DIGITAL IMAGE FUNDAMENTALS

Introduction of Digital Image Processing

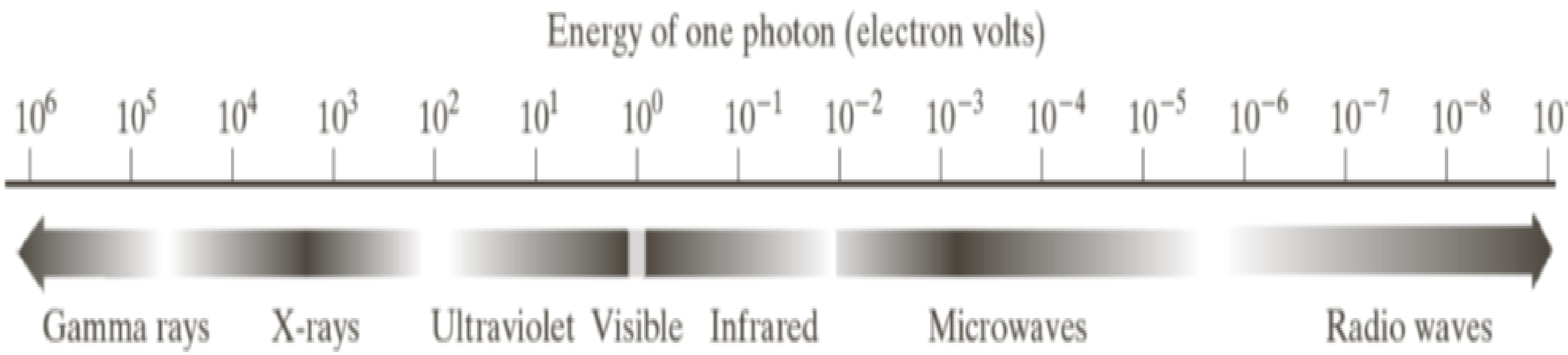
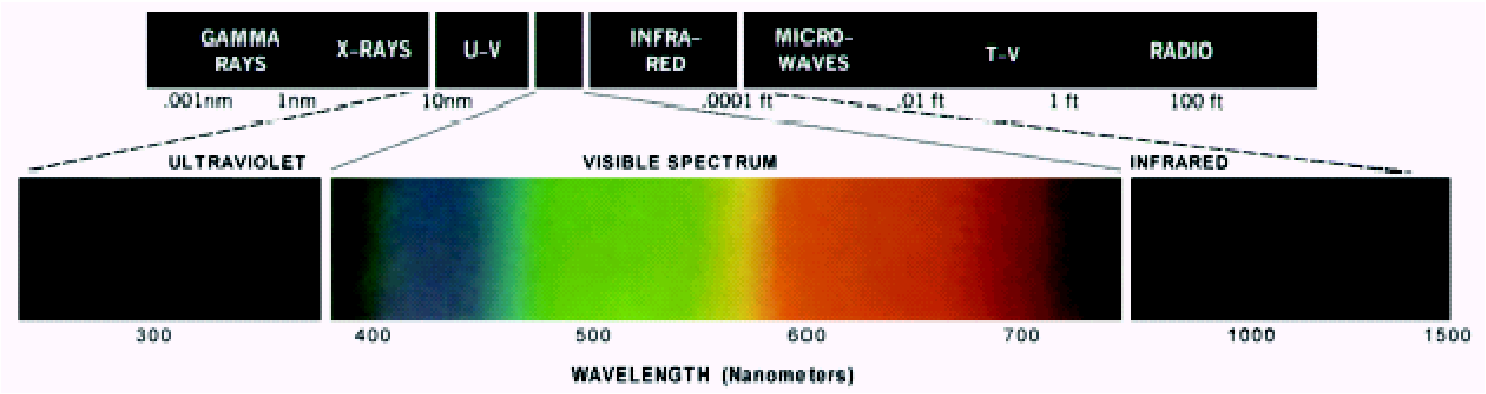


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

- An image is not limited to visible light
- Digital image processing mainly uses: Visible, Infrared, X-rays, Microwaves
- Visible spectrum: Violet ≈ 400 nm and Red ≈ 700 nm
- As wavelength increases, energy decreases

Light And The Electromagnetic Spectrum

- Light is just a particular part of the electromagnetic spectrum that can be sensed by the human eye
- The electromagnetic spectrum is split up according to the wavelengths of different forms of energy



Electromagnetic Spectrum (With Units)

- The electromagnetic spectrum can be defined in terms of wavelength, frequency, or photon energy.
- The electromagnetic spectrum spans from high-energy gamma rays with very high-energy, short wavelengths to low-energy radio waves with long wavelengths, with frequency and energy inversely proportional to wavelength.
- As frequence increases, wavelength decreases, energy increases

$$f = \frac{c}{\lambda} \qquad E = hf = \frac{hc}{\lambda} \qquad \lambda \uparrow \Rightarrow \text{Energy} \downarrow$$

Where:

- $c = 3 \times 10^8$ m/s $\quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
- $h = 6.626 \times 10^{-34}$ J·s $\quad 10^6 \text{ eV} = 1.6 \times 10^{-13} \text{ J}$

- *h* is the planck's constant that relates the energy of a photon J·s (joule–second)

Electromagnetic Spectrum (With Units)

Gamma Rays

- Wavelength (λ):

  < 0.01 nm $=< 10^{-11}$ m

- Frequency (f):

  > 3×10^{19} Hz

- Energy (E):

  > 100 keV (kilo–electron volts)

- **Applications:** Cancer treatment, nuclear imaging

X-Rays

- Wavelength (λ):

  0.01 nm – 10 nm

  10^{-11} m – 10^{-8} m

- Frequency (f):

  $3 \times 10^{16} - 3 \times 10^{19}$ Hz

- Energy (E):

  100 eV – 100 keV

- **Applications:** Medical imaging, CT scans

Electromagnetic Spectrum (With Units)

Ultraviolet (UV)

Wavelength (λ):

10 nm – 400 nm

10^{-8} m – 4×10^{-7} m

Frequency (f):

7.5×10^{14} – 3×10^{16} Hz

Energy (E):

3 eV – 100 eV

Applications: Sterilization, forensics

Visible Light

- **Wavelength (λ):**

  400 nm – 700 nm

  4×10^{-7} m – 7×10^{-7} m
- **Frequency (f):**

  4.3×10^{14} – 7.5×10^{14} Hz
- **Energy (E):**

  1.8 eV – 3.1 eV
- **Applications:** Human vision, cameras

Electromagnetic Spectrum (With Units)

Infrared (IR)

- **Wavelength (λ):**

  700 nm – 1 mm

  7×10^{-7} m – 10^{-3} m

- **Frequency (f):**

  3×10^{11} – 4.3×10^{14} Hz

- **Energy (E):**

  0.001 eV – 1.8 eV

- **Applications:** Thermal imaging, night vision

Microwaves

- **Wavelength (λ):**

  1 mm – 1 m

  10^{-3} m – 1 m

- **Frequency (f):**

  3×10^{8} – 3×10^{11} Hz

- **Energy (E):**

  10^{-6} – 10^{-3} eV

- **Applications:** Radar, satellite imaging

Electromagnetic Spectrum (With Units)

Radio Waves

- **Wavelength (λ):**

  > 1 m

- **Frequency (f):**

  $< 3 \times 10^8$ Hz

- **Energy (E):**

  $< 10^{-6}$ eV

- **Applications:** MRI, radio astronomy

Brightness Adaptation & Discrimination

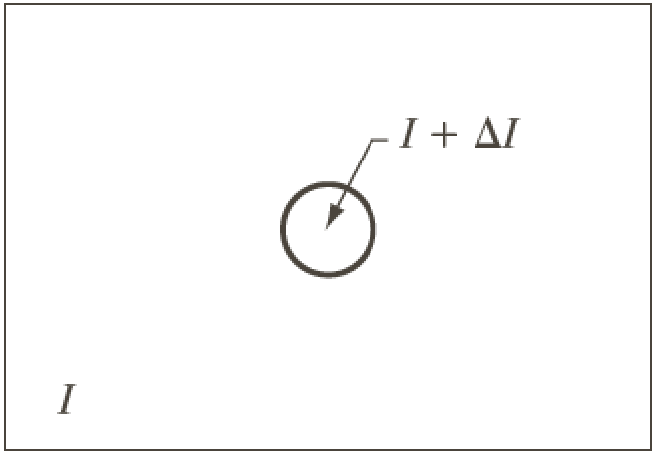
The human visual system can perceive approximately 10^{10} different light intensity levels.

However, at any one time we can only discriminate between a much smaller number – *brightness adaptation.*

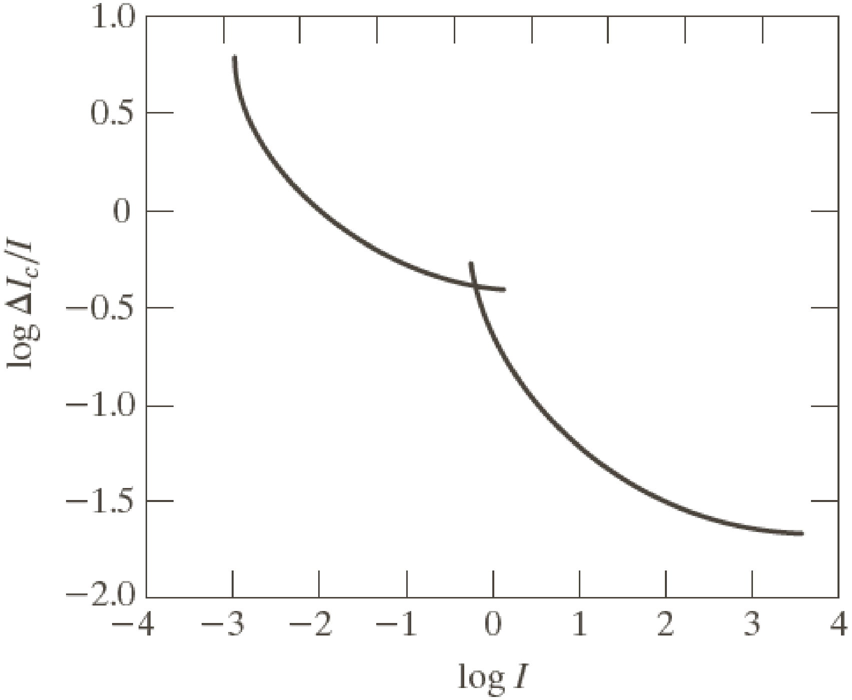
Similarly, the *perceived intensity* of a region is related to the light intensities of the regions surrounding it.

Brightness Adaptation & Discrimination (cont…)

Weber ratio

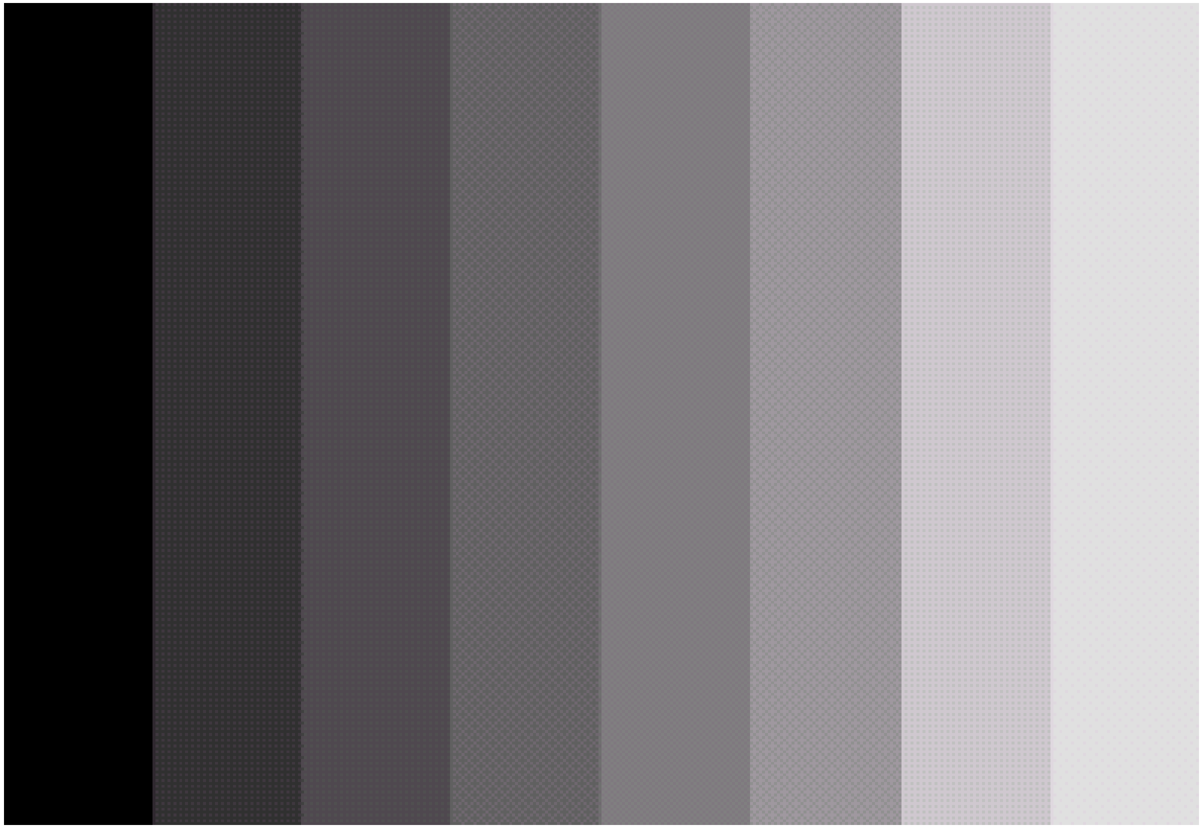


Intensity in millilambert



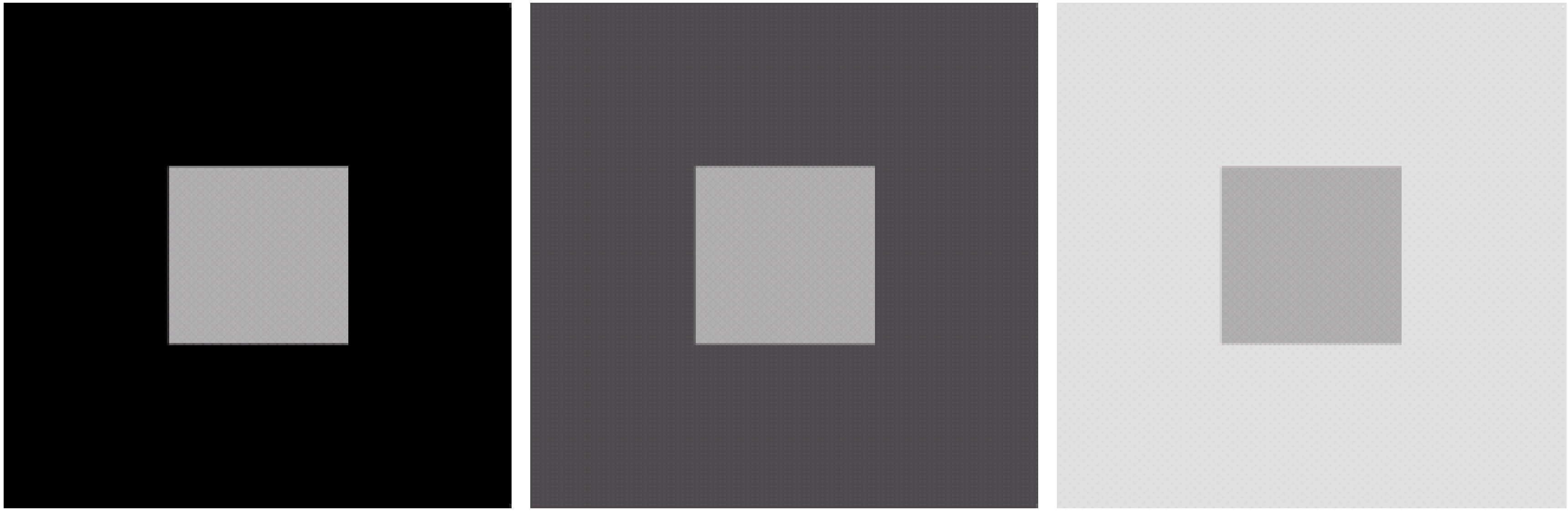
Perceived brightness is not the simple function of intensity. It overshoots or undershoots at boundaries.

