

# Regular Grammar

By

NAYAN KUMAR


# Regular Grammar

Regular grammar is a type of grammar that describes a regular language.

A regular grammar is a mathematical object,  $G$ , which consists of four components,  $G = (V, T, P, S)$ , where

- **V**: non-empty, finite set of non-terminal symbols,
- **T**: a finite set of terminal symbols, or alphabet, symbols,
- **P**: a set of grammar rules, each of one having one of the forms
  - $A \rightarrow aB$
  - $A \rightarrow a$
  - $A \rightarrow \epsilon$ , Here  $\epsilon$ =empty string,  $A, B \in N$ ,  $a \in \Sigma$
- $S \in V$  is the start symbol.

# DFA to REGULAR GRAMMAR

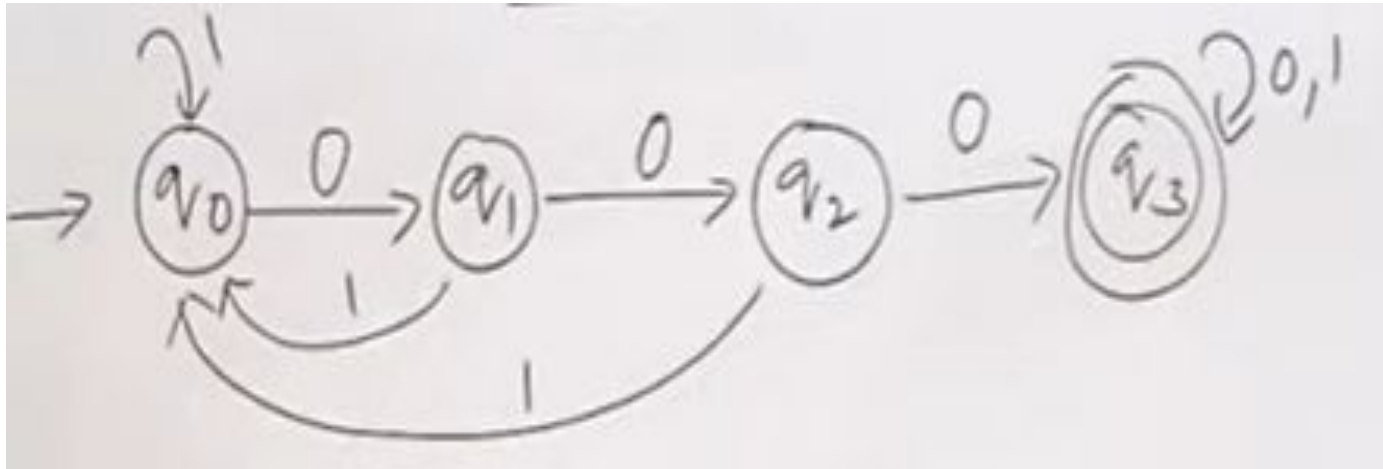
- 
1.  $A_i \rightarrow aA_j$  , if  $(q_i, a) = q_j$  where  $q_j \notin F$
  2.  $A_i \rightarrow aA_j$  and  $A \rightarrow a$  are the production rules, if  $(q_i, a) = q_j$  where  $q_j \in F$

