normally,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

ie Euclidean Distance

(other distances → Manhattan distance
→ minkowski distance
etc.

male = 0

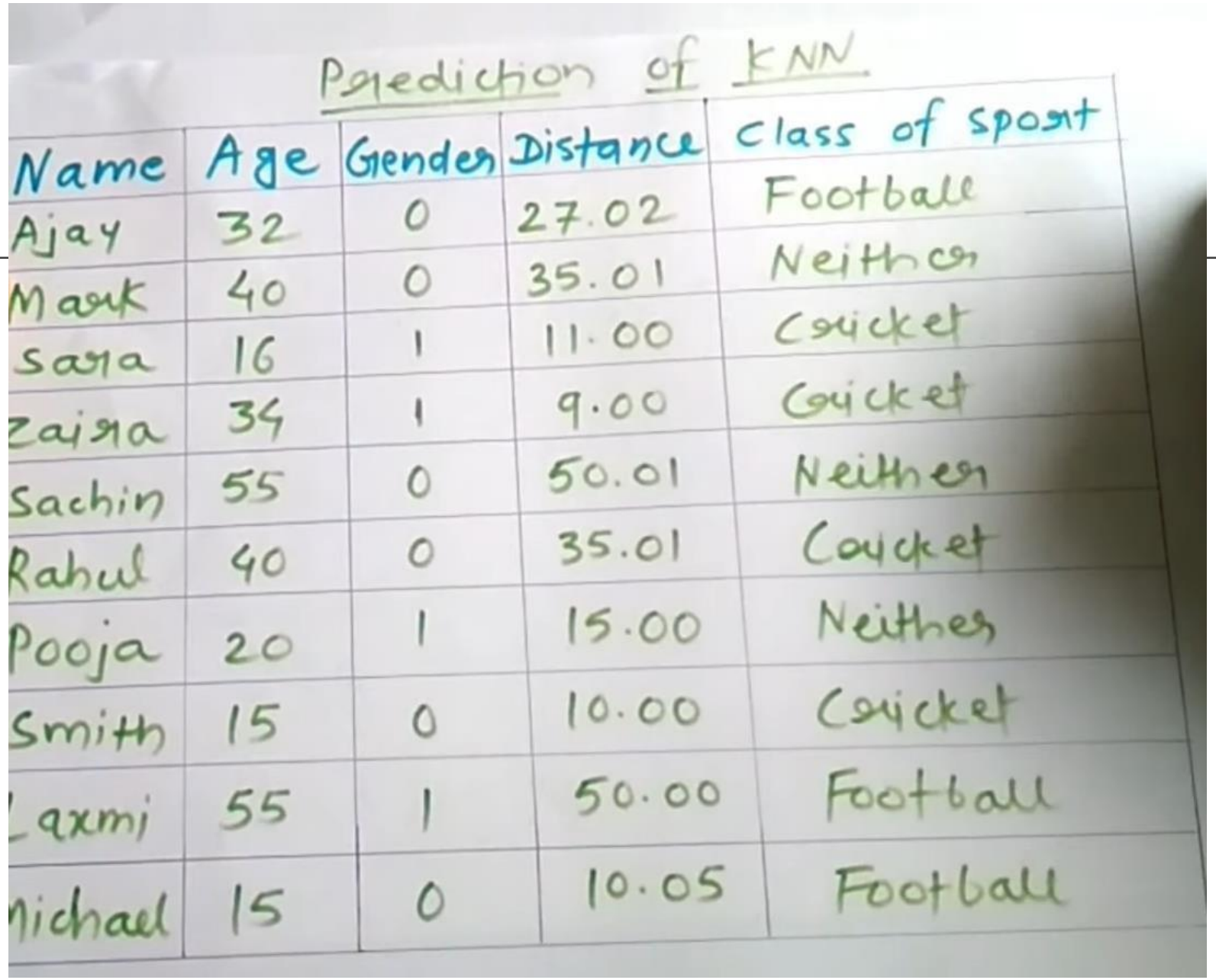
Female = 1

Ajay male = 0 Age = 32

$$= \sqrt{(5-32)^2 + (1-0)^2}$$

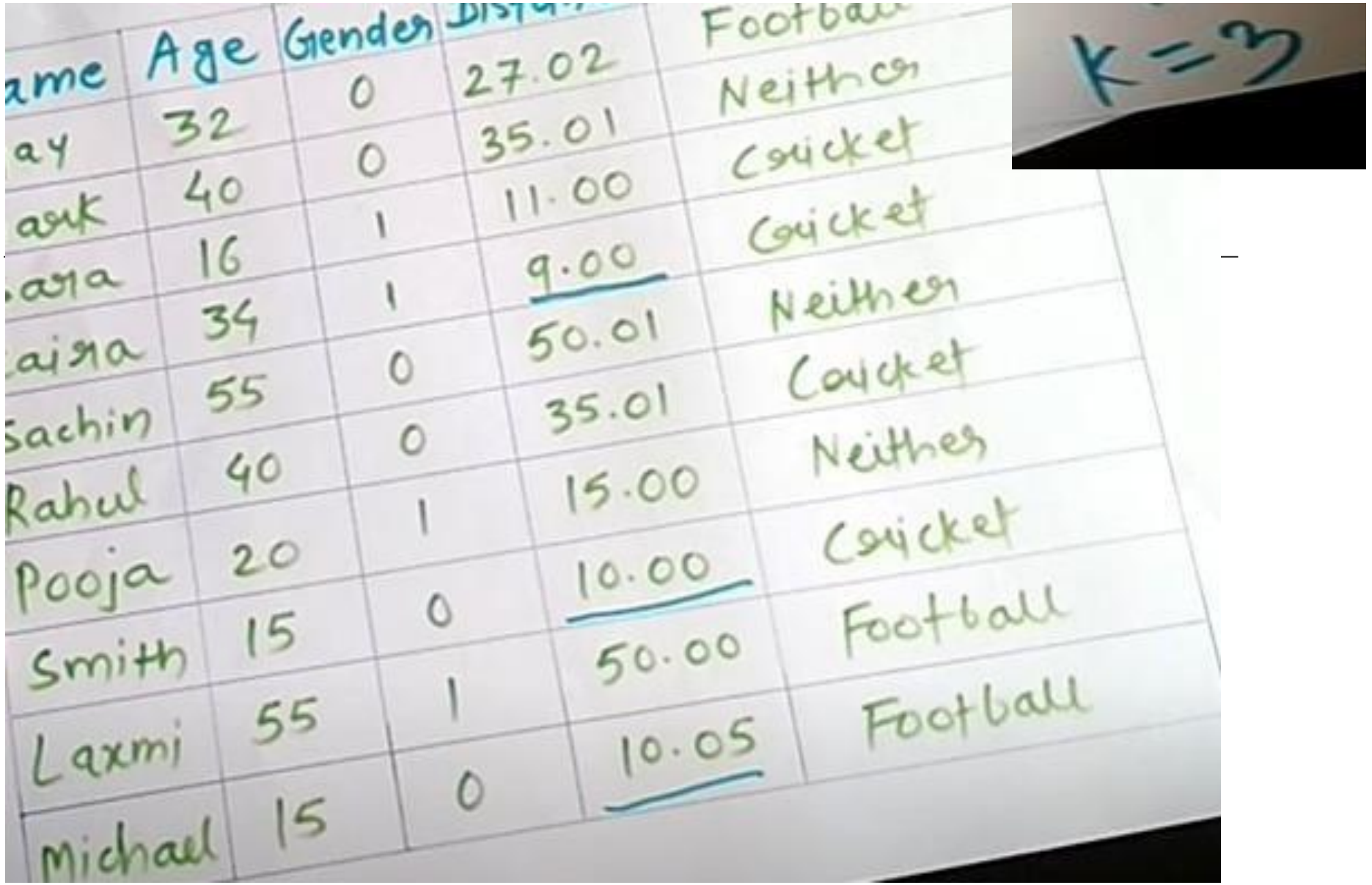
$$= \sqrt{729 + 1}$$

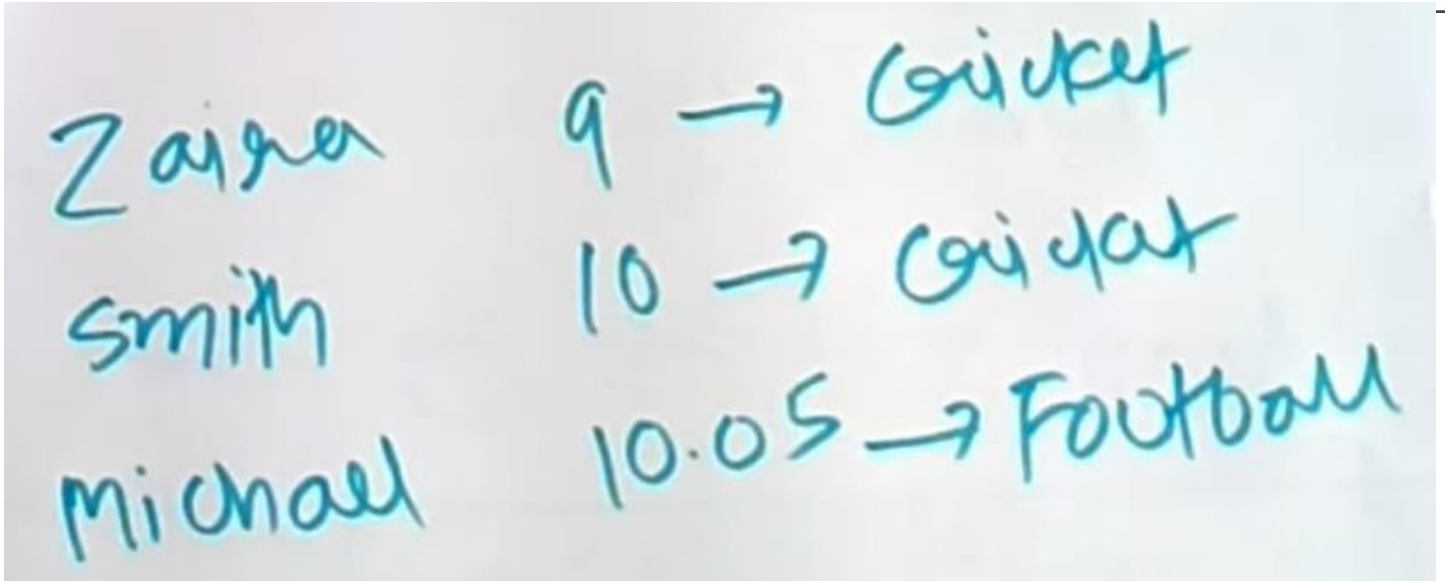
$$= 27.02$$

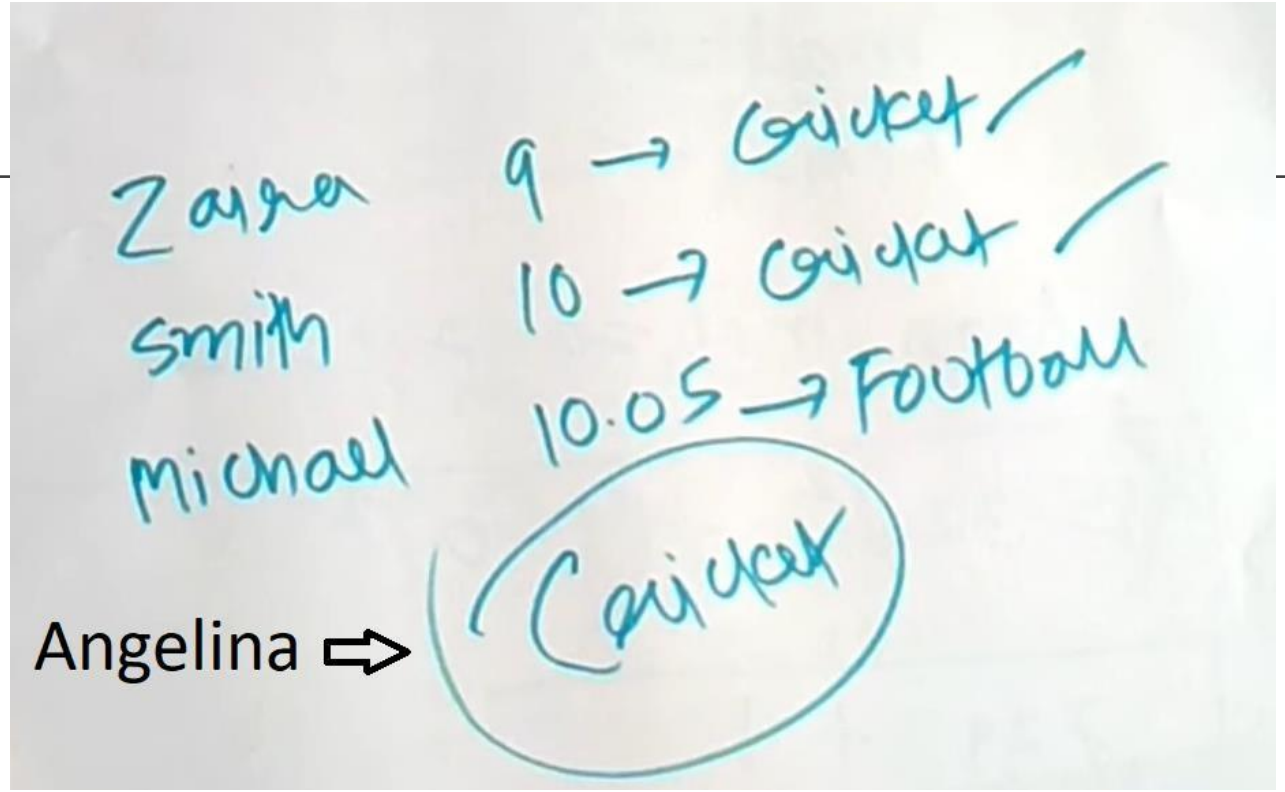


Prediction of KNN

Name	Age	Gender	Distance	Class of sport
Ajay	32	0	27.02	Football
Mark	40	0	35.01	Neither
sara	16	1	11.00	Cricket
Zaira	34	1	9.00	Cricket
Sachin	55	0	50.01	Neither
Rahul	40	0	35.01	Cricket
Pooja	20	1	15.00	Neither
Smith	15	0	10.00	Cricket
Laxmi	55	1	50.00	Football
Michael	15	0	10.05	Football


K=3
ame Age Gender
ay 32 0 27.02 Footba
ark 40 0 35.01 Neither
ara 16 1 11.00 Cricket
aira 34 1 9.00 Cricket
Sachin 55 0 50.01 Neither
Rahul 40 0 35.01 Cricket
Pooja 20 1 15.00 Neither
Smith 15 0 10.00 Cricket
Laxmi 55 1 50.00 Football
Michael 15 0 10.05 Football


Zaira 9 → Cricket
Smith 10 → Cricket
Michael 10.05 → Football


Zayra 9 → Cricket
smith 10 → Cricket
Michael 10.05 → Football
Angelina ⇨
Cricket

KNN Question

A dataset contains information about houses and their prices. Each house is described using **4 features**:

Training Data:

House	Size (sqft)	Bedrooms	Age (years)	Distance (km)	Price
A	1200	2	10	5	L
B	1500	3	5	3	H
C	800	2	20	10	L
D	1800	4	3	2	H
E	1000	3	15	7	L

A new house has the following features:

- Size = 1300 sqft
- Bedrooms = 3
- Age = 8 years
- Distance = 4 km

1. Using the K-Nearest Neighbors (KNN) algorithm with
- K = 3

Predict whether the new house belongs to **High (H)** or **Low (L)** price category.

Distance Calculations (Euclidean)

A (L): (1200,2,10,5)

$$\sqrt{(100)^2 + (1)^2 + (-2)^2 + (-1)^2} = \sqrt{10000 + 1 + 4 + 1} = \sqrt{10006} \approx 100.03$$

B (H): (1500,3,5,3)

$$\sqrt{(-200)^2 + 0^2 + 3^2 + 1^2} = \sqrt{40000 + 0 + 9 + 1} = \sqrt{40010} \approx 200.02$$

C (L): (800,2,20,10)

$$\sqrt{500^2 + 1^2 + (-12)^2 + (-6)^2} = \sqrt{250000 + 1 + 144 + 36} = \sqrt{250181} \approx 500.18$$

D (H): (1800,4,3,2)

$$\sqrt{(-500)^2 + (-1)^2 + 5^2 + 2^2} = \sqrt{250000 + 1 + 25 + 4} = \sqrt{250030} \approx 500.03$$

E (L): (1000,3,15,7)

$$\sqrt{300^2 + 0^2 + (-7)^2 + (-3)^2} = \sqrt{90000 + 0 + 49 + 9} = \sqrt{90058} \approx 300.10$$

Nearest 3 Neighbors

House	Distance	Class
A	100.03	L
B	200.02	H
E	300.10	L

Majority Voting

L, H, L → **2 L vs 1 H**

Final Answer

Predicted class = Low (L)

You are a business analyst at a retail company and want to predict the monthly sales revenue of new stores based on their size (in square feet) and the number of employees.

Dataset:

Suppose you have the following training dataset:

Store Size (sq ft)	Number of Employees	Monthly Sales Revenue ($)
1500	5	30000
2000	7	50000
2500	10	70000
3000	15	80000
3500	20	100000

Goal:

Predict the monthly sales revenue for a new store with a size of 2800 sq ft and 12 employees.

Calculate Distances:

- **For Store 1 (1500 sq ft, 5 employees):** Distance = sqrt((1500 - 2800)² + (5 - 12)²)
  Distance = sqrt((1300)² + (-7)²)
  Distance = sqrt(1690000 + 49)
  Distance ≈ 1300.02
- **For Store 2 (2000 sq ft, 7 employees):** Distance = sqrt((2000 - 2800)² + (7 - 12)²)
  Distance = sqrt((800)² + (-5)²)
  Distance = sqrt(640000 + 25)
  Distance ≈ 800.02
- **For Store 3 (2500 sq ft, 10 employees):** Distance = sqrt((2500 - 2800)² + (10 - 12)²)
  Distance = sqrt((-300)² + (-2)²)
  Distance = sqrt(90000 + 4)
  Distance ≈ 300.01

- **For Store 4 (3000 sq ft, 15 employees):** Distance = sqrt((3000 - 2800)² + (15 - 12)²)
  Distance = sqrt((200)² + (3)²)
  Distance = sqrt(40000 + 9)
  Distance ≈ 200.02
- **For Store 5 (3500 sq ft, 20 employees):** Distance = sqrt((3500 - 2800)² + (20 - 12)²)
  Distance = sqrt((700)² + (8)²)
  Distance = sqrt(490000 + 64)
  Distance ≈ 700.04

Step 1: Data Preparation

Prepare your data using the two features (store size and number of employees) to find the K nearest neighbors for the new store.

Step 2: Choosing K

Choose k = 3. This means you will consider the 3 closest neighbors to make your prediction.

Step 3: Distance Calculation

Calculate the Euclidean distance between the new store and each store in the dataset using the formula:

$$\text{Distance} = \text{sqrt}((X1 - X2)^2 + (Y1 - Y2)^2)$$

Where:

- X1 and Y1 are the features of the new store (size and employees).
- X2 and Y2 are the features of each training store.

Step 4: Finding Neighbors

Now, we have calculated the distances for each store. Here are the distances we obtained:

- Store 1: 1300.02
- Store 2: 800.02
- Store 3: 300.01
- Store 4: 200.02
- Store 5: 700.04

Now, we will select the three closest neighbors (smallest distances):

1. Store 4 (200.02)
2. Store 3 (300.01)
3. Store 5 (700.04)

Step 5: Target Value Prediction

Next, we will take the monthly sales revenues of these three nearest neighbors and calculate their average.

- Store 4 Revenue: $80,000
- Store 3 Revenue: $70,000
- Store 5 Revenue: $100,000

Step 5: Target Value Prediction

Next, we will take the monthly sales revenues of these three nearest neighbors and calculate their average.

- Store 4 Revenue: $80,000
- Store 3 Revenue: $70,000
- Store 5 Revenue: $100,000

Average Revenue = (80000 + 70000 + 100000) / 3
Average Revenue = 250000 / 3
Average Revenue ≈ 83333.33

The predicted monthly sales revenue for the new store (2800 sq ft and 12 employees) is approximately **$83,333.33**.