Brightness Adaptation & Discrimination (cont…)



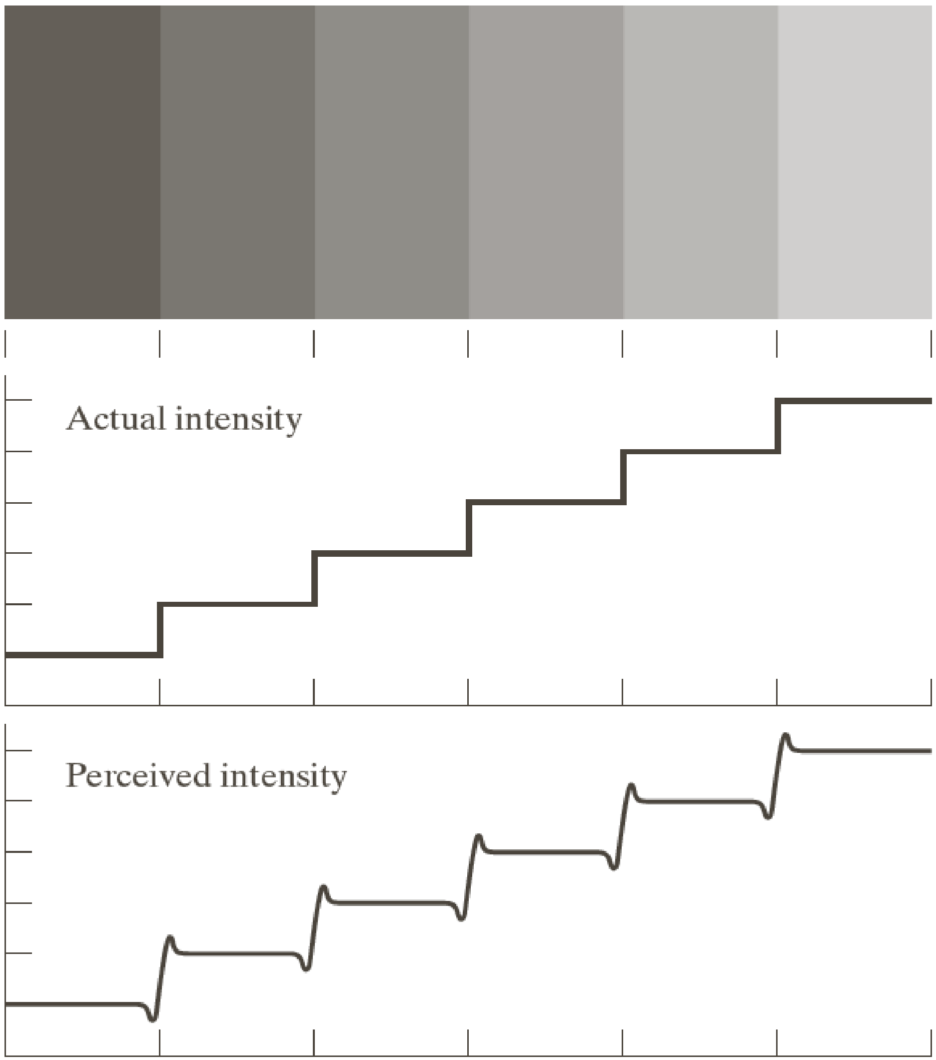
An example of Mach bands

Brightness Adaptation & Discrimination (cont…)



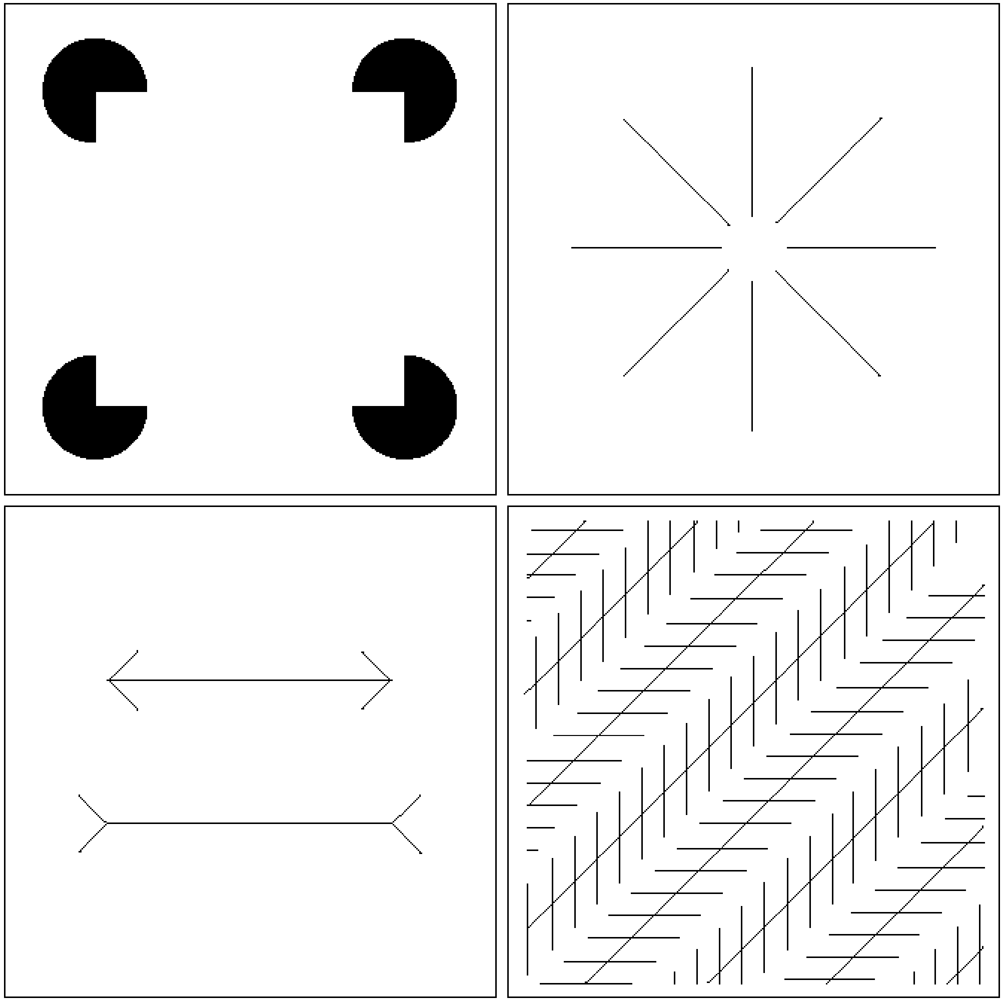
An example of *simultaneous contrast*

Brightness Adaptation & Discrimination (cont…)



Optical Illusions

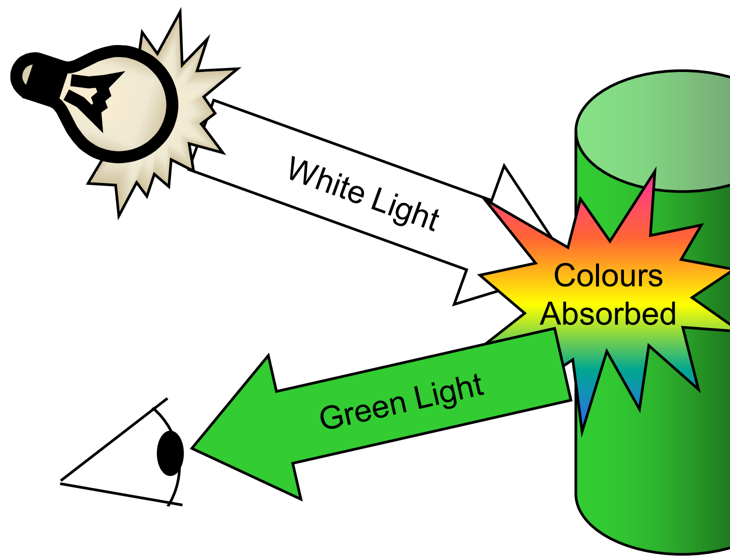
Our visual systems play lots of interesting tricks on us



Reflected Light

The colours that we perceive are determined by the nature of the light reflected from an object

For example, if white light is incident onto a green object most wavelengths are absorbed, while green light is reflected from the object



Digital Image

In the following slides we will consider what is involved in capturing a digital image of a real-world scene

- Image sensing and representation
- Sampling and quantisation
- Resolution

Image Representation

Before we discuss image acquisition recall that a digital image is composed of M rows and N columns of pixels each storing a value

Pixel values are most often grey levels in the range 0-255(black-white)

We will see later on that images can easily be represented as matrices

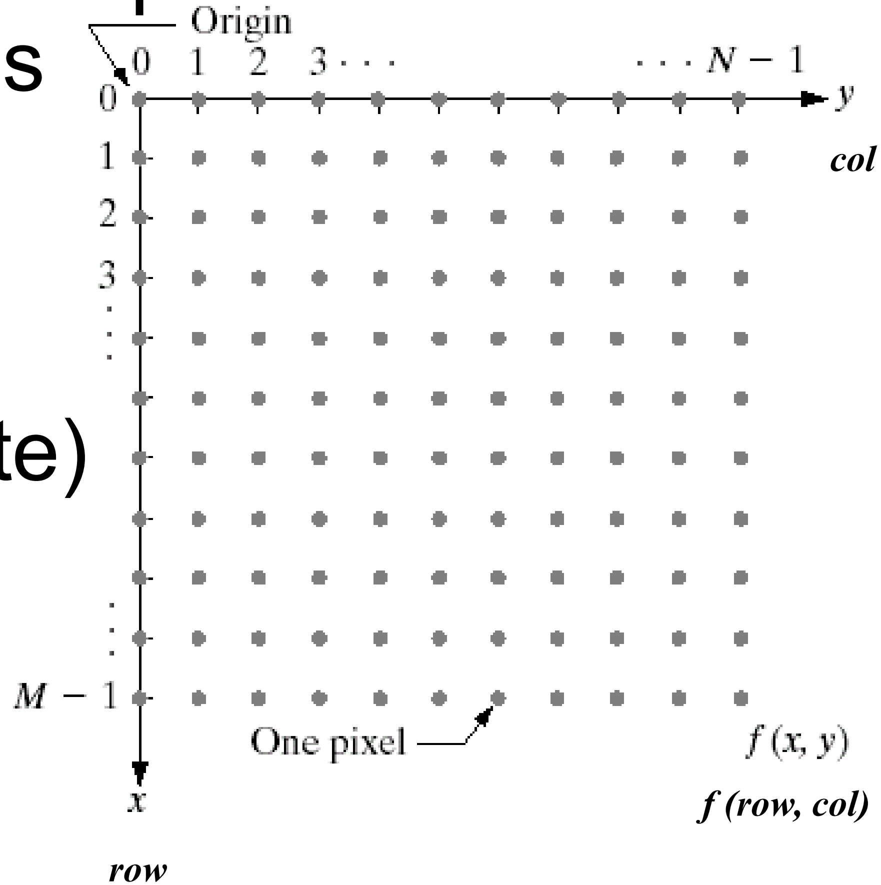
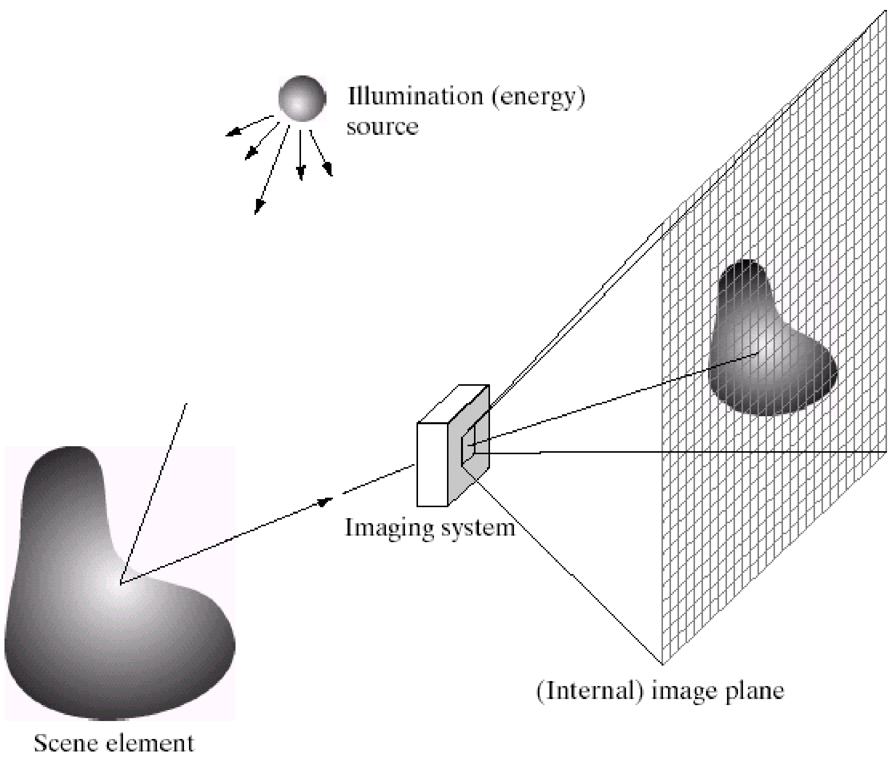


Image Acquisition

Images are typically generated by *illuminating* a *scene* and absorbing the energy reflected by the objects in that scene

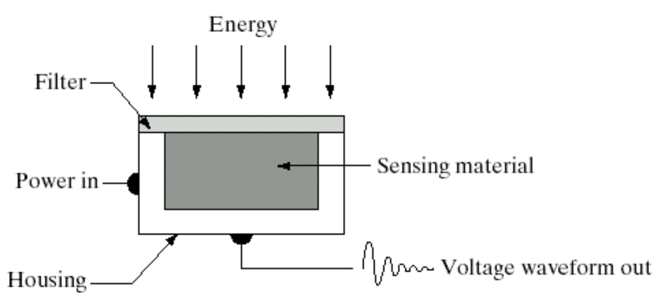


- Typical notions of illumination and scene can be different:
  - X-rays of a skeleton
  - Ultrasound of an unborn baby
  - Electro-microscopic images of molecules