## Example-1

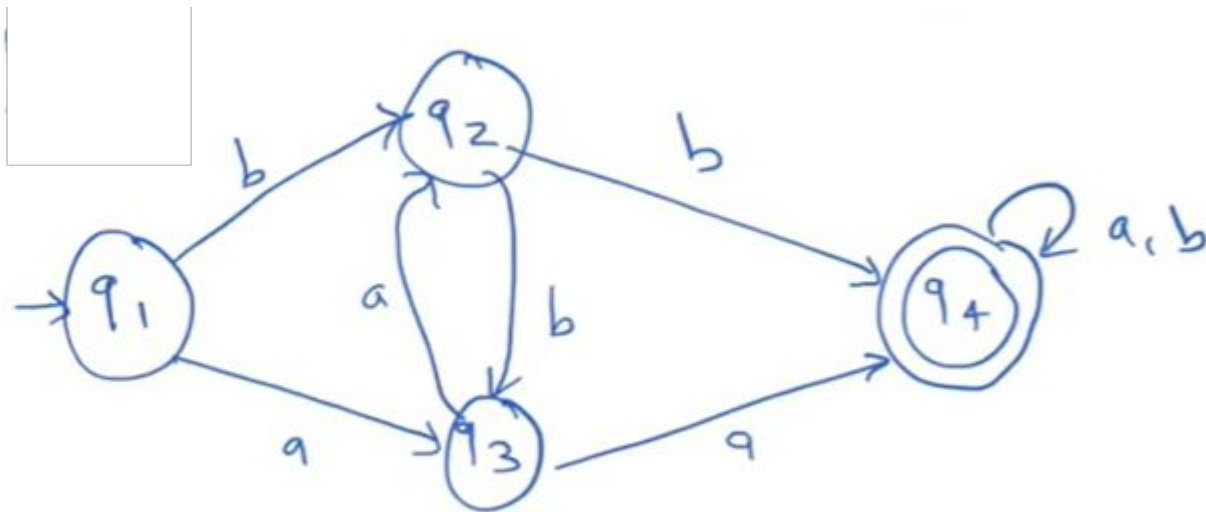
Construct RG from given DFA:



## Example-2



## Example-3





## REGULAR GRAMMAR to DFA

1. Each productions  $Ai \rightarrow aAj$  , induces a transition from **qi** to **qj** with label **a**.
2. Each production  $Ak \rightarrow a$  induces a transition from **qk** to **qf** with label '**a**'.



## **Example-4**

Construct a finite automata recognizing  $L(G)$  where  $G$  is the grammar

$$S \rightarrow aS \mid bA \mid b$$
$$A \rightarrow aA \mid bS \mid a$$





## **Example-5**

Construct a finite automata recognizing  $L(G)$  where  $G$  is the grammar

$S \rightarrow aS \mid bS \mid aA$

$A \rightarrow bB$

$B \rightarrow C$

$C \rightarrow a$

## Left Linear Regular Grammar

In this type of regular grammar, all the non-terminals on the left-hand side exist at the leftmost place, i.e; left ends.

$A \rightarrow a, A \rightarrow \mathbf{B}a, A \rightarrow \epsilon$ , where

A and B are non-terminals,

a is terminal, and

$\epsilon$  is empty string

### Example

$S \rightarrow \mathbf{B}00 \mid \mathbf{S}11$

$B \rightarrow \mathbf{B}0 \mid \mathbf{B}1 \mid 0 \mid 1$

where

S and B are non-terminals, and

0 and 1 are terminals

## Right Linear Regular Grammar

In this type of regular grammar, all the non-terminals on the right-hand side exist at the rightmost place, i.e; right ends

$A \rightarrow a, A \rightarrow aB, A \rightarrow \epsilon$  where,

A and B are non-terminals,

a is terminal, and

$\epsilon$  is empty string

### Example

$S \rightarrow 00B \mid 11S$

$B \rightarrow 0B \mid 1B \mid 0 \mid 1$

where,

S and B are non-terminals, and

0 and 1 are terminals



## Examples-6

Find a right linear grammar (RLG) for the language

$$L = \{a^n b^m : n + m \text{ is odd}\}.$$

Convert the RLG to left linear grammar (LLG)