Common Distance Measures in KNN

1. Manhattan Distance (L1)

$$D = \sum |x_i - y_i|$$

Moves like city blocks (up–down, left–right).

Example:

Points (2,3) and (5,7)

$$|2 - 5| + |3 - 7| = 3 + 4 = 7$$

2. Euclidean Distance (L2)

$$D = \sqrt{\sum (x_i - y_i)^2}$$

Straight-line distance.

Example:

$$\sqrt{(3^2 + 4^2)} = 5$$

3. Minkowski Distance (General Form)

$$D = \left(\sum |x_i - y_i|^p \right)^{1/p}$$

- p = 1 → Manhattan
- p = 2 → Euclidean

4. Chebyshev Distance

$$D = \max(|x_i - y_i|)$$

Only the largest difference matters.

Example:

(2,5) and (6,9) → max(4,4) = 4

5. Hamming Distance

Used for categorical/binary data.

Counts mismatches.

Example:

10101 and 10011 → 2 differences

6. Cosine Distance

$$D = 1 - \frac{A \cdot B}{|A||B|}$$

Measures angle, not magnitude.

Used in text data.

7. Jaccard Distance

$$D = 1 - \frac{|A \cap B|}{|A \cup B|}$$

Used for sets/binary features.

Distance Type	Best For
Euclidean	Continuous numeric data
Manhattan	Grid-like movement
Minkowski	General purpose
Chebyshev	Max difference matters
Hamming	Binary/categorical data
Cosine	Text / high-dimensional
Jaccard	Set/binary features

Curse of Dimensionality

The **curse of dimensionality** refers to the problems that arise when working with data that has **too many features (dimensions)**.

As the number of features increases, the data becomes:

- Very **sparse**
- Harder to analyze
- Less meaningful for distance-based methods like KNN, K-means, etc.

Effect on Machine Learning

- Distance-based models (KNN, clustering) perform poorly
- Models overfit easily
- Training becomes slow and expensive
- Visualization becomes difficult

How to Handle It

- Feature selection (remove unnecessary features)
- Dimensionality reduction (PCA, LDA, Autoencoders)
- Regularization
- Collect more data if possible

The curse of dimensionality means that as the number of features increases, data becomes sparse, distances become meaningless, and learning becomes difficult without a very large amount of data.

Simple Example

Imagine you are placing points in space.

Case 1: 1 Feature (1D)

Suppose feature = Height

Range = 0 to 10

To cover this line well, you may need only **10 points**.

```lua
0----1----2----3----4----5----6----7----8----9----10
```

Points are close and easy to compare.

Case 2: 2 Features (2D)

Features = Height and Weight

Range = 0 to 10 for both

Now space is a square:

Area = 10 × 10 = 100

To cover this space well, you may need about **100 points**.

Case 3: 3 Features (3D)

Features = Height, Weight, Age

Range = 0 to 10

Now space is a cube:

Volume = 10 × 10 × 10 = 1000

You now need about **1000 points**.

Case 4: 10 Features

Range = 0 to 10 for each feature

Space size = 10^{10}

You would need **10,000,000,000 points** to cover the space well — which is impossible in practice.

Applications of K-Nearest Neighbors (KNN)

KNN is a simple, instance-based learning algorithm mainly used for classification and regression.

1. Pattern Recognition

- Handwritten digit recognition
- Face recognition
- Signature verification

2. Recommendation Systems

- Suggesting movies, songs, or products based on similar users
- E-commerce product recommendations

3. Medical Diagnosis

- Disease classification based on patient symptoms
- Cancer detection using medical data

4. Image and Video Analysis

- Image classification
- Object recognition
- Color quantization

5. Text and Document Classification

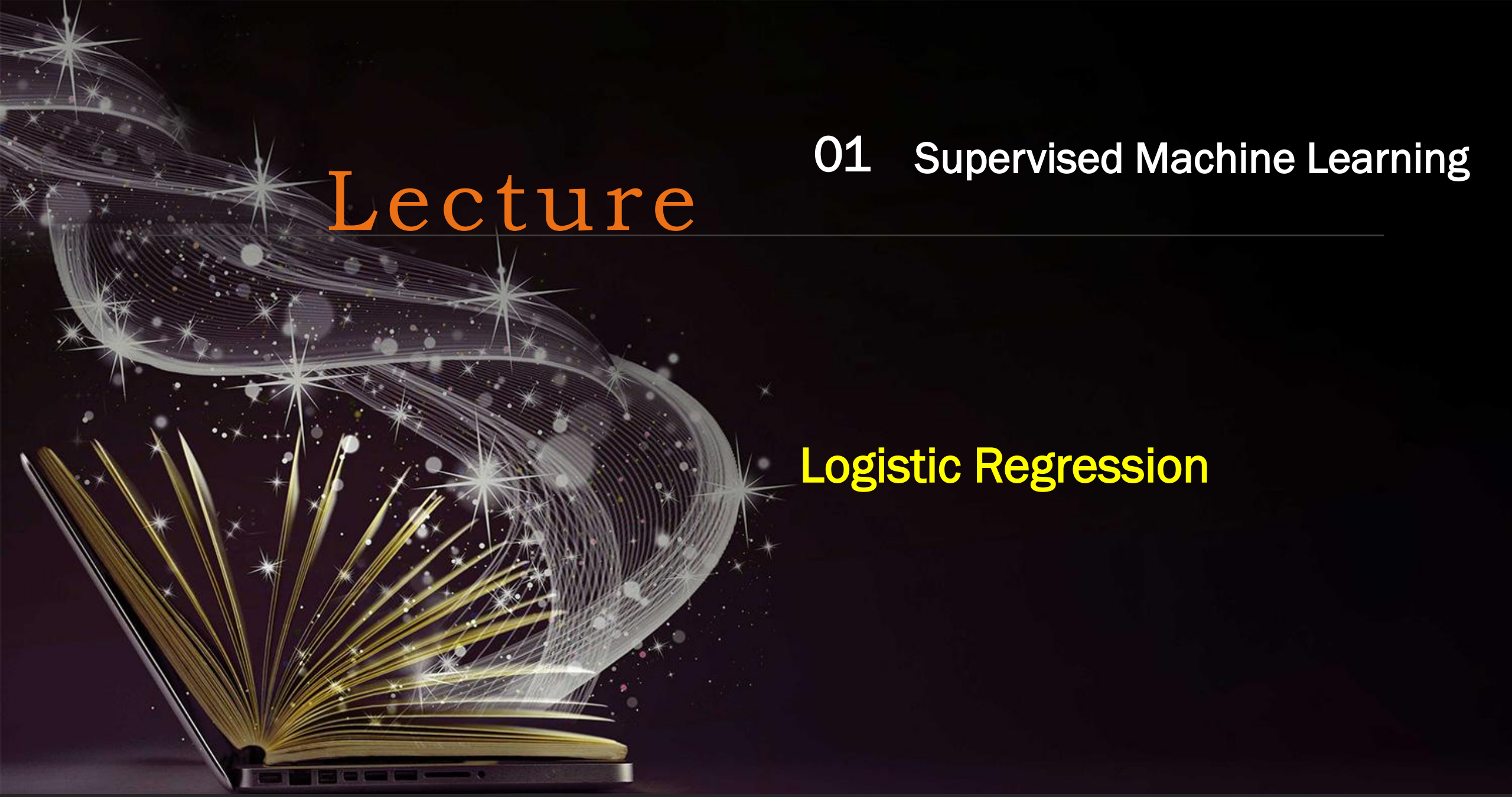
- Spam detection
- News article classification
- Language detection