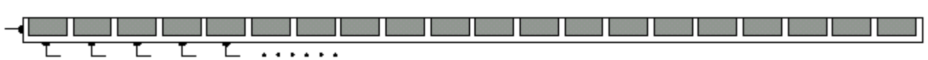
Image Sensing

Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage

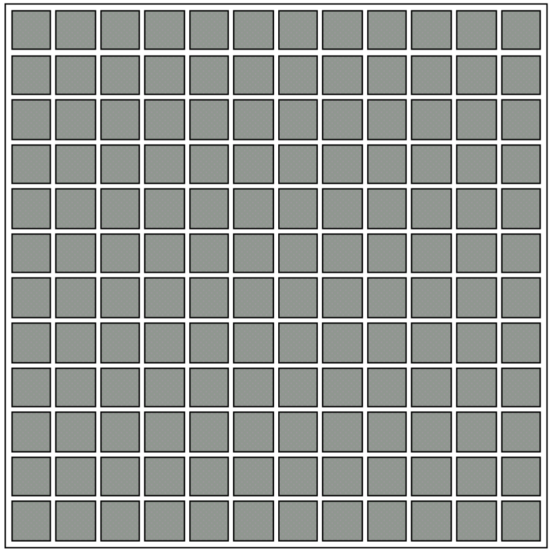
Collections of sensors are arranged to capture images



Imaging Sensor

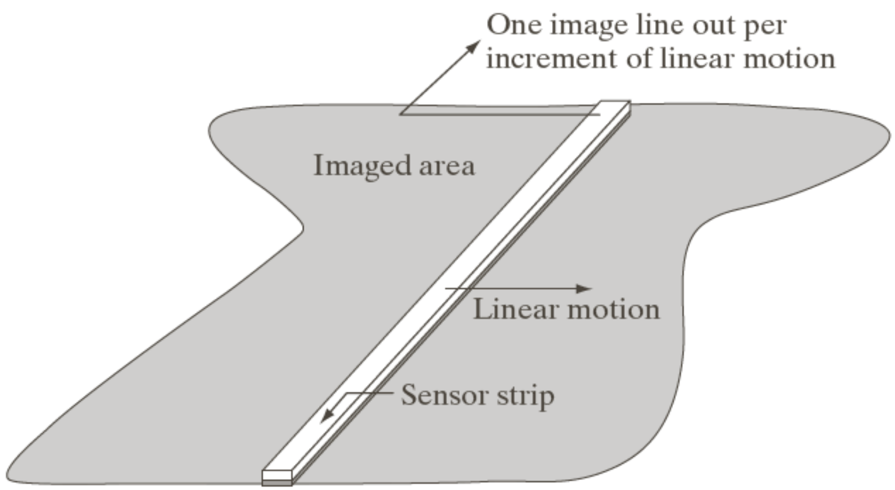


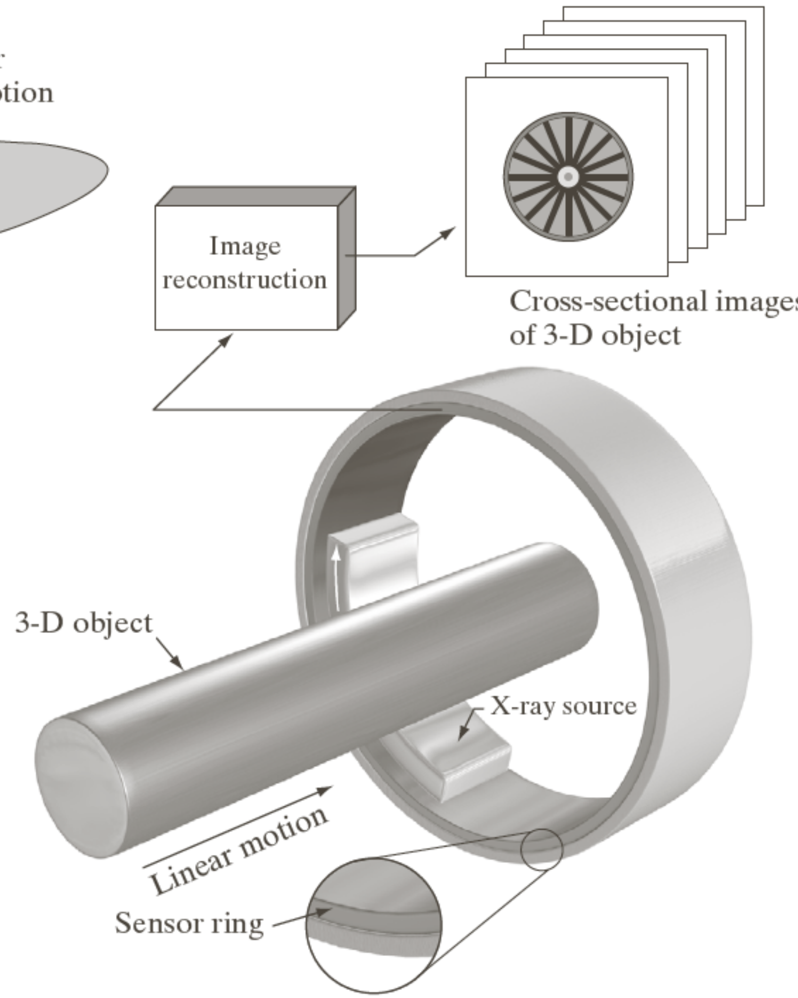
Line of Image Sensors



Array of Image Sensors

Image Sensing





Using Sensor Strips and Rings

Image Sampling And Quantisation

A digital sensor can only measure a limited number of **samples** at a **discrete** set of energy levels

Quantisation is the process of converting a continuous **analogue** signal into a digital representation of this signal

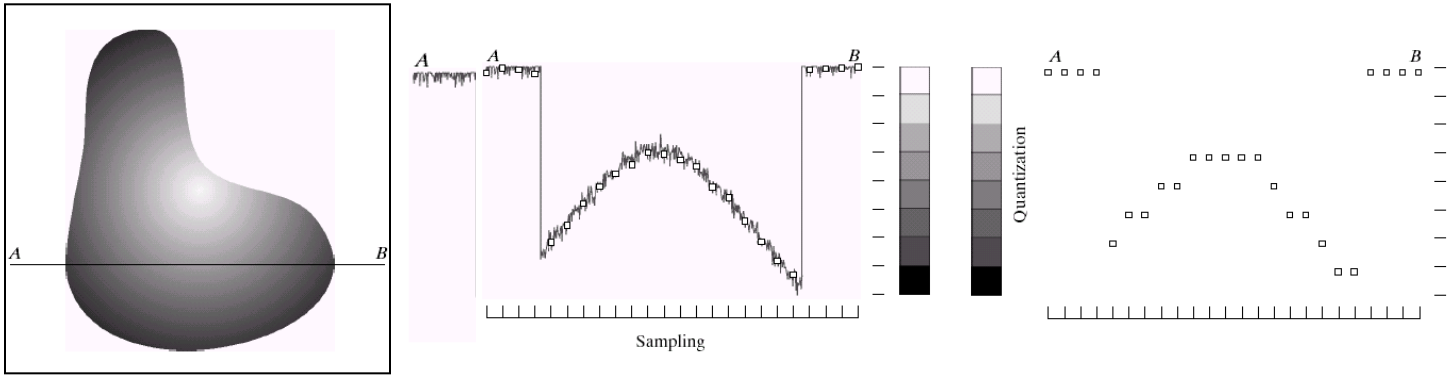


Image Sampling And Quantisation

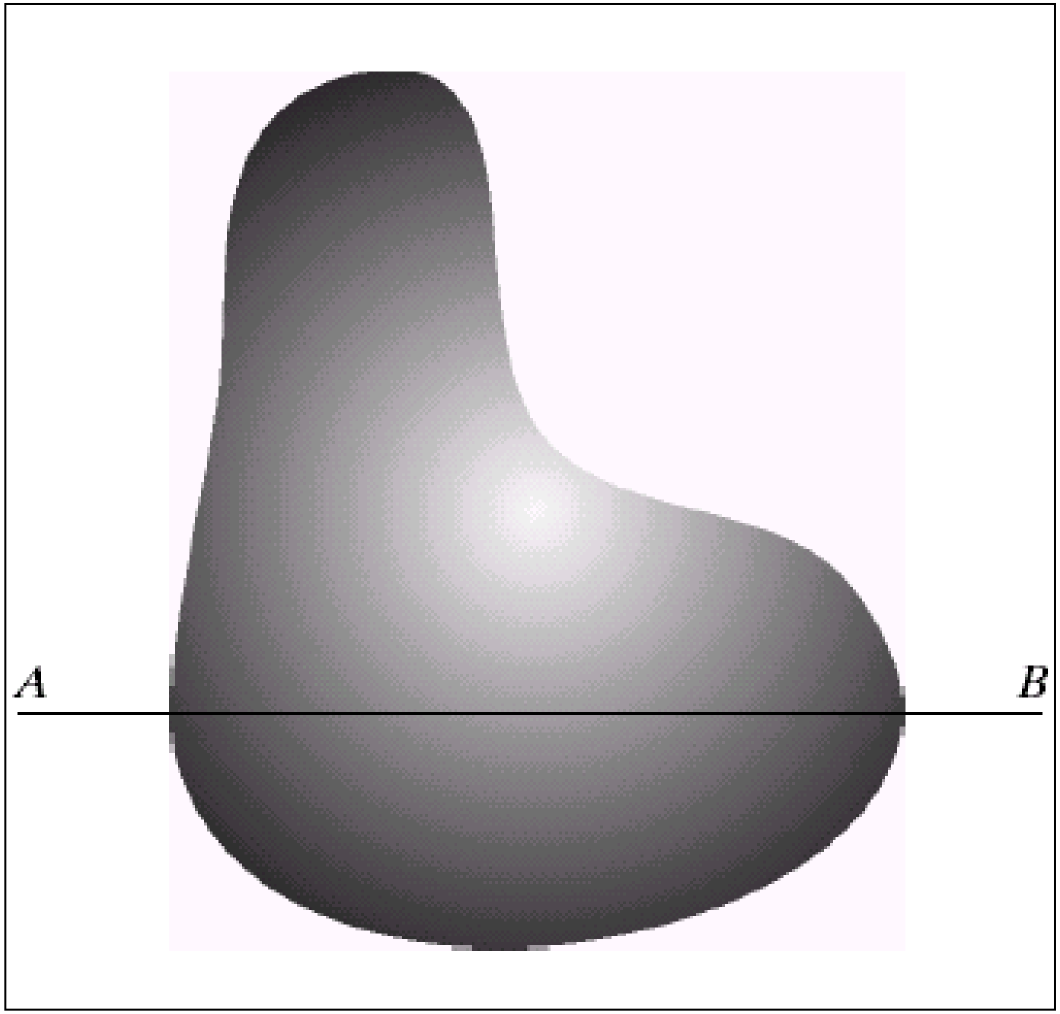
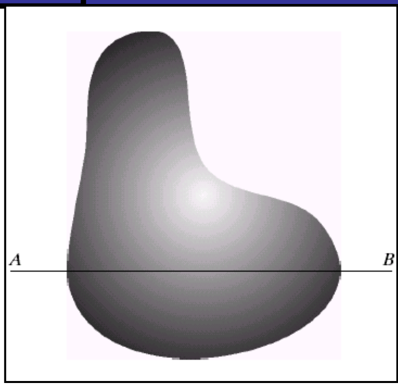


Image Sampling And Quantisation



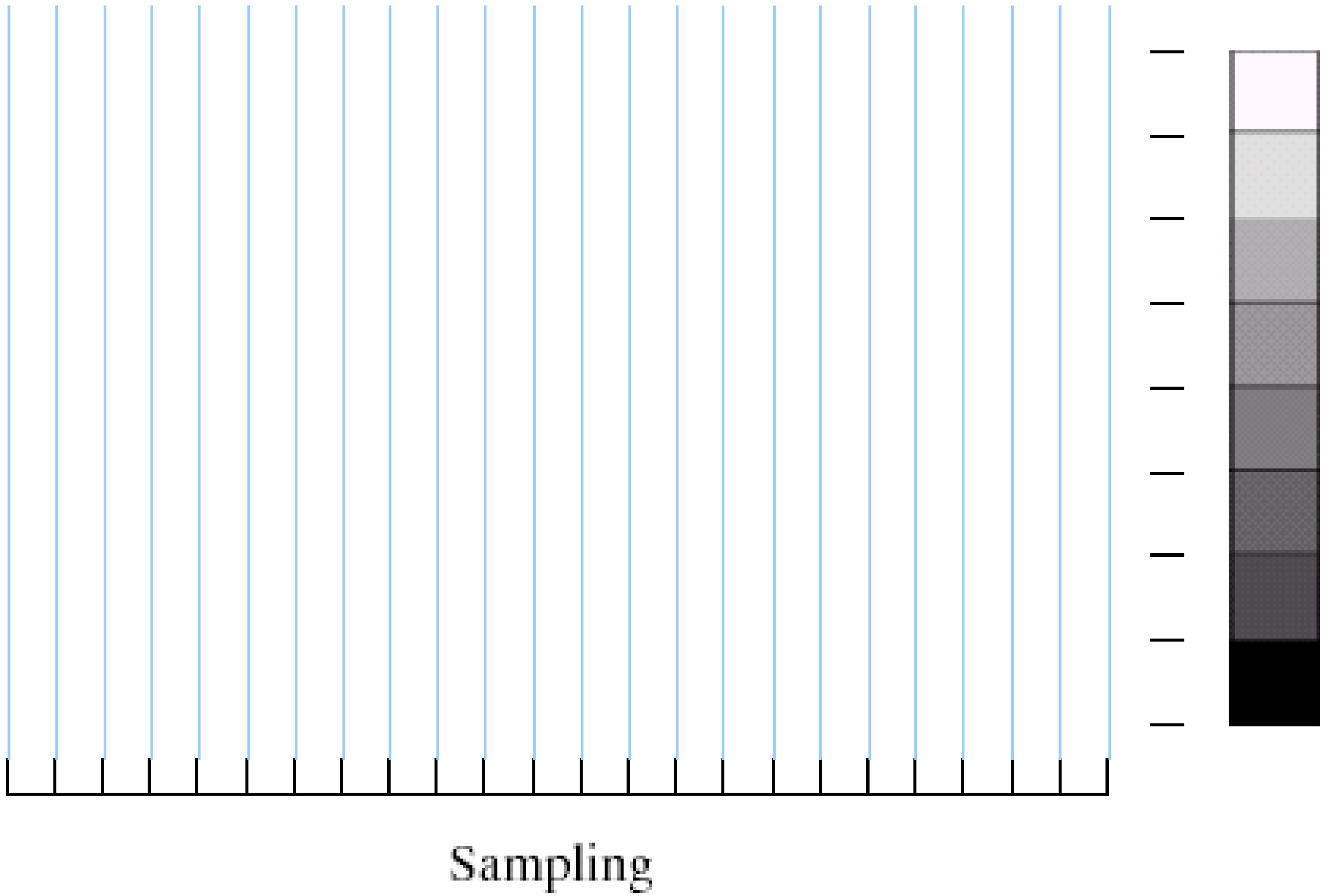
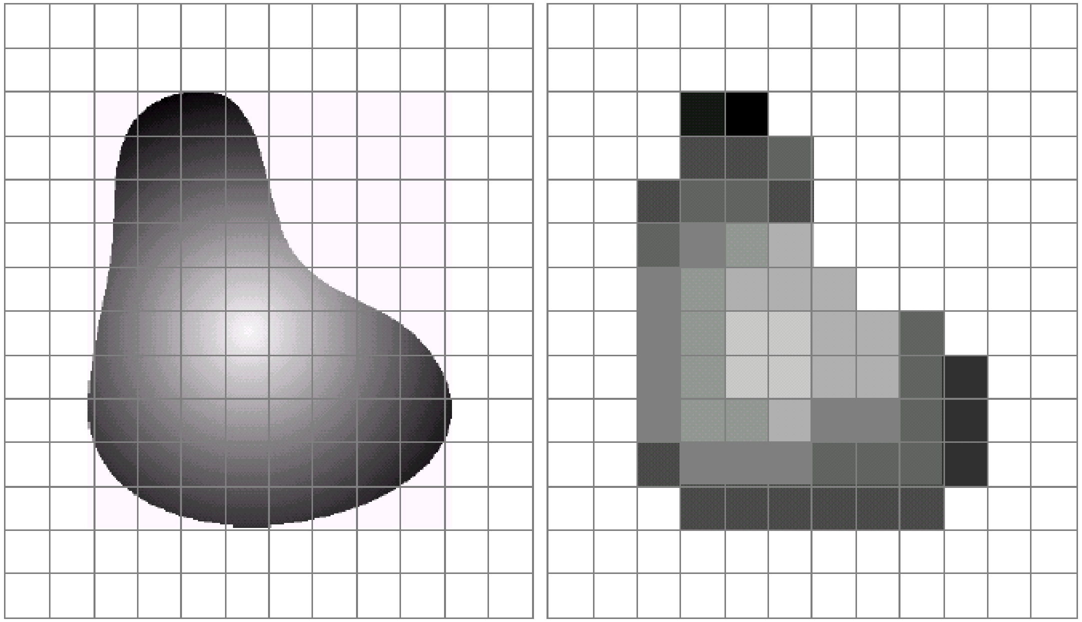