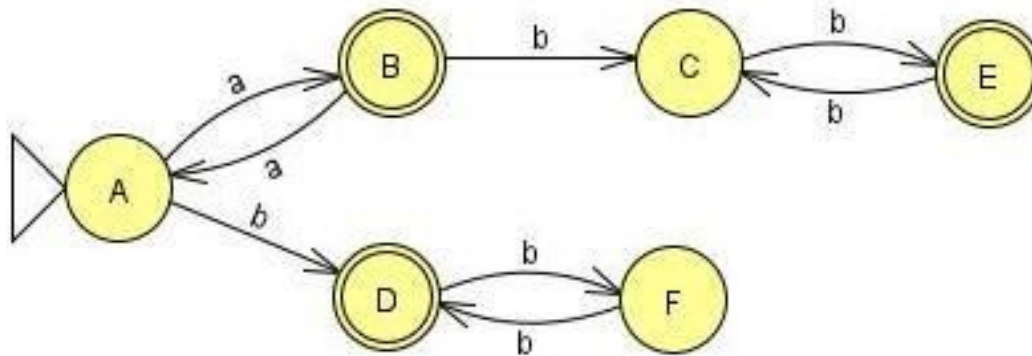
## Examples-6

Find a right linear grammar (RLG) for the language

$$L = \{a^n b^m : n + m \text{ is odd}\}.$$

Convert the RLG to left linear grammar (LLG)

STEP-1 [Construct the DFA]



# Examples-6

**STEP-2** [Find RLG from the FA]

Right Linear Grammar:

$A \rightarrow aB \mid bD$

$B \rightarrow bC \mid aA \mid$

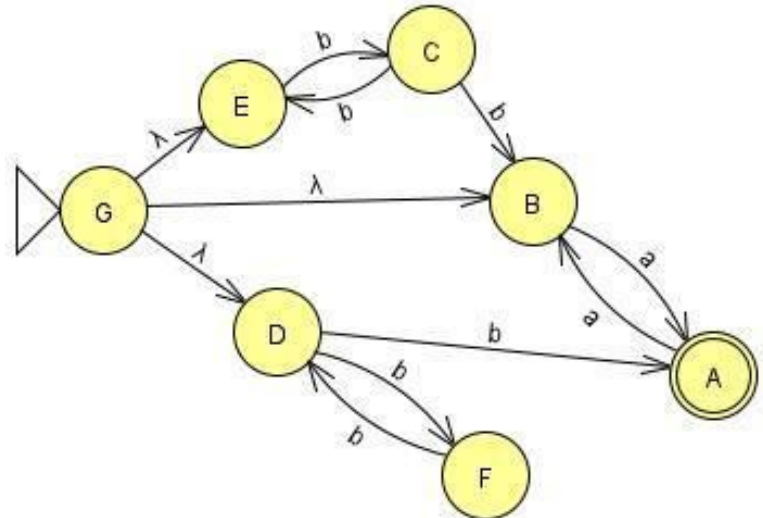
$C \rightarrow bE$

$D \rightarrow bF \mid$

$E \rightarrow bC \mid$

$F \rightarrow bD$

**STEP-3** [Find the reverse FA]



## Examples-6

**STEP-4** [Right Linear Grammar for the Reverse FA]

$G \rightarrow B \mid D \mid E$

$B \rightarrow aA$

$D \rightarrow bA \mid bF$

$E \rightarrow bC$

$A \rightarrow aB \mid \epsilon$

$C \rightarrow bE \mid bB$

$F \rightarrow bD$

**STEP-5** [Left Linear Grammar for the given Language]

$G \rightarrow B \mid D \mid E$

$B \rightarrow Aa$

$D \rightarrow Ab \mid Fb$


$E \rightarrow Cb$

$A \rightarrow Ba \mid \epsilon$

$C \rightarrow Eb \mid Bb$

$F \rightarrow Db$

## Examples-7



Construct a DFA that accepts the language generated by the following grammar. Write its regular expression.