6. Finance

- Credit scoring
- Risk assessment
- Fraud detection

7. Anomaly Detection

- Detecting unusual patterns in network traffic
- Identifying outliers in data

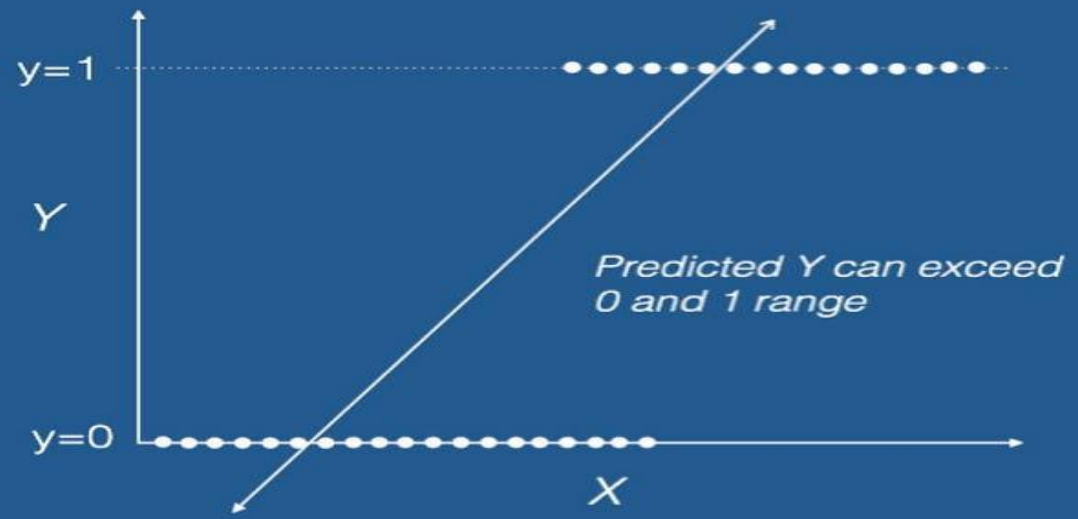


Lecture

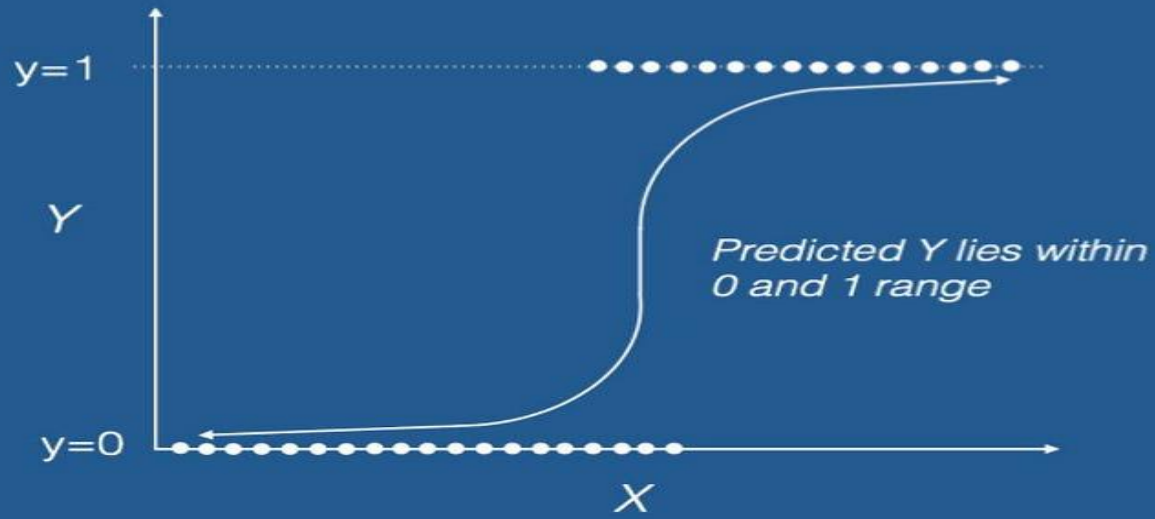
01 Supervised Machine Learning

Logistic Regression

Linear Regression



Logistic Regression



Hours Study	Pass (1) / Fail (0)
29	0
15	0
33	1
28	1
39	1

Logistic regression analysis is used to examine the association of (categorical or continuous) independent variable(s) with one dichotomous dependent variable. This is in contrast to linear regression analysis in which the dependent variable is a continuous variable.

Definition of the logistic function [edit]

An explanation of logistic regression can begin with an explanation of the standard logistic function. The logistic function is a sigmoid function, which takes any real input t, and outputs a value between zero and one.[2] For the logit, this is interpreted as taking input log-odds and having output probability. The *standard* logistic function $\sigma : \mathbb{R} \to (0,1)$ is defined as follows:

$$\sigma(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$$

A graph of the logistic function on the *t*-interval (−6,6) is shown in Figure 1.

Let us assume that t is a linear function of a single explanatory variable x (the case where t is a *linear combination* of multiple explanatory variables is treated similarly). We can then express t as follows:

$$t = \beta_0 + \beta_1 x$$

And the general logistic function $p : \mathbb{R} \to (0,1)$ can now be written as:

$$p(x) = \sigma(t) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

In the logistic model, $p(x)$ is interpreted as the probability of the dependent variable Y equaling a success/case rather than a failure/non-case. It is clear that the response variables Y_i are not identically distributed: $P(Y_i = 1 \mid X)$ differs from one data point X_i to another, though they are independent given design matrix X and shared parameters β.[11]

A company wants to predict whether a customer will buy a product (1 = Buy, 0 = Not Buy) based on one feature:

$x =$ number of hours spent on the website.

The logistic regression model is:

$$P(y = 1|x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

Given:

$\beta_0 = -4$

$\beta_1 = 1.2$

Question:

1. If a customer spends $x = 3$ hours on the website, calculate the probability that the customer will buy the product.
2. Based on a threshold of 0.5, predict whether the customer will buy or not.

Given model:

$$P(y=1|x) = \frac{1}{1+e^{-(\beta_0+\beta_1 x)}}$$

$\beta_0 = -4,\ \beta_1 = 1.2,\ x = 3$

Step 1: Compute the linear term

$$z = \beta_0 + \beta_1 x = -4 + (1.2 \times 3) = -4 + 3.6 = -0.4$$

Step 2: Apply logistic function

$$P = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{0.4}}$$

$$e^{0.4} \approx 1.4918$$

$$P = \frac{1}{1+1.4918} = \frac{1}{2.4918} \approx 0.401$$

Step 3: Prediction using threshold 0.5

$$0.401 < 0.5 \Rightarrow \text{Predict: Not Buy } (0)$$

Logistic Regression Problem Solution

- The dataset of pass or fail in an exam of 5 students is given in the table.
- Use logistic regression as classifier to answer the following questions.

1. Calculate the probability of pass for the student who studied 33 hours.

2. At least how many hours student should study that makes he will pass the course with the probability of more than 95%.

Hours Study	Pass (1) / Fail (0)
29	0
15	0
33	1
28	1
39	1

Assume the model suggested by the optimizer for odds of passing the course is,

$$\log(odds) = -64 + 2 * hours$$

In logistic regression, log-odds refers to the logarithm of the odds of an event occurring. The odds of an event are the ratio of the probability of the event occurring to the probability of it not occurring.

$$\log(odds) = -64 + 2 * hours$$

Odds and Log-Odds

Given a probability p of an event occurring:

$$\text{Odds} = \frac{p}{1-p}$$

The **log-odds** (also called the **logit**) is the natural logarithm of the odds:

$$\text{Log-Odds} = \log\left(\frac{p}{1-p}\right)$$

Logistic Regression and Log-Odds

In logistic regression, the relationship between the independent variables (predictors) and the dependent variable (binary outcome) is modeled in terms of log-odds. The logistic regression model can be written as:

$$\text{Log-Odds} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k$$

where:

- β_0 is the intercept,
- $\beta_1, \beta_2, \ldots, \beta_k$ are the coefficients of the independent variables $x_1, x_2, \ldots, x_k$.

The model expresses the log-odds of the dependent variable being 1 (the event of interest) as a linear combination of the predictors. The coefficients represent the change in log-odds for a one-unit change in the corresponding predictor.

Converting Log-Odds to Probability

The logistic function (or sigmoid function) can be used to convert log-odds back to the probability p:

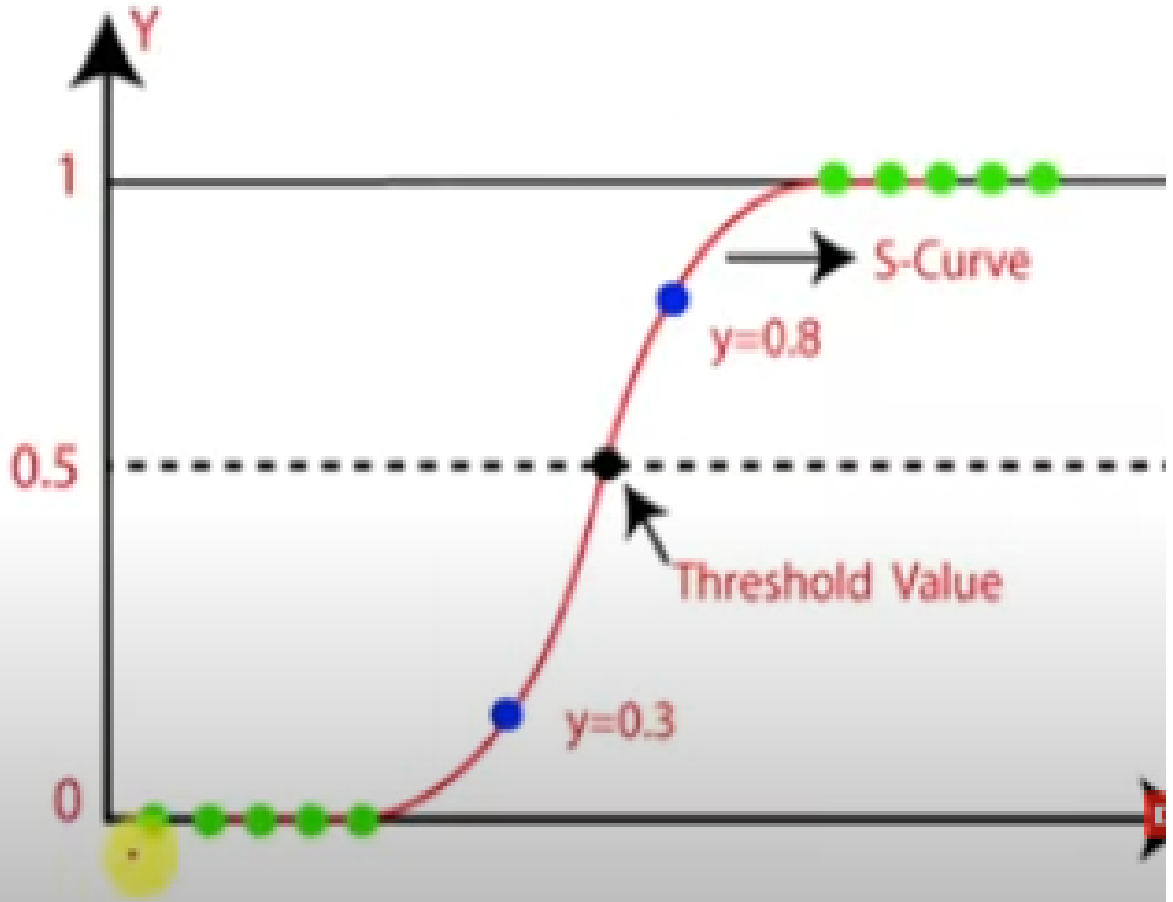
$$p = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k)}}$$

This function ensures that the predicted probabilities lie between 0 and 1.

In summary, in logistic regression, the log-odds are the core concept for modeling the relationship between predictors and the binary outcome.

- We use Sigmoid Function in logistic regression

- $$s(x) = \frac{1}{1+e^{-x}}$$



1. Calculate the probability of pass for the student who studied 33 hours.

- $p = \frac{1}{1+e^{-z}}$ $\qquad s(x) = \frac{1}{1+e^{-x}}$
- $z = -64 + 2 * 33 = -64 + 66 = 2$
- $p = \frac{1}{1+e^{-2}} = 0.88$

Hours Study	Pass (1) / Fail (0)
29	0
15	0
33	1
28	1
39	1

$$\log(odds) = z = -64 + 2 * hours$$

That is, if student studies 33 hours, then there is **88% chance** that the student will pass the exam

2. At least how many hours student should study that makes he will pass the course with the probability of more than 95%.