Image Sampling And Quantisation (cont…)

Digitizing the coordinate values is **Sampling**

Digitizing the amplitude values is **Quantization**



Remember that a digital image is always only an **approximation** of a real world scene

Image Representation

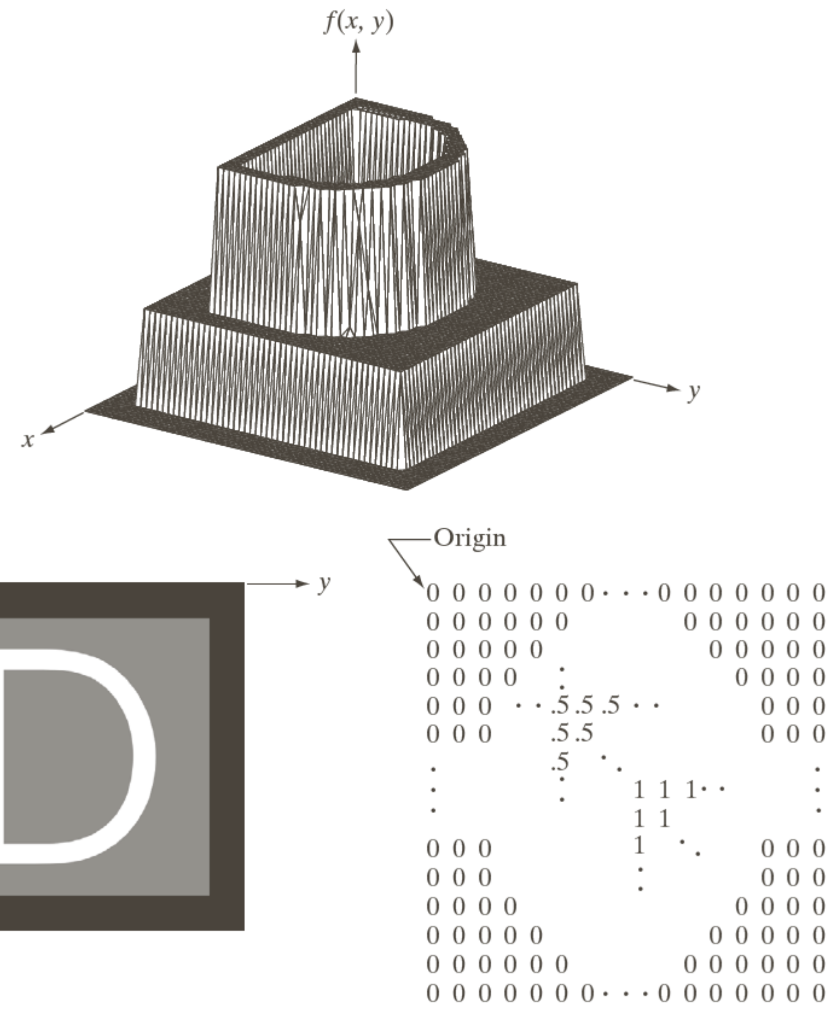


Image Representation

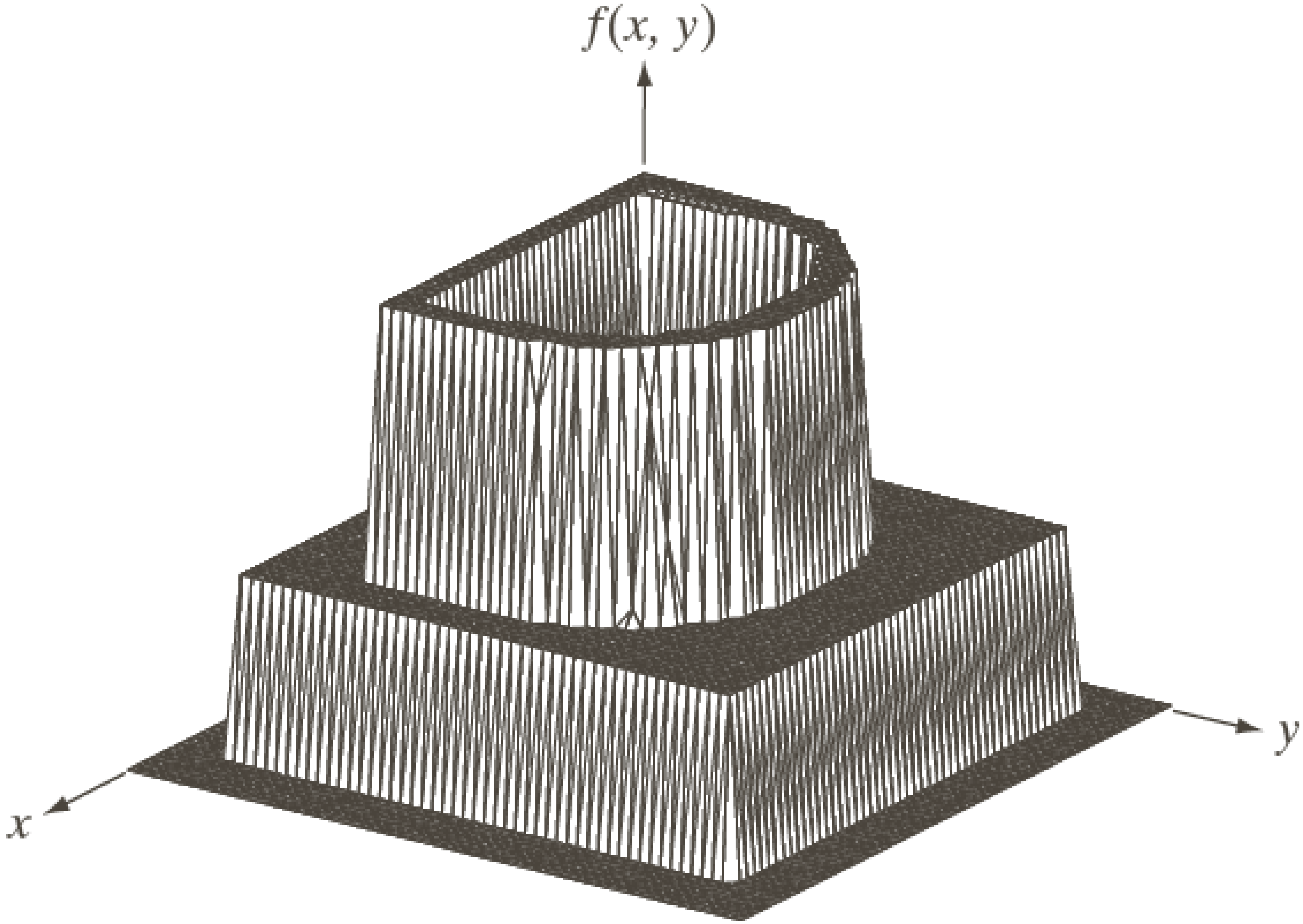


Image Representation

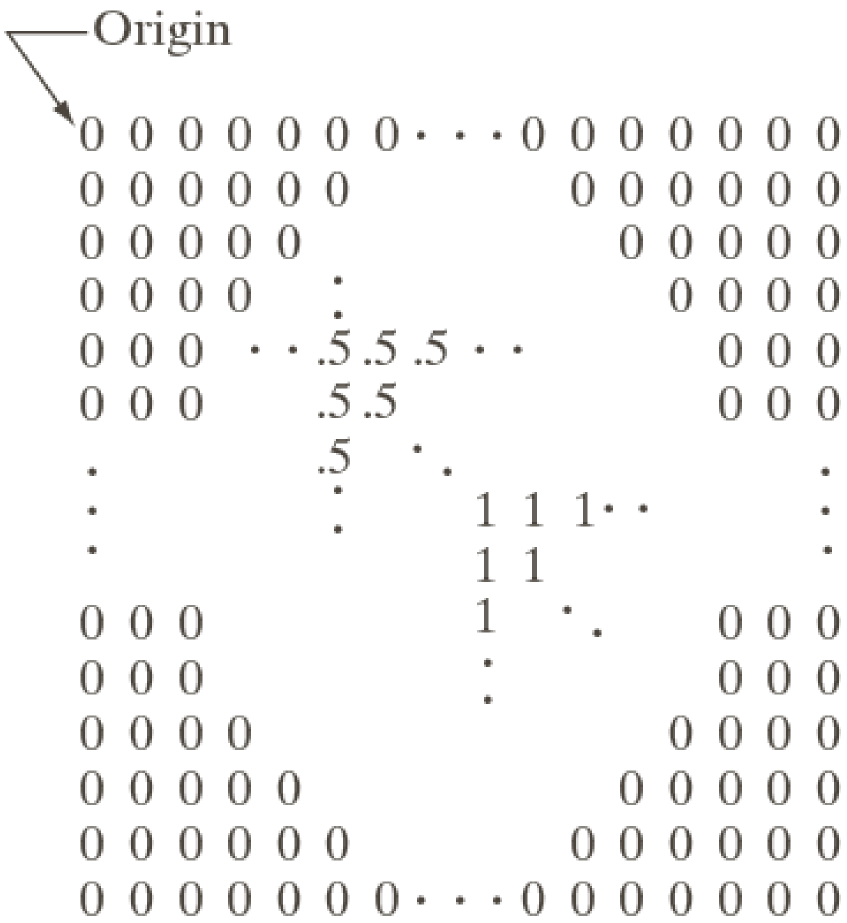
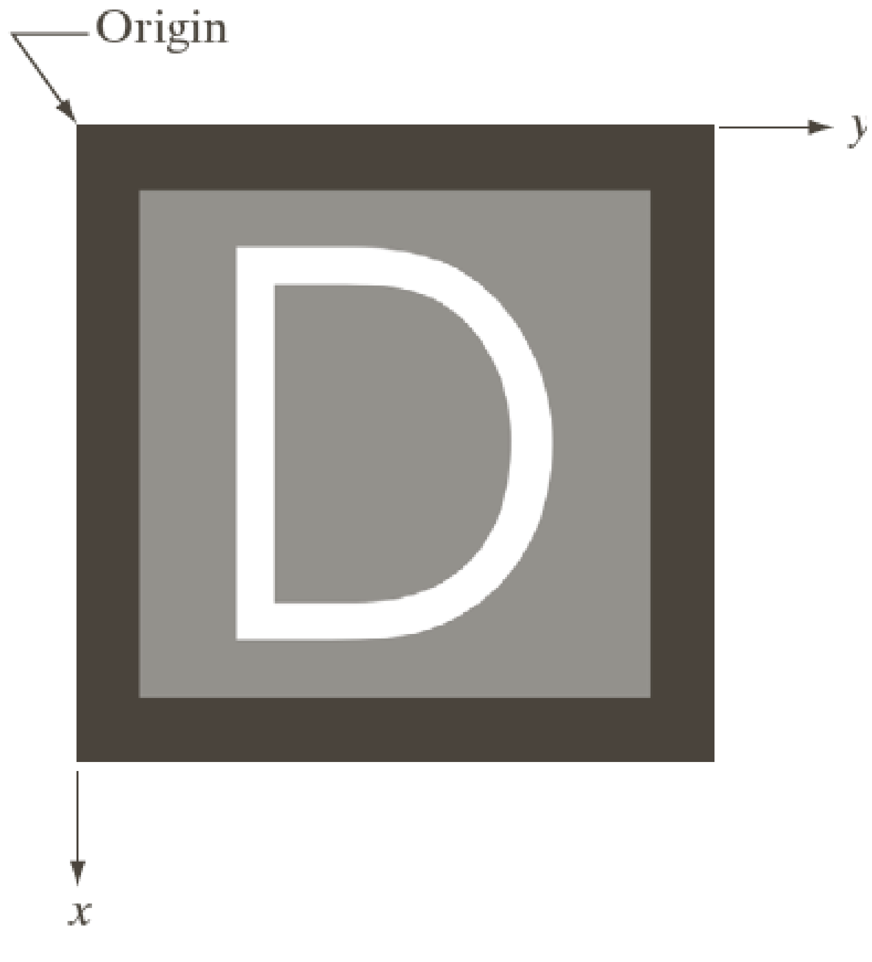