$$S \rightarrow abS|A$$

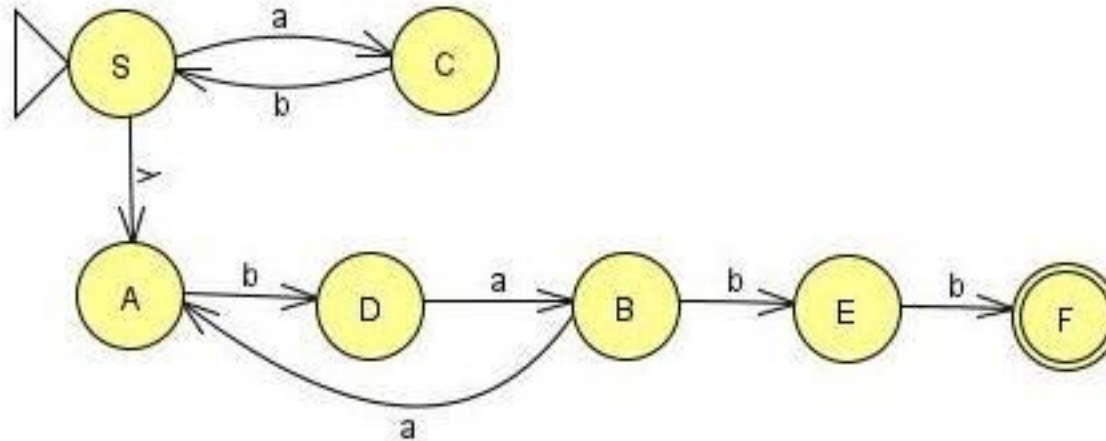
$$A \rightarrow baB$$

$$B \rightarrow aA|bb$$

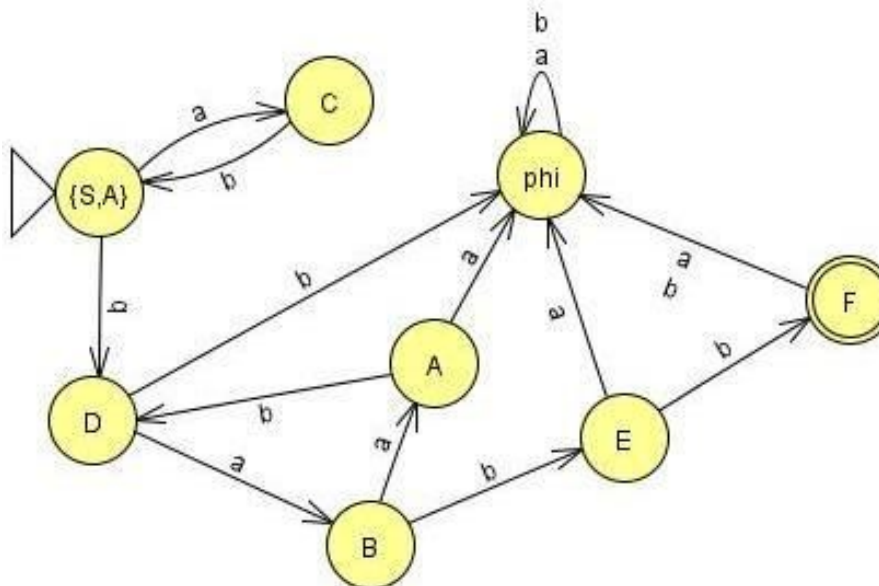


# Examples-7

## STEP-1 [The constructed NFA is]



## STEP-2 [The constructed DFA]



## STEP-3 [Regular Exp ]

The desired RE is:

**$(ab)^*(baa)^*babb$**

## Left Linear Regular Grammar

In this type of regular grammar, all the non-terminals on the left-hand side exist at the leftmost place, i.e; left ends.

$A \rightarrow a, A \rightarrow Ba, A \rightarrow \epsilon$ , where

A and B are non-terminals,

a is terminal, and

$\epsilon$  is empty string

### Example

$S \rightarrow B00 \mid S11$

$B \rightarrow B0 \mid B1 \mid 0 \mid 1$

where

S and B are non-terminals, and

0 and 1 are terminals