Hours Study	Pass (1) / Fail (0)
29	0
15	0
33	1
28	1
39	1

- $\boldsymbol{p = \frac{1}{1+e^{-z}} = 0.95}$
- $0.95 * (1 + e^{-z}) = 1$
- $0.95 * e^{-z} = 1 - 0.95$
- $e^{-z} = \frac{0.05}{0.95} = 0.0526$
- $\ln(e^{-z}) = \ln(0.0526)$

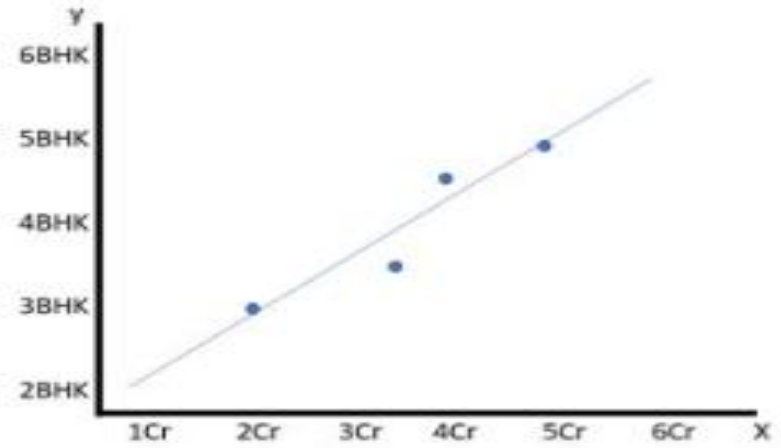
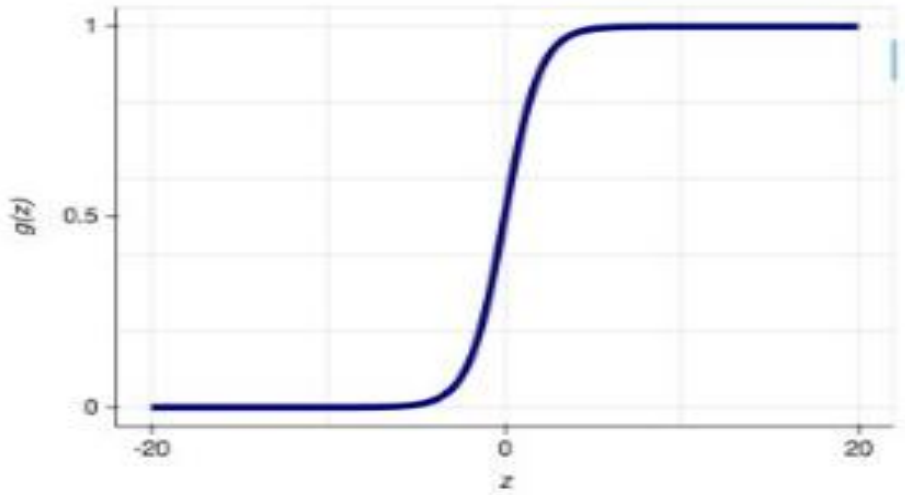
$\boldsymbol{\ln(e^x) = x}$

$-z = ln(0.0526) = -2.94$

- $z = 2.94$
- $\log(odds) = z = -64 + 2 * hours$
- $2.94 = -64 + 2 * hours$
- $2 * hours = 2.94 + 64$
- $2 * hours = 66.94$
- $hours = \frac{66.94}{2}$
- $\boldsymbol{hours = 33.47\ Hours}$

Hours Study	Pass (1) / Fail (0)
29	0
15	0
33	1
28	1
39	1

- The student should study **at least 33.47 hours**, so that he will pass the exam with more than 95% probability

Linear Regression	Logistic Regression
Target is an interval variable	Target is discrete (binary or ordinal) variable
Predicted values are the mean of the target variable at the given values of the input variable	Predicted values are the probability of the particular levels of the given values of the input variable
Solve regression problems	Solve classification problems
Example : What is the Temperature?	Example : Will it rain or not?
Graph is straight line	Graph is S-curve
	

Use Logistic Regression

- The student dataset has entrance mark based on the historic data of those who are selected or not selected.
- Based on the logistic regression, the values of the learnt parameters are $\beta_0 = 1$ and $\beta_1 = 8$.
- Assuming marks of x = 60, compute the resultant class.

If we assume the threshold value as 0.5, then it is observed that 0.44 < 0.5, therefore, the candidate with marks 60 is not selected.

answer

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}$$

$$\beta_0 + \beta_1 x = 481$$

$$p(x) = \frac{1}{1 + e^{-481}} = 0.44$$

Applications of Logistic Regression

Logistic Regression is mainly used for binary and multiclass classification problems.

1. Healthcare

- Disease prediction (diabetes, heart disease)
- Patient risk assessment

2. Marketing

- Customer churn prediction
- Purchase prediction
- Campaign response analysis

3. Finance

- Loan approval prediction
- Credit risk modeling
- Fraud detection

4. Education

- Student performance prediction
- Dropout prediction

5. Social Media and Web Analytics

- Click-through rate (CTR) prediction
- Ad conversion prediction

6. Cybersecurity

- Spam filtering
- Intrusion detection

7. Human Resources

- Employee attrition prediction
- Resume screening

8. Manufacturing

- Quality control (defective vs non-defective)
- Predictive maintenance

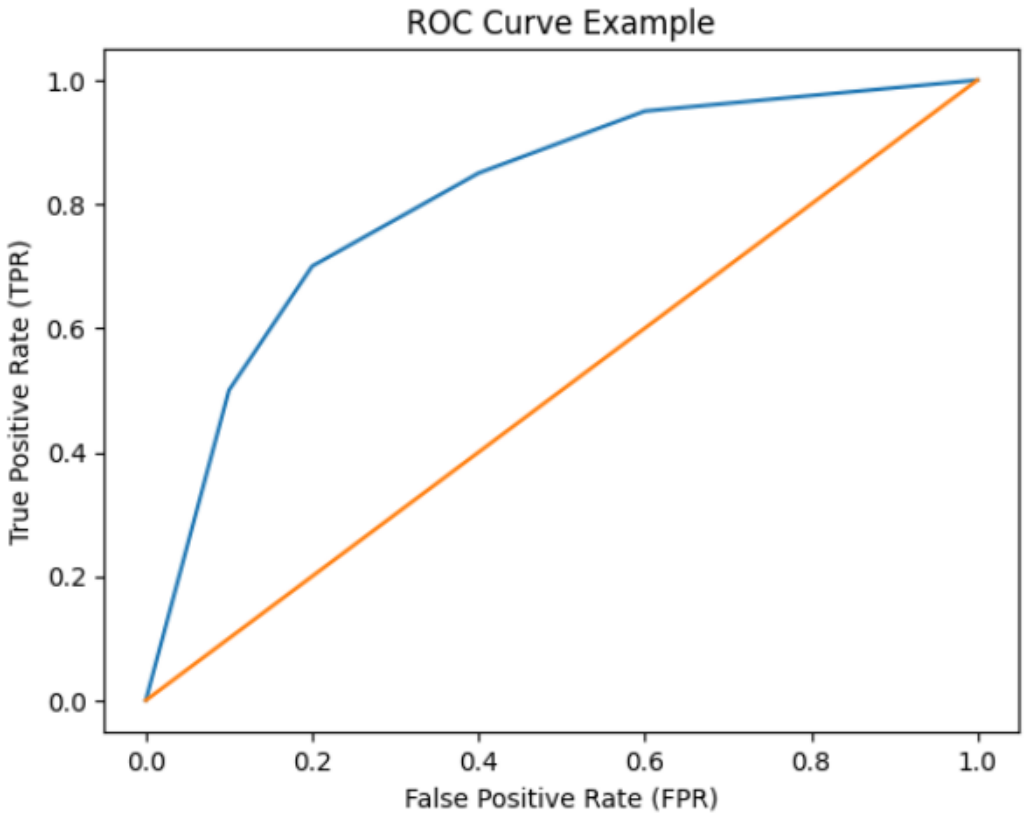
Performance Analysis

Error Analysis - Train/Test Split, validation set, Accuracy, Precision, Recall, F-measure, ROC curve, Confusion Matrix

ROC Curve (Receiver Operating Characteristic Curve) is a graphical tool used to evaluate the performance of a classification model, especially for binary classification.

It shows the relationship between:

- **True Positive Rate (TPR)** or **Sensitivity** on the Y-axis
- **False Positive Rate (FPR)** on the X-axis



ROC Curve – Graph and Explanation

You can see the ROC curve plotted above.

How to read this graph:

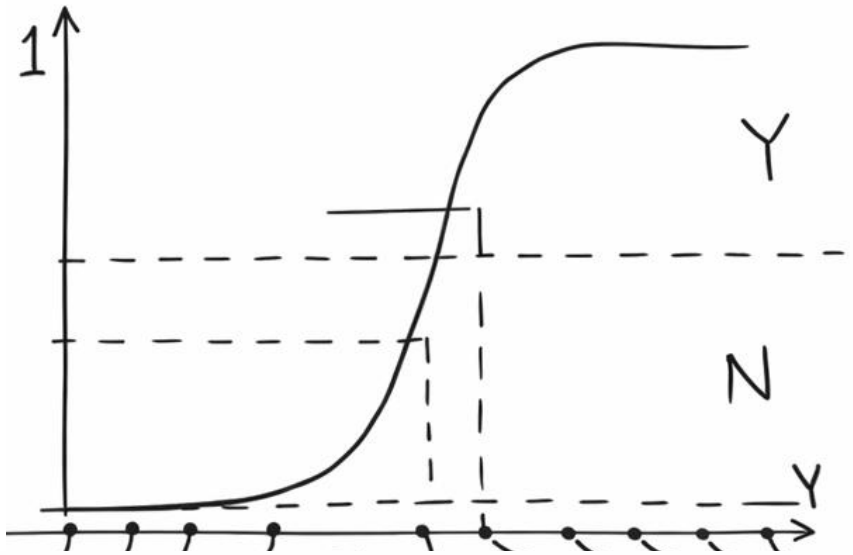
- **X-axis:** False Positive Rate (FPR)

  = FP / (FP + TN)

  → How many negatives are wrongly predicted as positive.
- **Y-axis:** True Positive Rate (TPR) or Sensitivity

  = TP / (TP + FN)