Image Representation



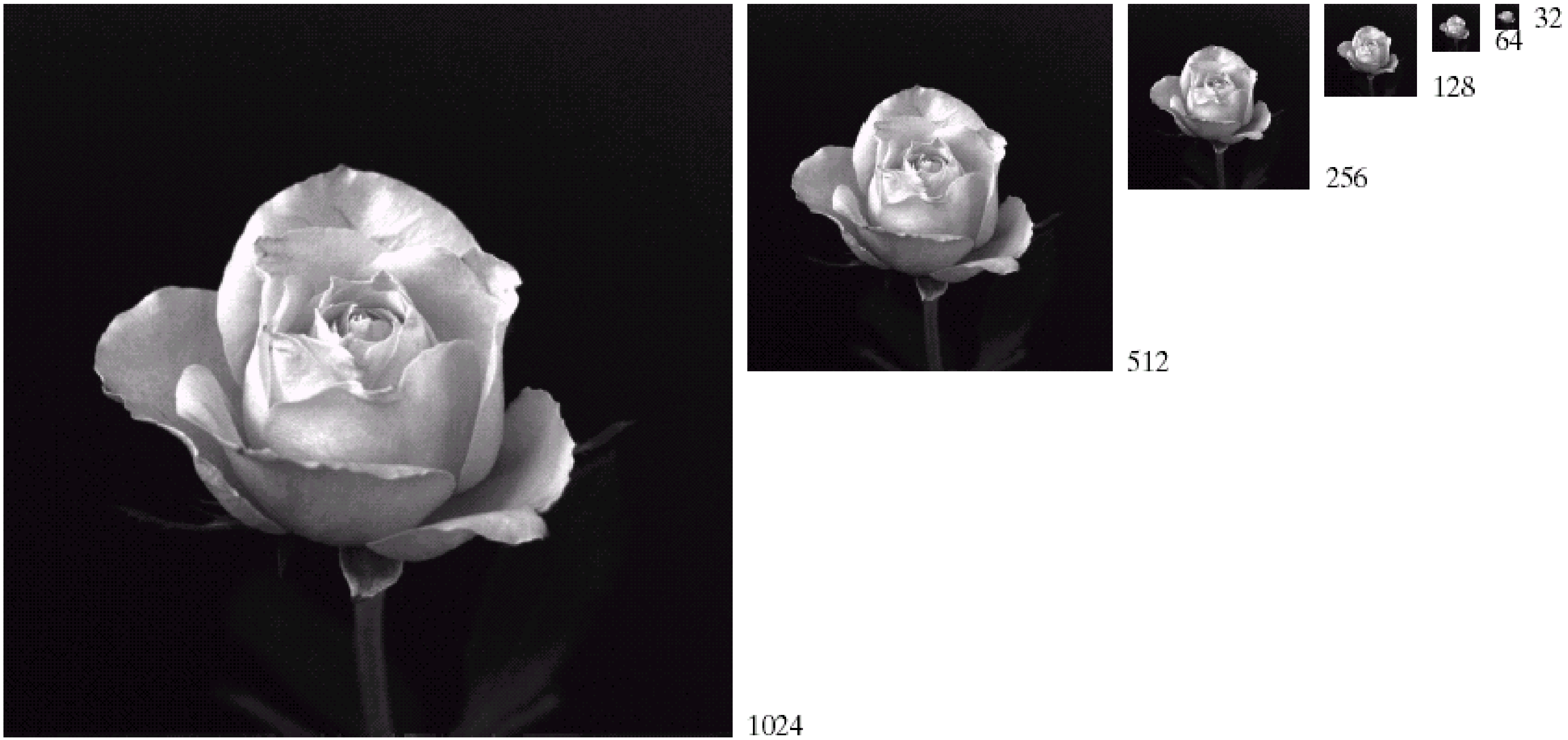
Spatial Resolution

The spatial resolution of an image is determined by how sampling was carried out

Spatial resolution simply refers to the smallest discernable detail in an image

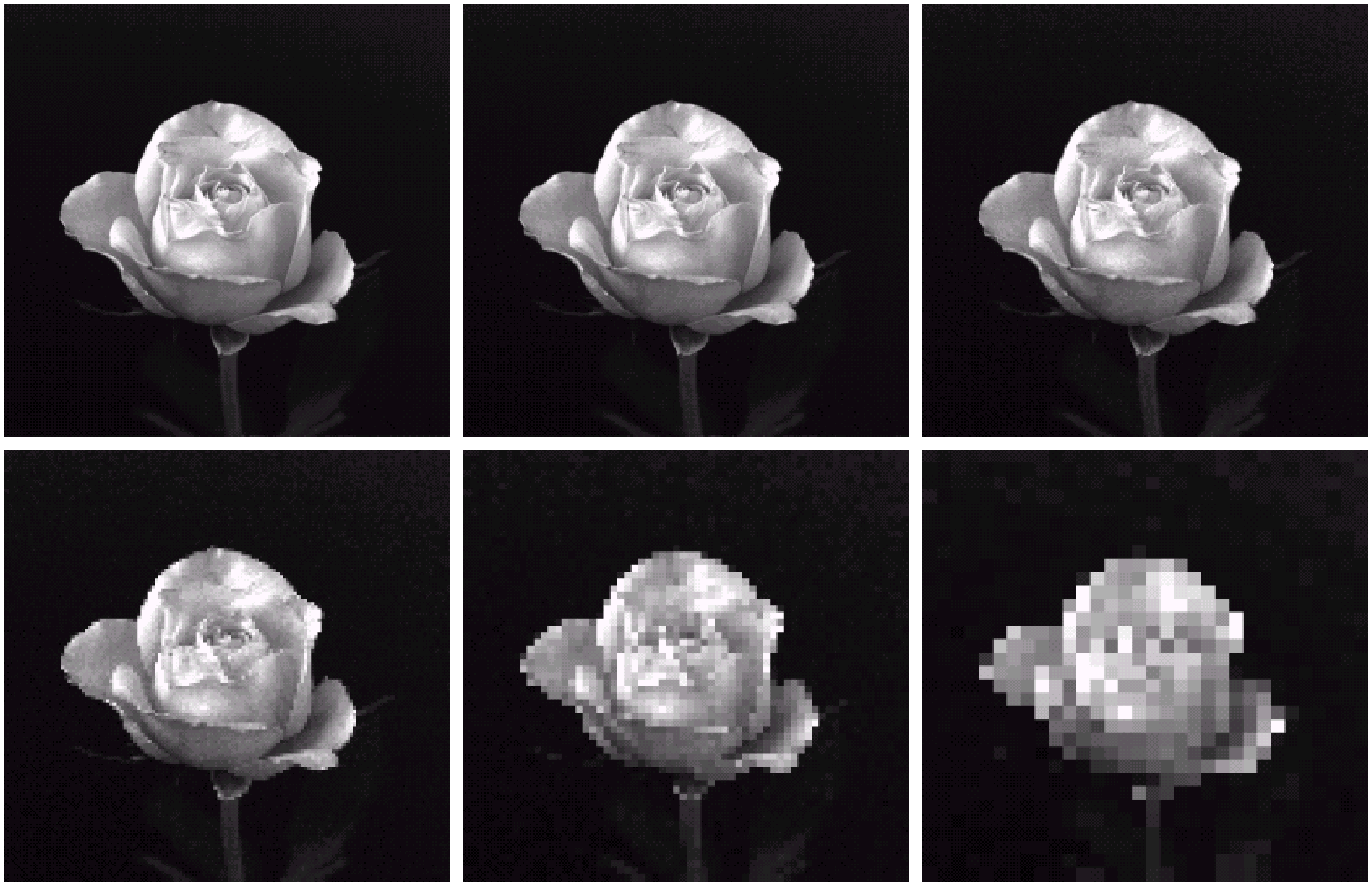
- Vision specialists will often talk about pixel size
- Graphic designers will talk about *dots per inch* (DPI)

Spatial Resolution (cont…)



Spatial Resolution (cont…)

1024 * 1024 | 512 * 512 | 256 * 256



128 * 128 | 64 * 64 | 32 * 32

Spatial Resolution (cont…)

Intensity Level Resolution

Intensity level resolution refers to the number of intensity levels used to represent the image

– The more intensity levels used, the finer the level of detail discernable in an image
– Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

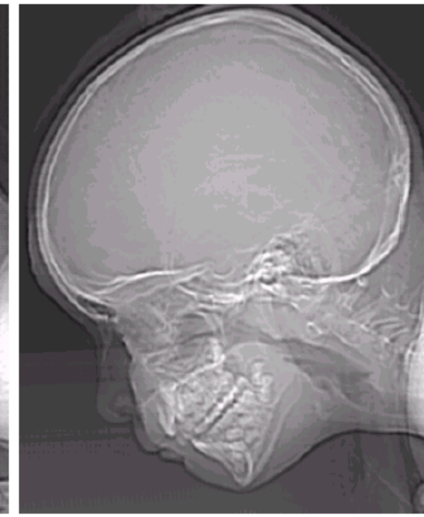
Intensity Level Resolution (cont…)

256 grey levels (8 bits per pixel)

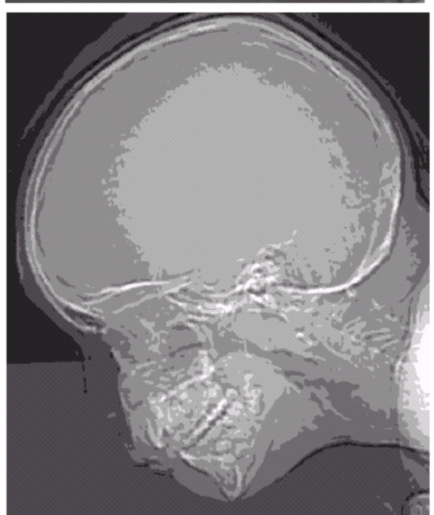
128 grey levels (7 bpp)

64 grey levels (6 bpp)

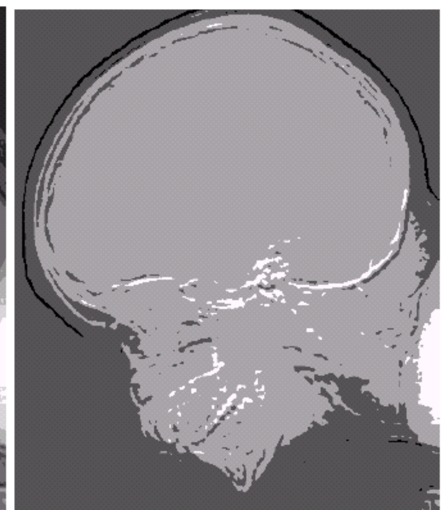
32 grey levels (5 bpp)



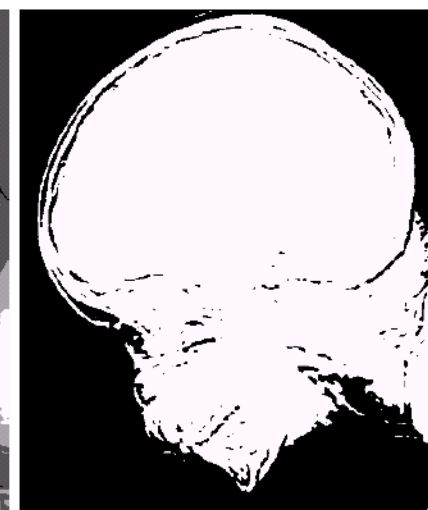
16 grey levels (4 bpp)



8 grey levels (3 bpp)

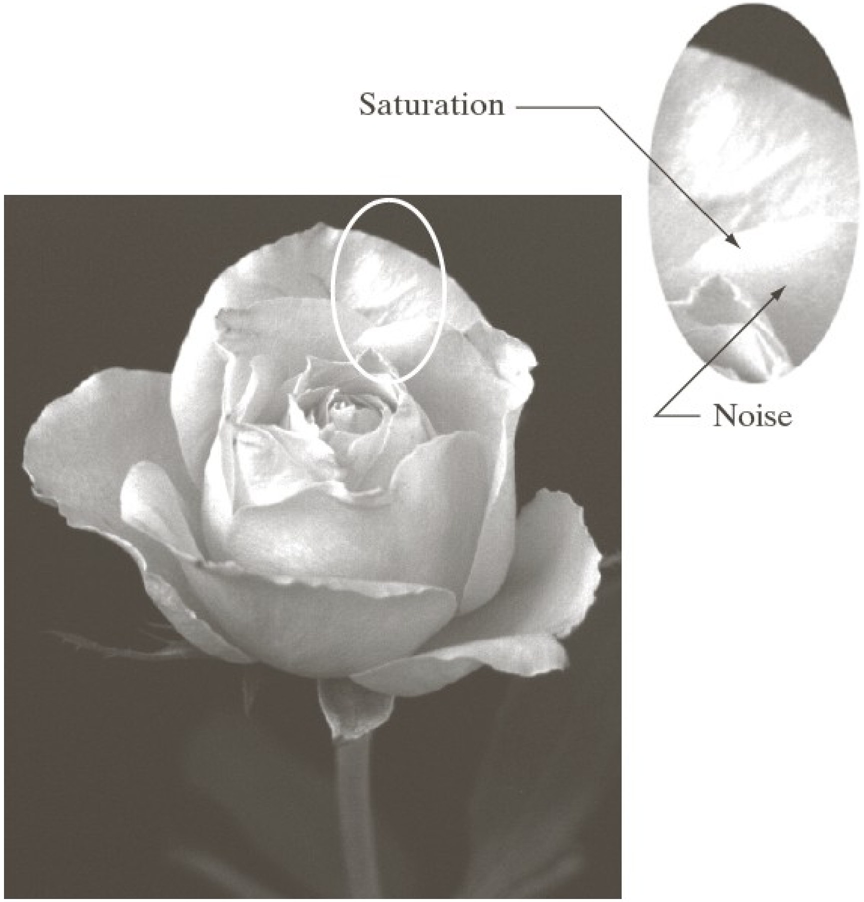


4 grey levels (2 bpp)



2 grey levels (1 bpp)

Saturation & Noise



Resolution: How Much Is Enough?

The big question with resolution is always *how much is enough*?

- This all depends on what is in the image and what you would like to do with it
- Key questions include
  - Does the image look aesthetically pleasing?
  - Can you see what you need to see within the image?

Resolution: How Much Is Enough? (cont…)

The picture on the right is fine for counting the number of cars, but not for reading the number plate

Intensity Level Resolution (cont…)

Low Detail

Medium Detail

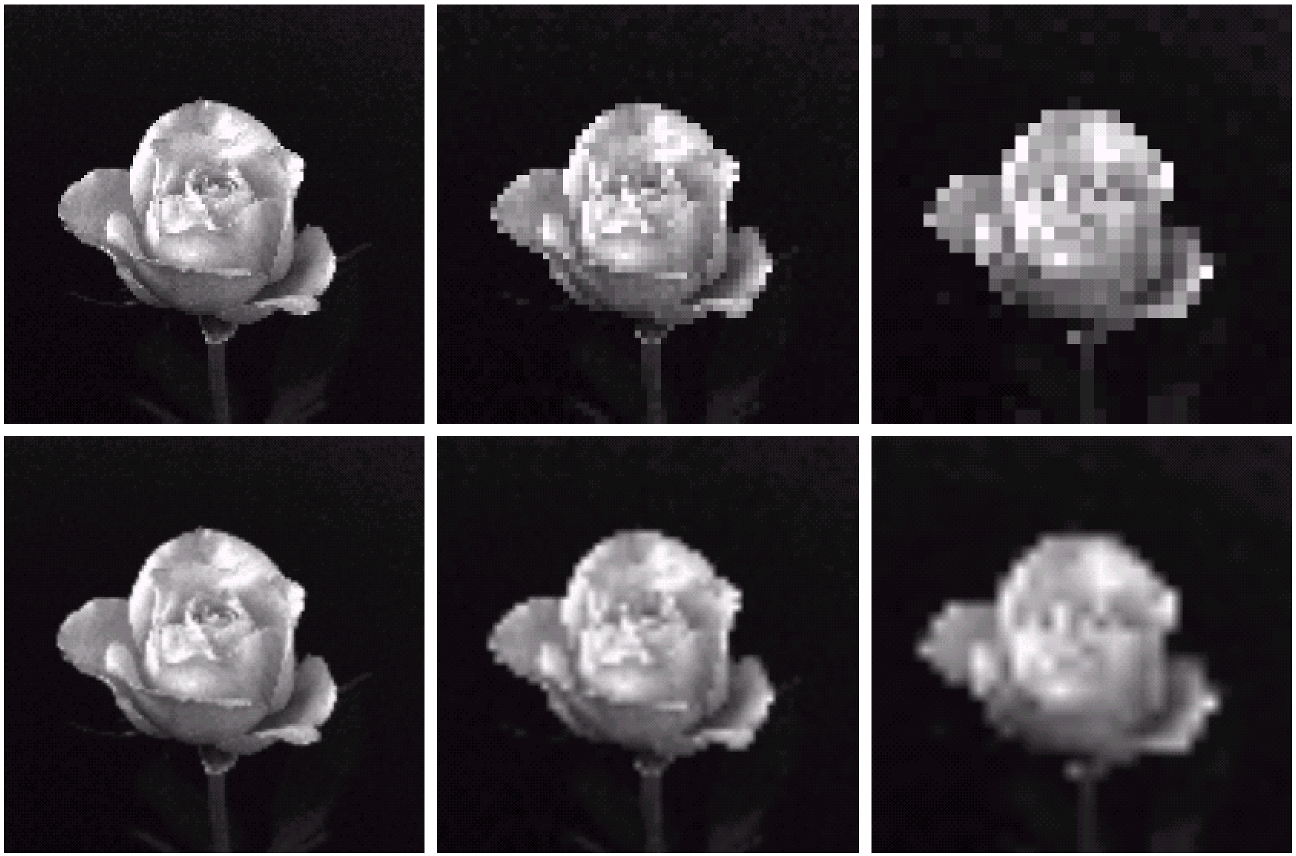
High Detail

Intensity Level Resolution (cont…)