  → How many actual positives are correctly predicted.



at T= 0.5

3 TP	2 FP
2 FN	3 TN

TPR= 3/5
FPR= 2/5

at T= 0

5 TP	0 FP
0 FN	5 TN

TPR= 5/5
FPR= 0/5

at T= 1

0 TP	5 FP
5 FN	0 TN

TPR= 0
FPR= 5/5

at T= 0.75

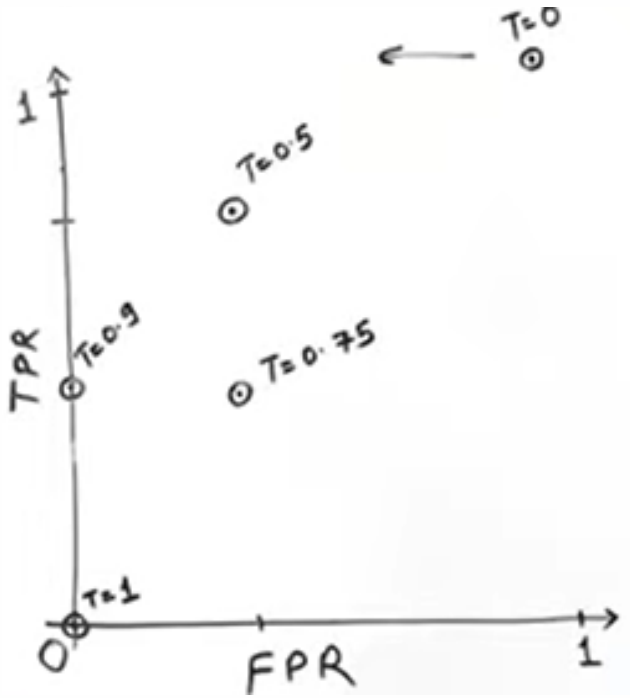
2 TP	2 FP
3 FN	3 TN

TPR= 2/5
FPR= 2/5

at T= 0.9

2 TP	0 FP
3 FN	5 TN

TPR= 2/5
FPR= 0/5



	Y	N
Y	TP	FP
N	FN	TN

$\rightarrow TPR = \frac{TP}{TP+FN}$

$\rightarrow FPR = \frac{FP}{FP+TN}$

Lines in the graph:

1. **Curved Line (Model Curve):**

   This shows the performance of a classifier at different thresholds.

   Each point corresponds to a different decision threshold.

2. **Diagonal Line:**

   This represents a **random classifier**.

   If your model lies close to this line, it is not useful.

Interpretation:

- The closer the curve is to the **top-left corner**, the better the model.
- A good model has:
  - High TPR (more correct positives)
  - Low FPR (fewer false alarms)

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- The closer the curve is to the **top-left corner**, the better the model.
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AUC (Area Under Curve):

- The area under this curve is called **AUC**.
- Range: 0 to 1
  - 0.5 → Random model
  - 1.0 → Perfect model
  - Higher AUC = Better classifier

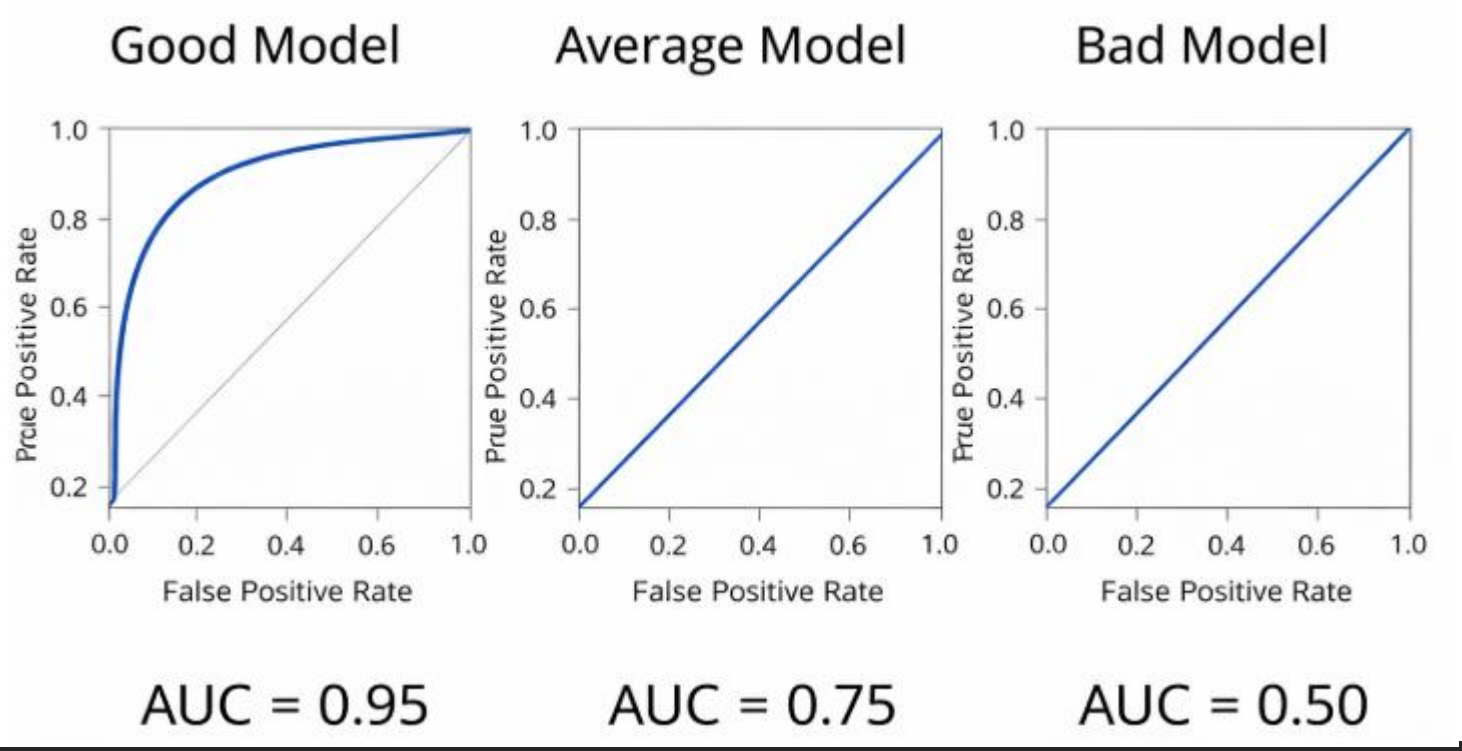
In simple words:

The ROC curve shows how well a model separates two classes by balancing:

- Catching positives correctly
- Avoiding false alarms

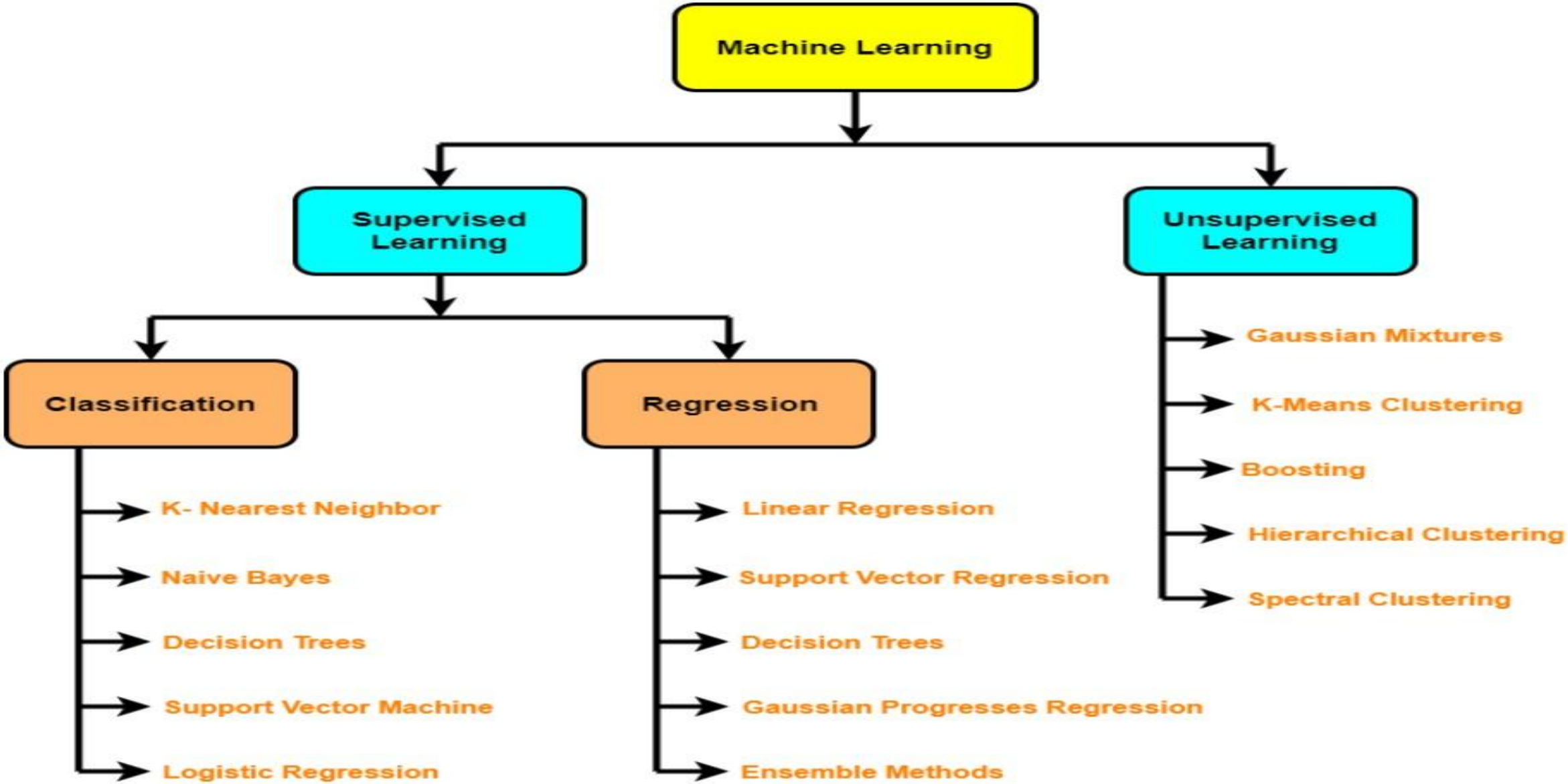
AUC (Area Under Curve):

- The area under this curve is called **AUC**.
- Range: 0 to 1
  - 0.5 → Random model
  - 1.0 → Perfect model
  - Higher AUC = Better classifier



17	Decision Tree: Introduction, Id3 Algorithm - 1
18	Decision Tree - Id3 Algorithm - 2
19	Decision Tree - Problem of Overfitting, Pre-pruning/post-pruning Decision Tree, Examples.
20	Support Vector Machine - Terminologies, Intuition, Learning, Derivation - 1
21	Support Vector Machine - Terminologies, Intuition, Learning, Derivation - 2
22	Support Vector Machine - KKT Condition - 3
23	Support Vector Machine - Kernel, Nonlinear Classification, and

25	Principal Component Analysis - Steps, merits, demerits, Intuition - 1
26	Principal Component Analysis - Steps, merits, demerits, Intuition - 2
27	Understanding and Implementing PCA using SVD for dimensionality reduction


Machine Learning
Supervised Learning
Unsupervised Learning
Classification
Regression
K- Nearest Neighbor
Naive Bayes
Decision Trees
Support Vector Machine
Logistic Regression
Linear Regression
Support Vector Regression
Decision Trees
Gaussian Progresses Regression
Ensemble Methods
Gaussian Mixtures
K-Means Clustering
Boosting
Hierarchical Clustering
Spectral Clustering

Naïve Bayes Classifier

What is Naive Bayes classifiers?

Naive Bayes classifiers are a collection of classification algorithms .

Works on the principle of conditional probability.

Naive Bayes classifiers are a collection of classification algorithms based on **Bayes' Theorem.** It is not a single algorithm but a family of algorithms where all of them share a common principle, i.e. every pair of features being classified is independent of each other.

The fundamental Naive Bayes assumption is that each feature makes an:

- independent
- equal

Conditional Probability

Conditional probability is the probability that an event happens **given that another event has already happened**.

It is written as:

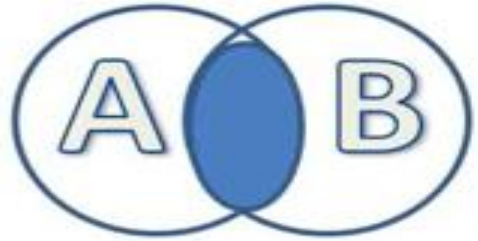
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

This means:

Probability of A **when B is known to have occurred**.

conditional probability

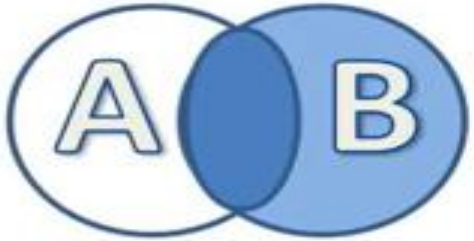
$P(A \cap B)$



Intersection of Events A and B

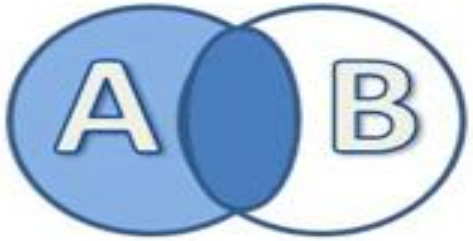
P(A ∩ B) is a probability of both events A and B occurring together.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



A for given B

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$



B for given A

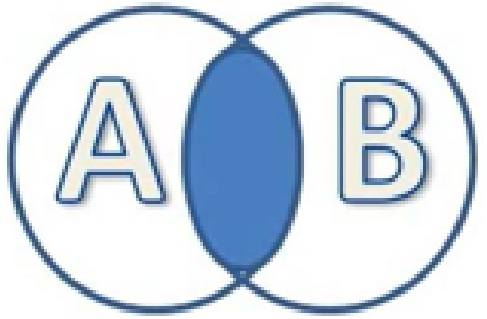
- $P(A \cap B)$ counts outcomes where both A and B happen.
- $P(B)$ counts all outcomes where B happens.

$$\frac{\text{Outcomes satisfying both A and B}}{\text{Outcomes satisfying B}}$$

What is Naive Bayes theorem?

$$P(A \cap B) = P(A)P(B \mid A)$$

$$P(A \cap B) = P(B)P(A \mid B)$$



Intersection of Events

- $P(A \cap B)$ counts outcomes where both A and B happen.

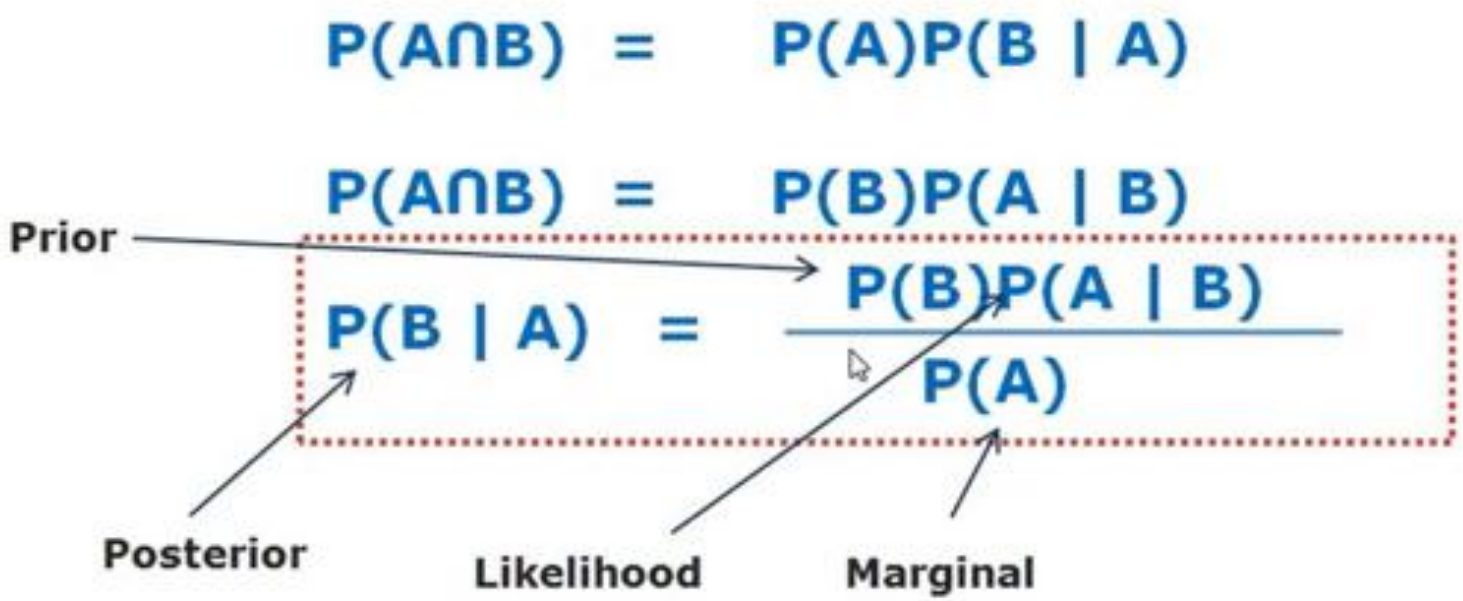
What is Naive Bayes theorem?

$$P(A \cap B) = P(B \mid A)\, P(A)$$

$$P(A \cap B) = P(A \mid B)\, P(B)$$

$$P(B \mid A)\, P(A) = P(B) P(A \mid B)$$

What is Naive Bayes theorem?



Baye's theorem gives the relationship between the probabilities of A and B, and the conditional probabilities of A given B and B given A.

Bayes' Theorem

Given a hypothesis H and evidence E, Bayes' theorem states that the relationship between the probability of the hypothesis before getting the evidence $P(H)$ and the probability of the hypothesis after getting the evidence $P(H|E)$ is

$$P(H|E) = \frac{P(E|H).P(H)}{P(E)}$$

Bayes' Theorem Proof

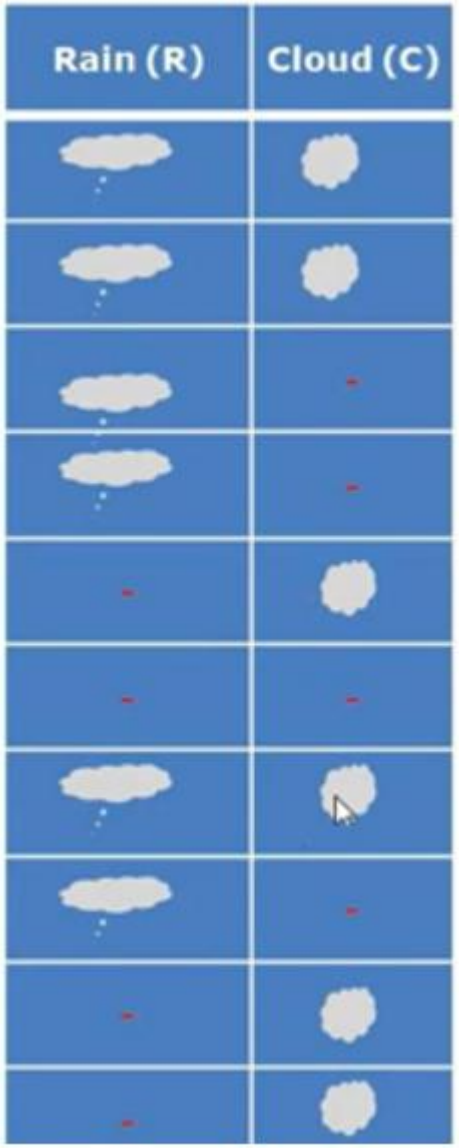
Likelihood
How probable is the evidence
Given that our hypothesis is true?