Intensity Level Resolution (cont…)

Intensity Level Resolution (cont…)

Interpolation (cont...)



a	b	c
d	e	f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Interpolation (cont...)

a b c
d e f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

Review

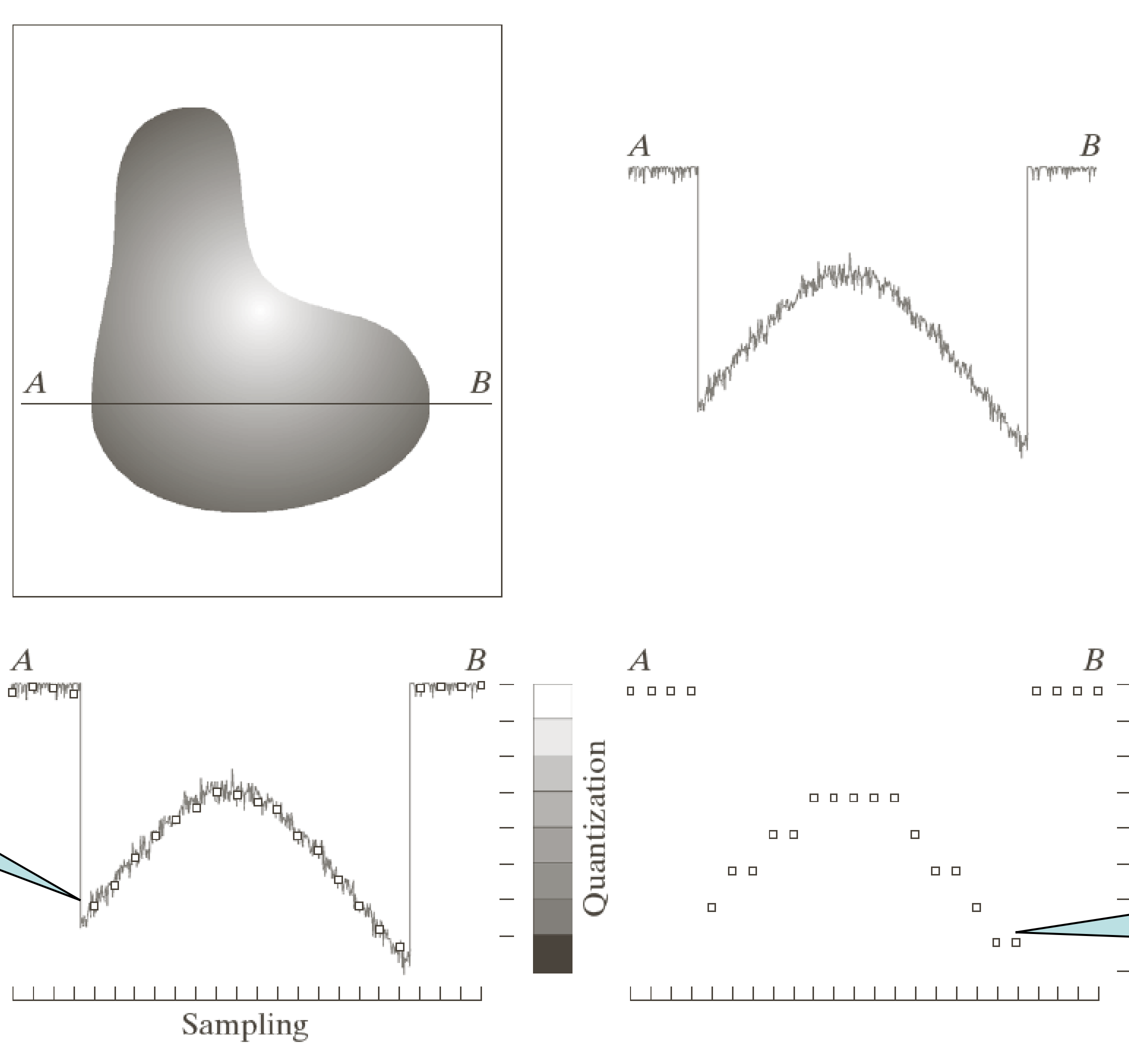


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Representing Digital Images

- The representation of an $M \times N$ numerical array as

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \ldots & f(0,N-1) \\ f(1,0) & f(1,1) & \ldots & f(1,N-1) \\ \ldots & \ldots & \ldots & \ldots \\ f(M-1,0) & f(M-1,1) & \ldots & f(M-1,N-1) \end{bmatrix}$$

Representing Digital Images

- The representation of an M × N numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Representing Digital Images

- The representation of an M × N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Representing Digital Images

- Discrete intensity interval $[0, L-1]$, $L=2^k$

- The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

Representing Digital Images

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  e.g., zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

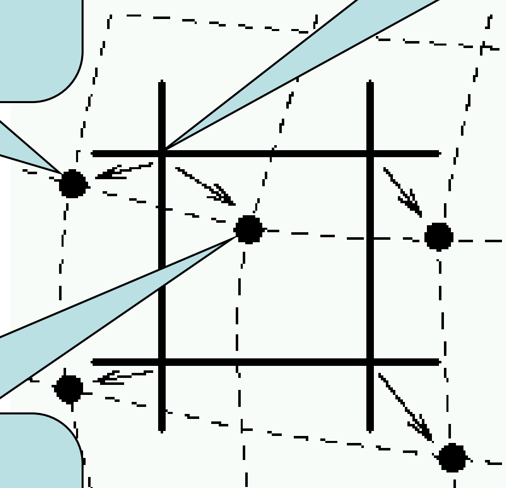
  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

  http://www.dpreview.com/learn/?/key=interpolation

Image Interpolation: Nearest Neighbor Interpolation

$f_1(x_2,y_2) =$
$f(\text{round}(x_2), \text{round}(y_2))$
$=f(x_1,y_1)$

$f(x_1,y_1)$



$f_1(x_3,y_3) =$
$f(\text{round}(x_3), \text{round}(y_3))$
$=f(x_1,y_1)$

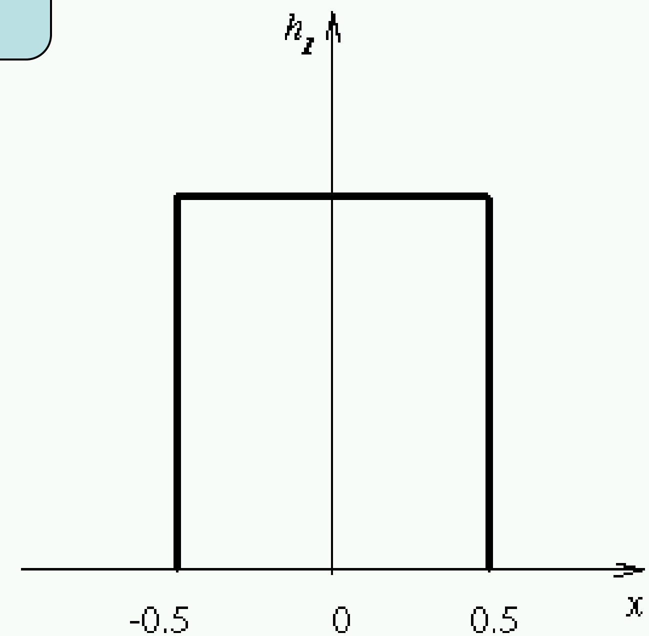
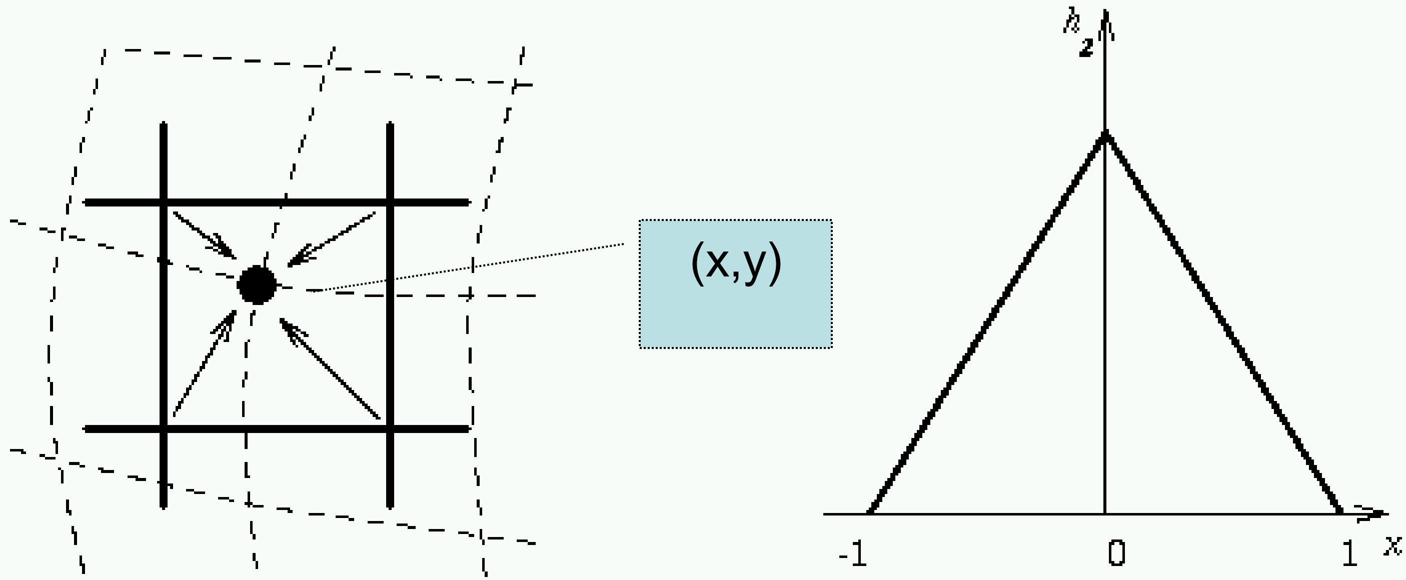


Image Interpolation: Bilinear Interpolation



$$
\begin{aligned}
&f_2(x, y) \\
&= (1-a)\bullet(1-b)\bullet f(l,k) + a\bullet(1-b)\bullet f(l+1,k) \\
&+(1-a)\bullet b\bullet f(l,k+1) + a\bullet b\bullet f(l+1,k+1) \\
&l = floor(x), k = floor(y), a = x-l, b = y-k.
\end{aligned}
$$

Image Interpolation: Bicubic Interpolation

- The intensity value assigned to point (x,y) is obtained by the following equation

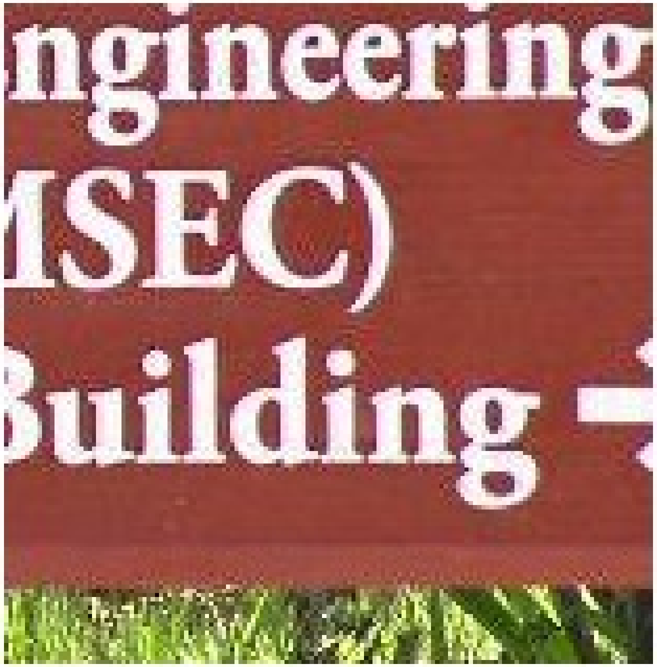
$$f_3(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x^i y^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

http://en.wikipedia.org/wiki/Bicubic_interpolation

Examples: Interpolation

Original Image



Examples: Interpolation

Nearest Neighbor Interpolation

Examples: Interpolation

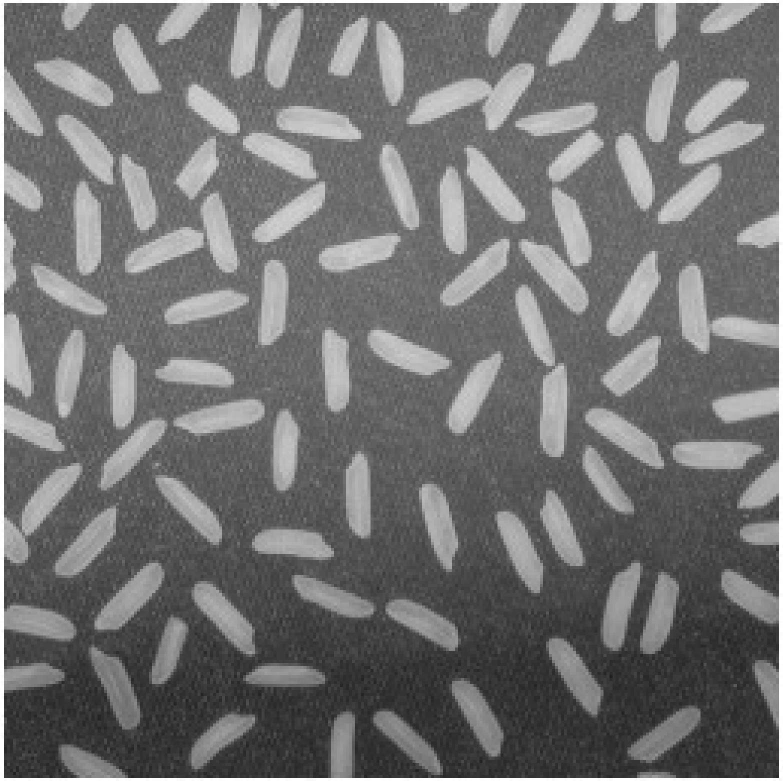
Bilinear Interpolation

Examples: Interpolation

Bicubic Interpolation

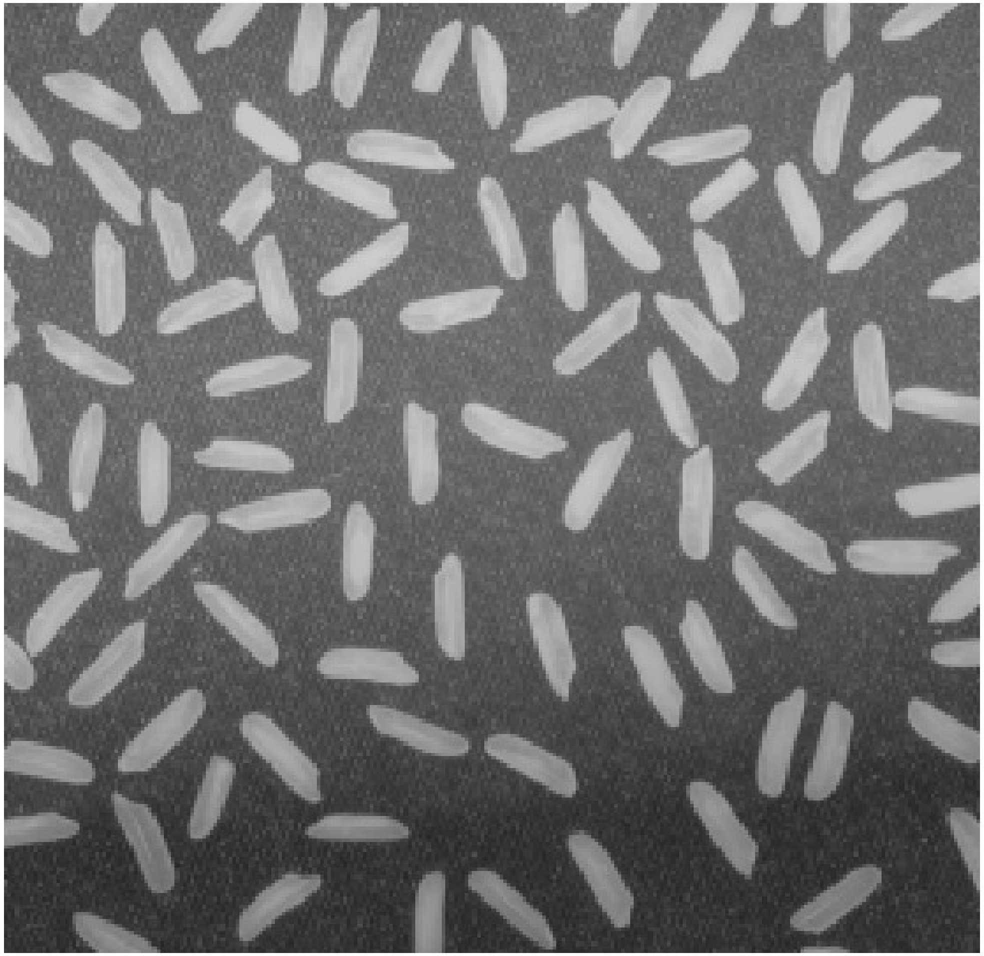
Examples: Interpolation

original image



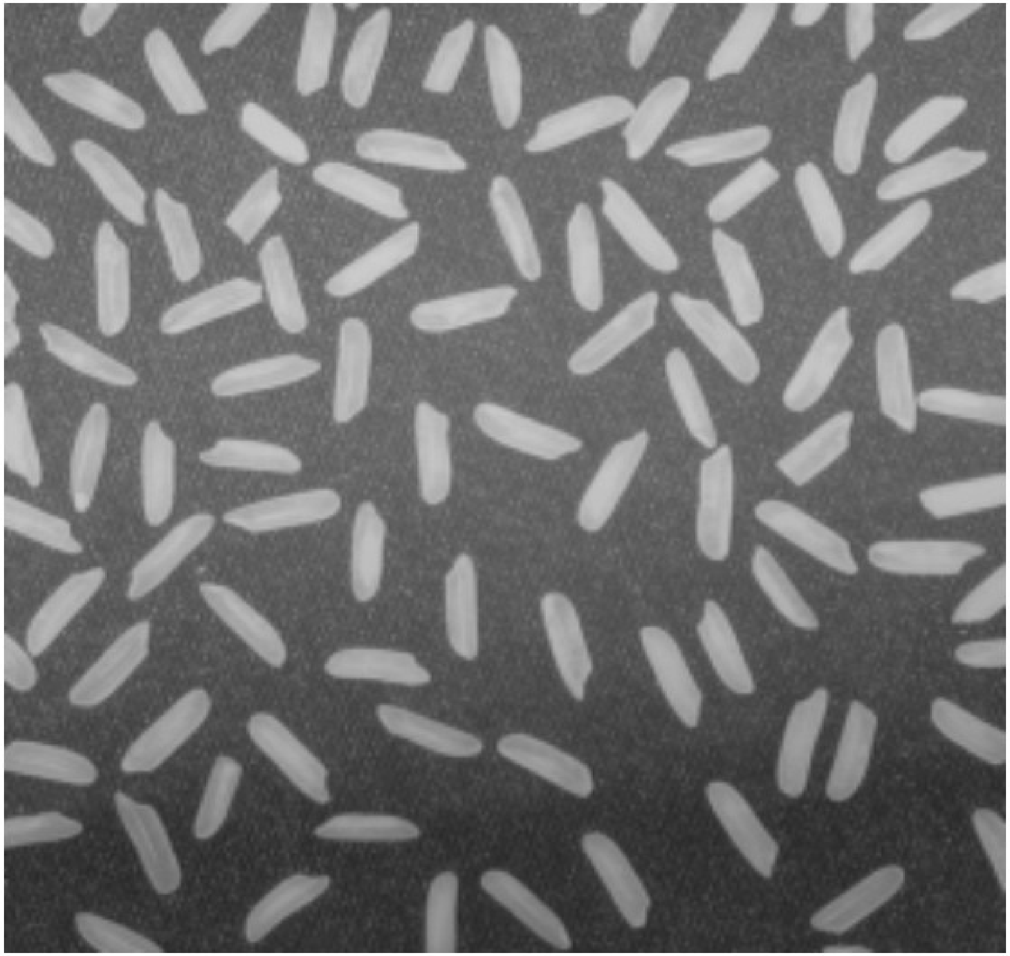
Examples: Interpolation

nearest



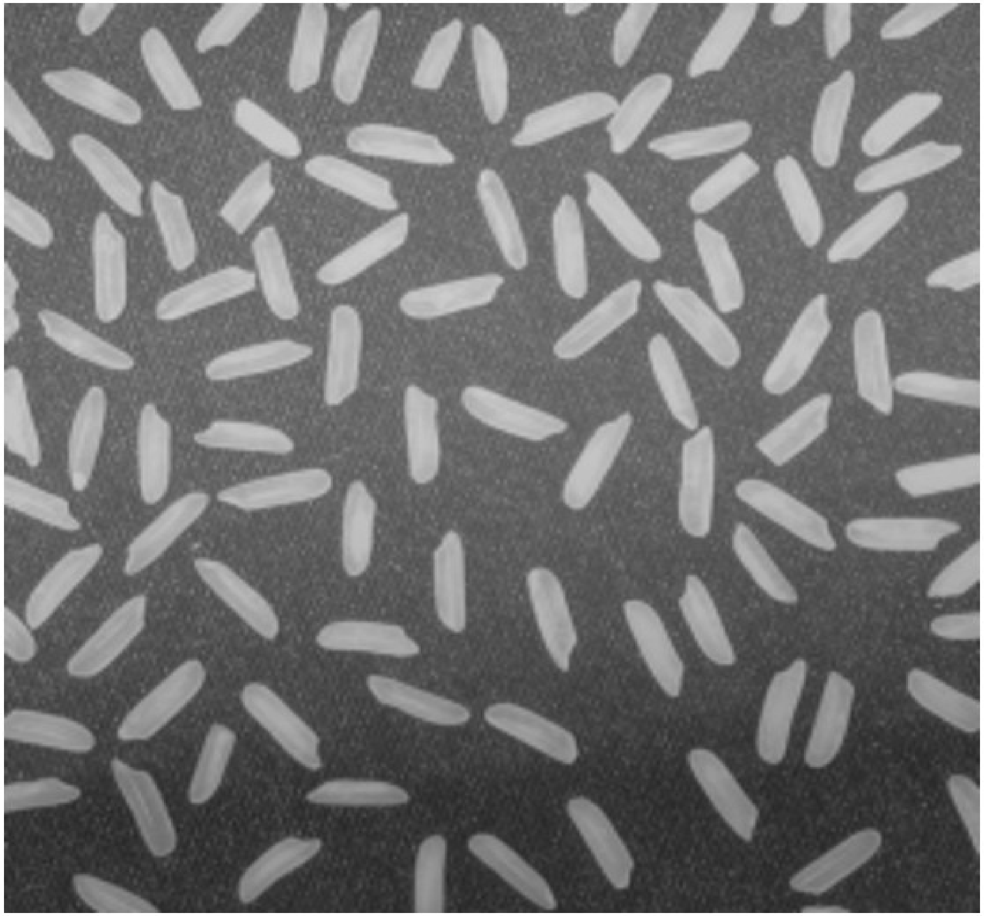
Examples: Interpolation

bilinear



Examples: Interpolation

bicubic



Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

- **Neighbors** of a pixel p at coordinates (x,y)

➢ **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
(x-1, y), (x+1, y), (x,y-1), and (x, y+1).

➢ **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
(x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

➢ **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
$N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Basic Relationships Between Pixels

- **Path**

➢ A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

➢ Here *n* is the *length* of the path.

➢ If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.

➢ We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

V = {1, 2}

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent **m-adjacent**

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

m-adjacent

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

- **Connected in S**

Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.
- If S has only one connected component, then S is called ***Connected Set*.**
- We call R a **region** of the image if R is a connected set
- Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

Basic Relationships Between Pixels

- **Boundary (or border)**

  - The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
  - If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

- **Foreground and background**

  - An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

  All the points in R_u is called **foreground**;

  All the points in $(R_u)^c$ is called **background**.

Question 1

- **In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 2

- **In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Part 1

Part 2

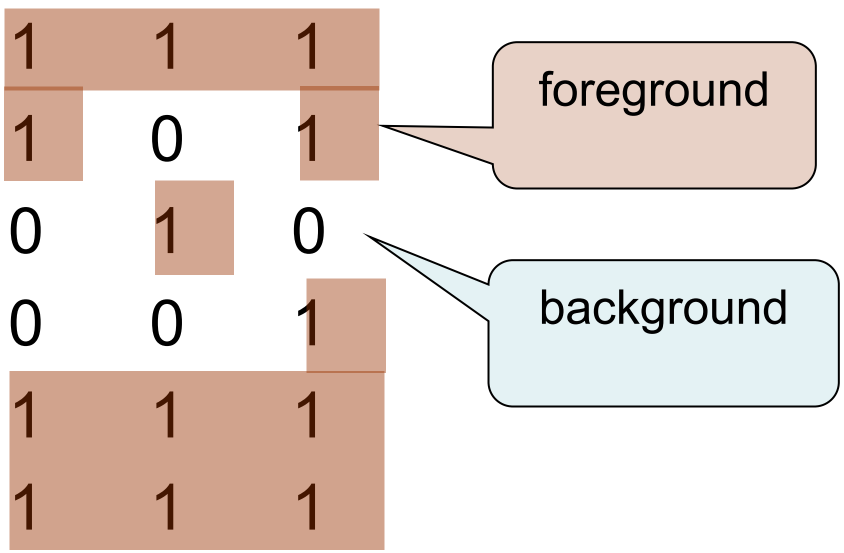
- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**



Question 3

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Distance Measures

- Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. D(p, q) ≥ 0 [D(p, q) = 0, iff p = q]

b. D(p, q) = D(q, p)

c. D(p, z) ≤ D(p, q) + D(q, z)

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. City Block Distance:

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. Chess Board Distance:

$D_8(p, q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Question 5

- **In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

- **In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 7

- **In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 8

- **In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	0	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Introduction to Mathematical Operations in DIP

- **Array vs. Matrix Operation**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \;.*\; B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- **Linear vs. Nonlinear Operation**

$$H\left[f(x,y)\right] = g(x,y)$$

$$H\left[a_i f_i(x,y) + a_j f_j(x,y)\right]$$

$$= H\left[a_i f_i(x,y)\right] + H\left[a_j f_j(x,y)\right]$$

Additivity

$$= a_i H\left[f_i(x,y)\right] + a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$= a_i g_i(x,y) + a_j g_j(x,y)$$

H is said to be a **linear operator;**

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations

- Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: f(x,y)

Noise: n(x,y) (at every pair of coordinates (x,y), the noise is uncorrelated and has zero average value)

Corrupted image: g(x,y)

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$\begin{aligned}
E\{\bar{g}(x,y)\} &= E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\} \\
&= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y) + n_i(x,y)\right]\right\} \\
&= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\} \\
&= f(x,y)
\end{aligned}$$

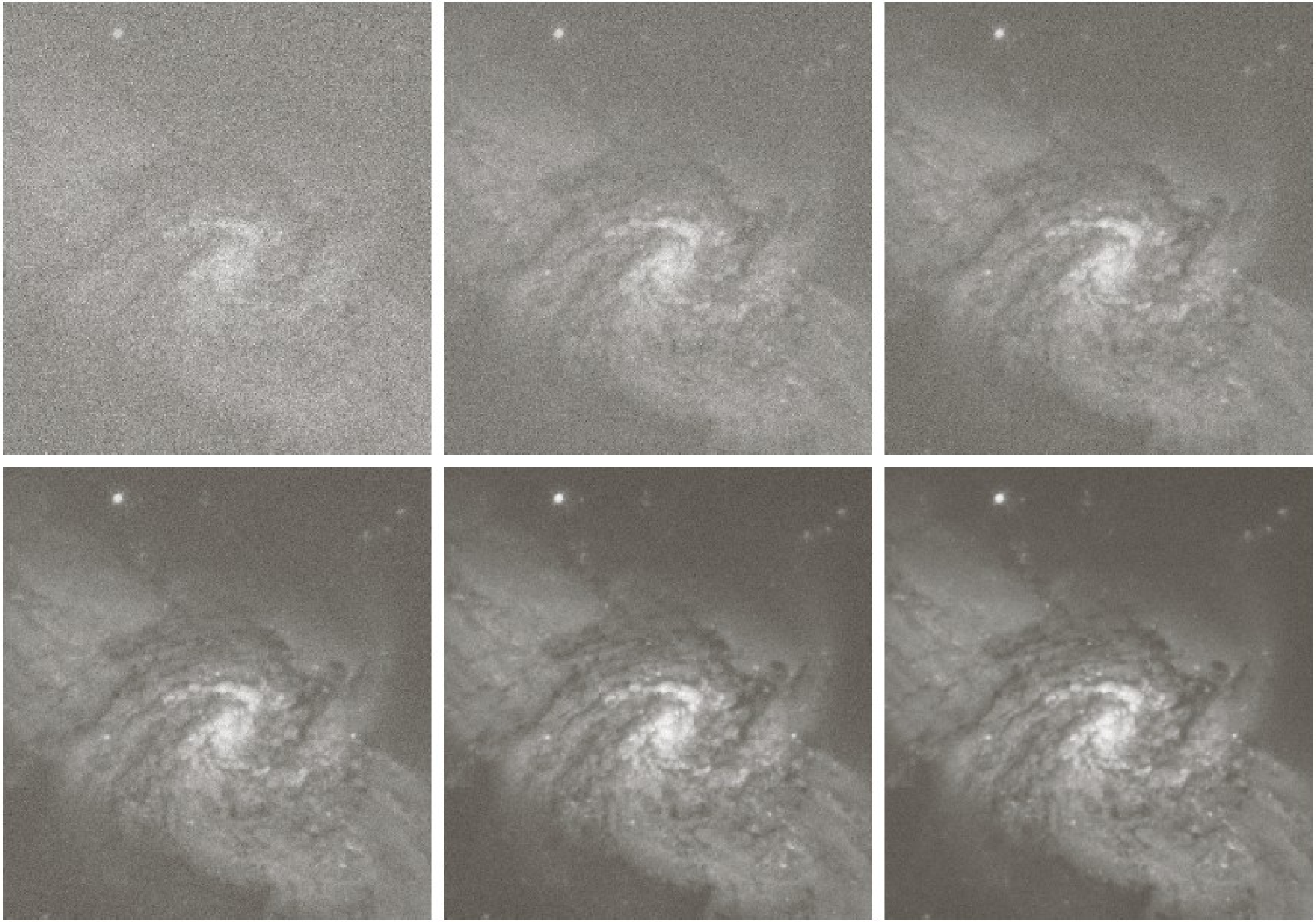
$$\sigma^2_{\bar{g}(x,y)} = \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)}$$

$$= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)} = \frac{1}{K}\sigma^2_{n(x,y)}$$