Prior
How probable was our hypothesis
Before observing the evidence?

$$P(H|E) = \frac{P(E|H).P(H)}{P(E)}$$

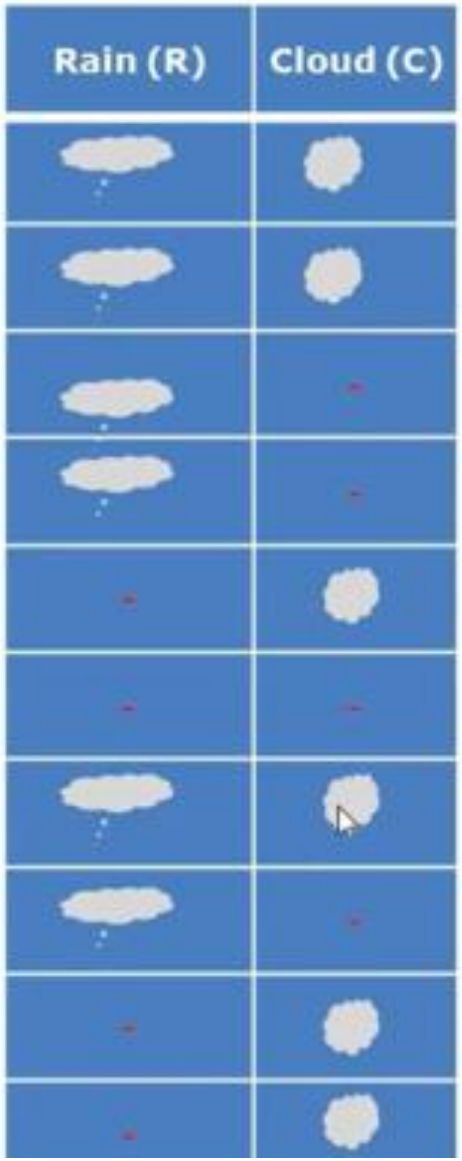
Posterior
How probable is our Hypothesis
Given the observed evidence?
(Not directly computable)

Marginal
How probable is the new evidence
Under all possible hypothesis?



$$P(R|C) =$$

Naive Bayes theorem



$$P(R|C) = \frac{P(C \mid R)\,P(R)}{P(C)} = \frac{(3/6)*(6/10)}{6/10}$$

$$P(R|C) = 0.5$$

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

NAIVE BAYES CLASSIFIER

Example - 1

$$P(PlayTennis = yes) = 9/14 = .64$$

$$P(PlayTennis = no) = 5/14 = .36$$

Outlook	Y	N
sunny	2/9	3/5
overcast	4/9	0
rain	3/9	2/5
Tempreature		
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Humidity	Y	N
high	3/9	4/5
normal	6/9	1/5
Windy		
Strong	3/9	3/5
Weak	6/9	2/5

NAIVE BAYES CLASSIFIER – Example -1

$$\langle Outlook = sunny, Temperature = cool, Humidity = high, Wind = strong \rangle$$

$$v_{NB} = \underset{v_j \in \{yes,no\}}{\text{argmax}} \; P(v_j) \prod_i \; P(a_i|v_j)$$

$$= \underset{v_j \in \{yes,no\}}{\text{argmax}} \; P(v_j) \quad P(Outlook = sunny|v_j) P(Temperature = cool|v_j)$$

$$\cdot \; P(Humidity = high|v_j) P(Wind = strong|v_j)$$

$$v_{NB}(yes) = P(yes) \; P(sunny|yes) \; P(cool|yes) \; P(high|yes) \; P(strong|yes) = .0053$$

$$v_{NB}(no) = P(no) \; P(sunny|no) \; P(cool|no) \; P(high|no) \; P(strong|no) = .0206$$

$$v_{NB}(yes) = \frac{v_{NB}(yes)}{v_{NB}(yes) + v_{NB}(no)} = 0.205 \qquad v_{NB}(no) = \frac{v_{NB}(no)}{v_{NB}(yes) + v_{NB}(no)} = 0.795$$

Color	Type	Origin	Stolen?
Red	Sports	Domestic	Yes
Red	Sports	Domestic	No
Red	Sports	Domestic	Yes
Yellow	Sports	Domestic	No
Yellow	Sports	Imported	Yes
Yellow	SUV	Imported	No
Yellow	SUV	Imported	Yes
Yellow	SUV	Domestic	No
Red	SUV	Imported	No
Red	Sports	Imported	Yes

$$\boldsymbol{New\ Instance} = (\boldsymbol{Red}, \boldsymbol{SUV}, \boldsymbol{Domestic})$$

$$p(Yes) = \frac{5}{10} = 0.5$$

$$p(No) = \frac{5}{10} = 0.5$$

Color	Yes	No
Red	3/5	2/5
Yellow	2/5	3/5

Type	Yes	No
Sports	4/5	2/5
SUV	1/5	3/5

Origin	Yes	No
Domestic	2/5	3/5
Imported	3/5	2/5

$$P(Yes|New\ Instance) = p(Yes) * P(Color = Red|Yes) * P(Type = SUV|Yes) * P(Origin = Domestic|Yes)$$

$$P(Yes|New\ Instance) = \frac{5}{10} * \frac{3}{5} * \frac{1}{5} * \frac{2}{5} = \frac{3}{125} = 0.024$$

$$P(No|New\ Instance) = p(No) * P(Color = Red|No) * P(Type = SUV|No) * P(Origin = Domestic|No)$$

$$P(No|New\ Instance) = \frac{5}{10} * \frac{2}{5} * \frac{3}{5} * \frac{3}{5} = \frac{9}{125} = 0.072$$

$$P(No|New\ Instance) > P(Yes|New\ Instance)$$

Why it is Called Naive Bayes?

It is named as "Naive" because it assumes the presence of one feature does not affect other features. The "Bayes" part of the name refers to its basis in Bayes'

You are given a small dataset on fruit attributes. We want to classify a new fruit with the attributes **{Long: Yes, Sweet: Yes, Yellow: No}** as either a **Banana** or **Orange** using the Naive Bayes algorithm.

Training Data

Fruit	Long	Sweet	Yellow	Total
Banana	400	350	450	500
Orange	0	150	300	300
Other	100	150	50	200
Total	500	650	800	1000

Solution Steps

The Naive Bayes formula is:

$$P(Class|Features) \propto P(Class) \times \prod P(Feature|Class)$$

We need to calculate the posterior probability for both "Banana" and "Orange" and select the class with the highest value.

1. Calculate Prior Probabilities

The prior probability is the overall probability of each class based on the training data.

- $P(\text{Banana}) = \text{Total Bananas}/\text{Total Fruits} = 500/1000 = 0.5$
- $P(\text{Orange}) = \text{Total Oranges}/\text{Total Fruits} = 300/1000 = 0.3$

2. Calculate Likelihoods (Conditional Probabilities)

Calculate the probability of each feature given the class label for the new fruit {Long: Yes, Sweet: Yes, Yellow: No}.

For Banana:

- $P(\text{Long: Yes}|\text{Banana}) = 400/500 = 0.8$
- $P(\text{Sweet: Yes}|\text{Banana}) = 350/500 = 0.7$
- $P(\text{Yellow: No}|\text{Banana}) = (500 - 450)/500 = 50/500 = 0.1$

For Orange:

- $P(\text{Long: Yes}|\text{Orange}) = 0/300 = 0.0$
- $P(\text{Sweet: Yes}|\text{Orange}) = 150/300 = 0.5$
- $P(\text{Yellow: No}|\text{Orange}) = (300 - 300)/300 = 0/300 = 0.0$

3. Calculate Unnormalized Posterior Probabilities

Multiply the prior probability by the likelihoods for each class.

- **Probability (Banana):**
  $$P(\text{Banana}) \times P(\text{Long: Yes}|\text{Banana}) \times P(\text{Sweet: Yes}|\text{Banana}) \times P(\text{Yellow: No}|\text{Banana})$$
  $$= 0.5 \times 0.8 \times 0.7 \times 0.1 = \mathbf{0.028}$$
- **Probability (Orange):**
  $$P(\text{Orange}) \times P(\text{Long: Yes}|\text{Orange}) \times P(\text{Sweet: Yes}|\text{Orange}) \times P(\text{Yellow: No}|\text{Orange})$$
  $$= 0.3 \times 0.0 \times 0.5 \times 0.0 = \mathbf{0.0}$$

4. Predict the Class

Compare the resulting probabilities. The class with the highest probability is the predicted class.

- Banana Probability: 0.028
- Orange Probability: 0.0

Since 0.028 > 0.0, the Naive Bayes classifier predicts that the fruit with attributes {Long: Yes, Sweet: Yes, Yellow: No} is a **Banana**.

Types of Naïve Bayes

Gaussian

Multinomial

Bernoulli

Bernoulli Naive Bayes

Here, the predictors are boolean variables. So, the only values you have are 'True' and 'False' (you could also have 'Yes' or 'No'). We use it when the data is according to multivariate Bernoulli distribution.

It is one of the prevalent **types of Naive Bayes model:** Its working is identical to the Multinomial classifier. However, the predictor variables are the independent Boolean variables. For example, it works as -a specific word exists or not in a document. Moreover, this model is famous for document classification tasks.

Multinomial Naive Bayes

People use this algorithm to solve document classification problems. For example, if you want to determine whether a document belongs to the 'Legal' category or 'Human Resources' category, you'd use this algorithm to sort it out. It uses the frequency of the present words as features.

This model is used when the data is multinomial distributed. Primarily, it is used for document classification problems. It denotes that a specific document belongs to which category (like Education, Politics, Sports, etc.). You can easily

understand these **types of Naive Bayes models** with an example.

Suppose there is a text document, and you want to extract all the distinctive words and prepare multiple features such that every feature signifies the word count in the document. The frequency is a feature to consider in this example. When you use multinomial Naive Bayes for this example, it neglects the non-occurrence of the features. Hence, if the e frequency is 0, the probability of occurrence of the particular feature is 0. It is one of those **types of Naive Bayes model:** that seamlessly works with text classification problems.

Gaussian Naive Bayes

If the predictors aren't discrete but have a continuous value, we assume that they are a sample from a gaussian distribution. It is among those types of Naive Bayes models that consider normal distribution. It assumes that the feature adopts a normal distribution. If predictors accept continuous values instead of discrete, the Gaussian Naive Bayes model assumes that such values are sampled through the Gaussian distribution. It is always better to first identify your problem and determine which is not a main type of Naive Bayes classifier.