Example: Addition of Noisy Images for Noise Reduction

- In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

- In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

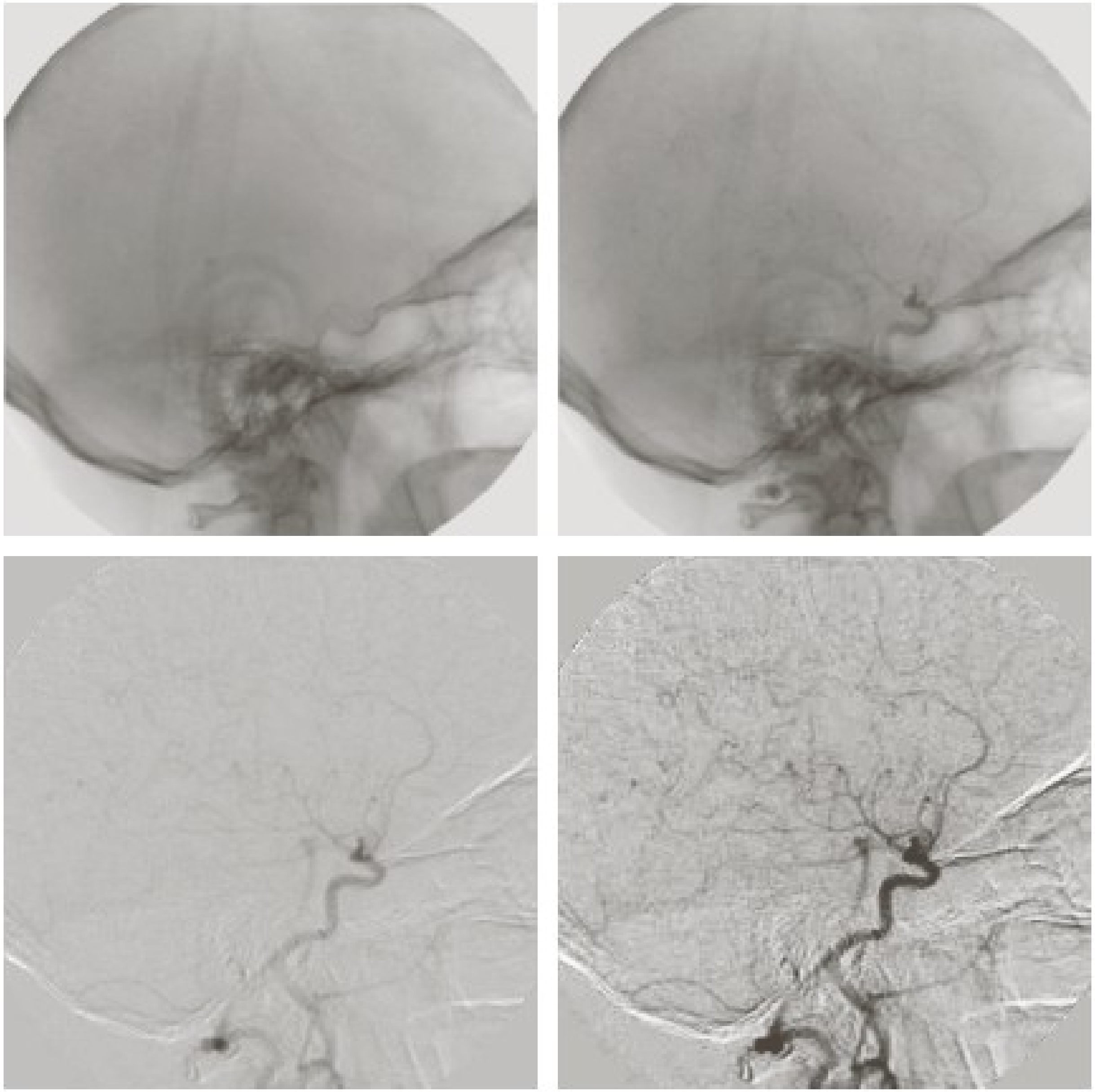
Mask h(x,y): an X-ray image of a region of a patient's body

Live images f(x,y): X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail g(x,y)

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

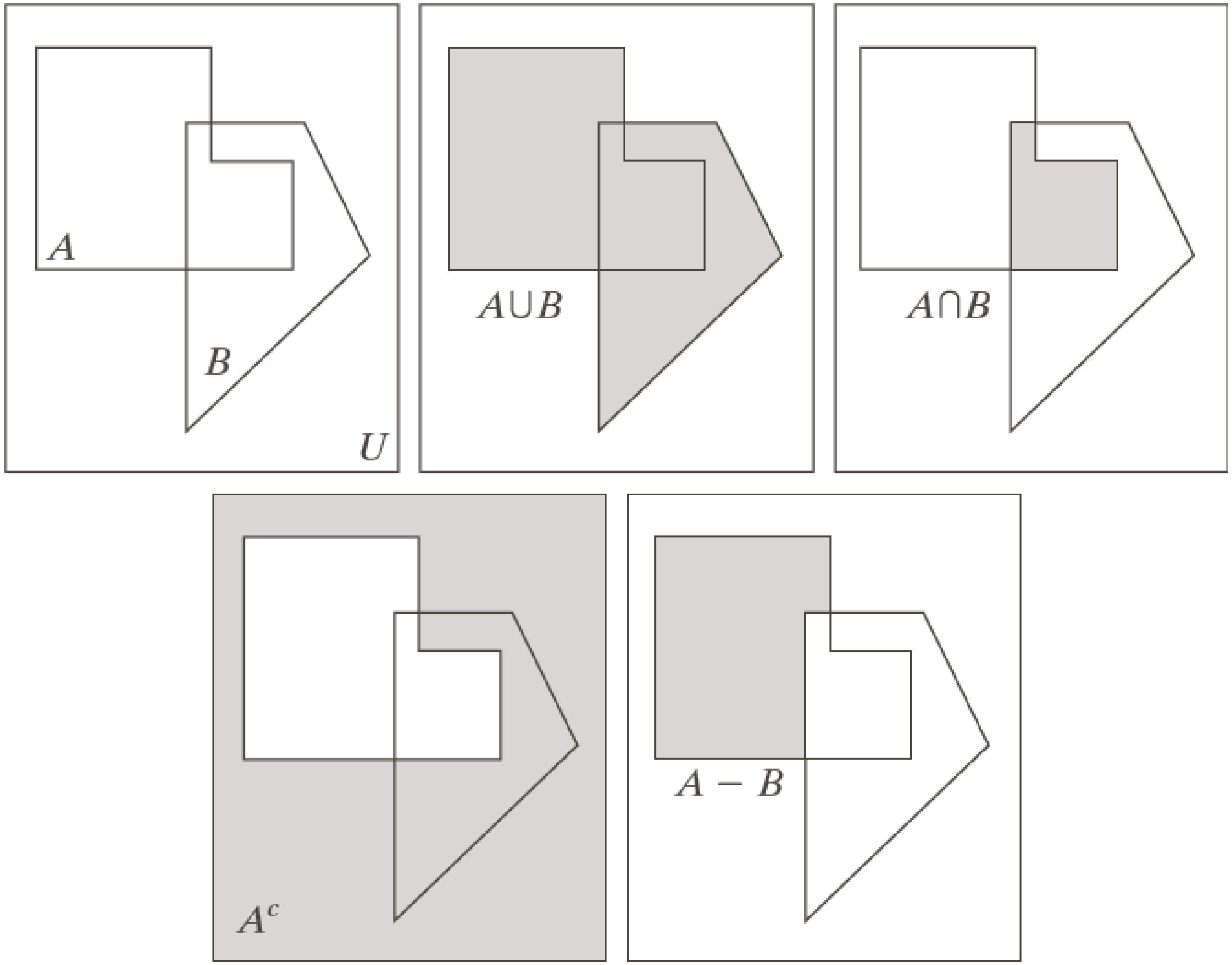
a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

An Example of Image Multiplication

- Z = immultiply(X, Y)
  - Multiplies each element in array X by the corresponding element in array Y, and an array Z of same size.
  - Original image I, I * I, I * 0.5

Set and Logical Operations



a b c
d e

FIGURE 2.31 (a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- Let A be the elements of a gray-scale image

  The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$$K = 2^k - 1; k \text{ is the number of intensity bits used to represent } z$$

g

Set and Logical Operations

- The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max_{z}(a, b) \mid a \in A, b \in B\}$$

Set and Logical Operations

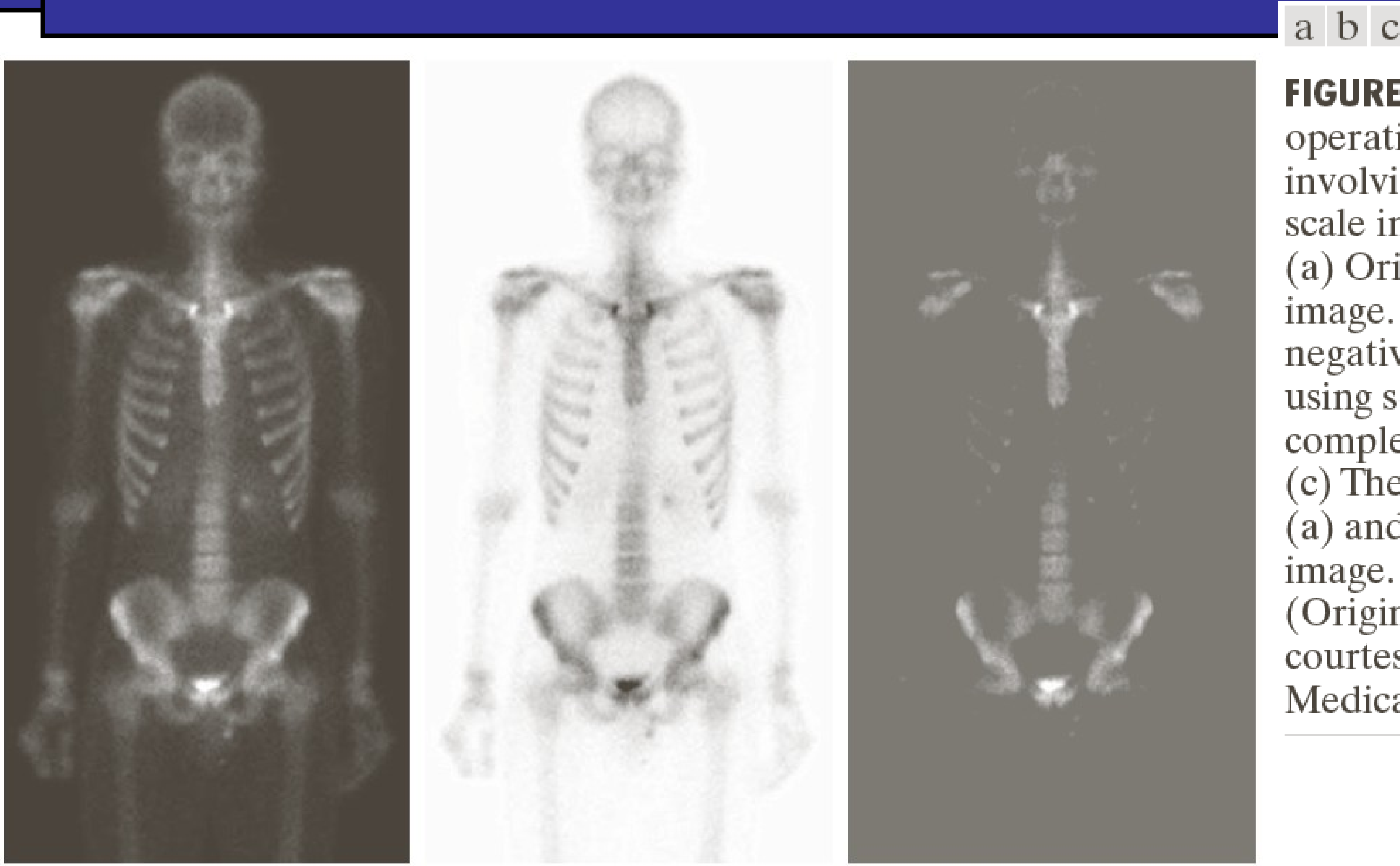


FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Set and Logical Operations

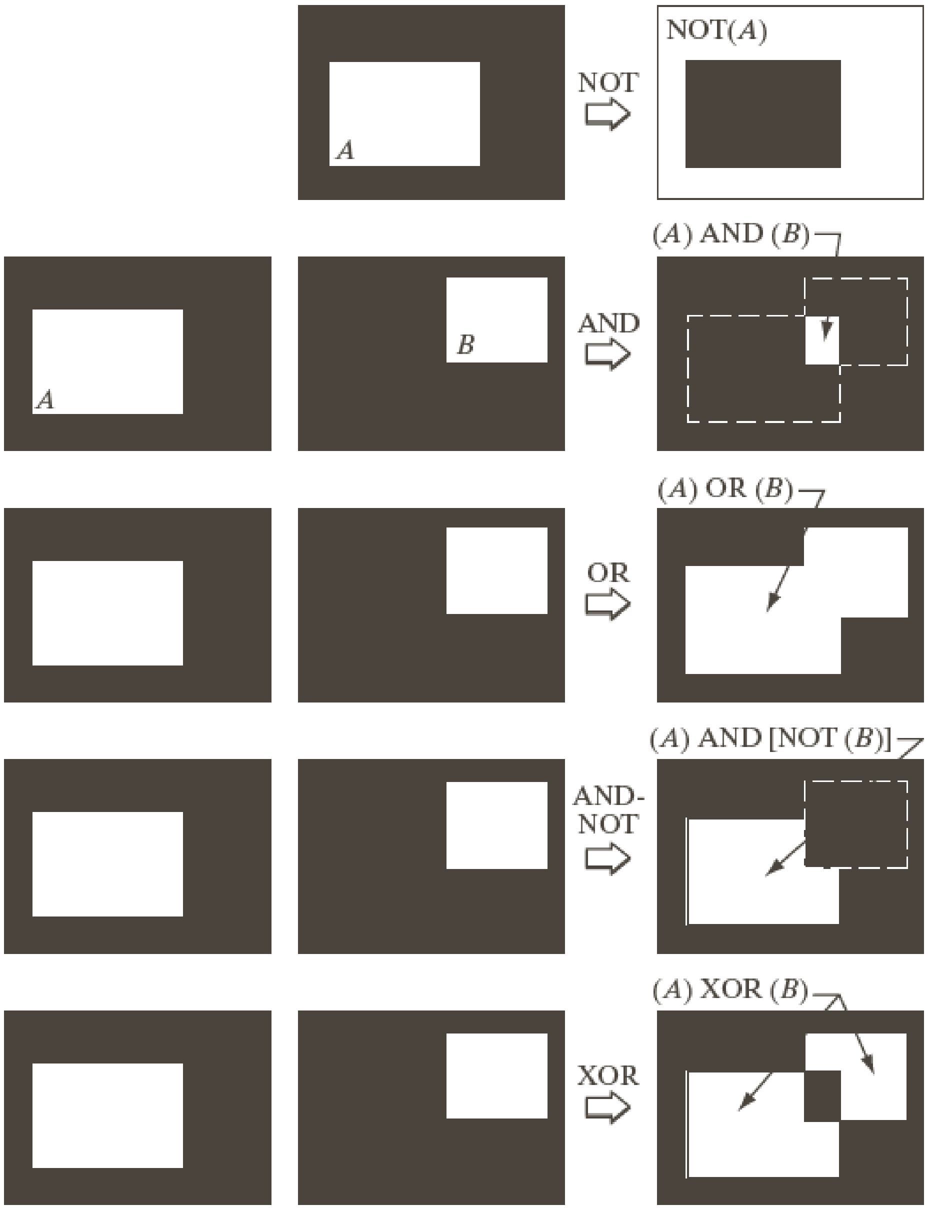


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.