CEE546 - Assignment 3

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Link to Part II of assignment: https://pastrana.xyz/equad_courtyard

Part I

1 Introduction

The goal of Part I is to use the dynamic relaxation (DR) method to calculate the internal forces in the elements of the Geiger Dome. A Matlab program, which executes the DR solver for a specific structural configuration specified in an input file, is provided as part of this assignment. The tension rods and compression struts are then designed according to Ultimate Limit States (ULS) conditions, ensuring that overall Serviceability Limit States (SLS) conditions are satisfied throughout the structure.

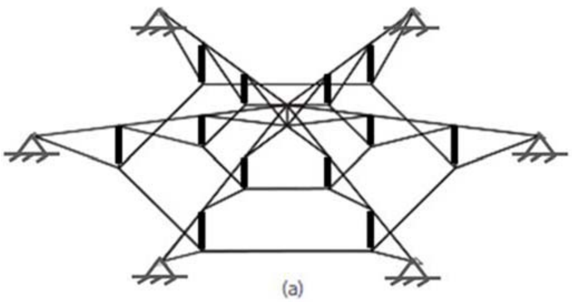
A starting input file, which describes the geometry and connectivity of the structure, is also provided. The goal is to complete this file by including accurate values for the following parameters:

1. Material properties (density and stiffness)
2. Level of prestress in tension members
3. Cross-sectional area of tension/compression elements

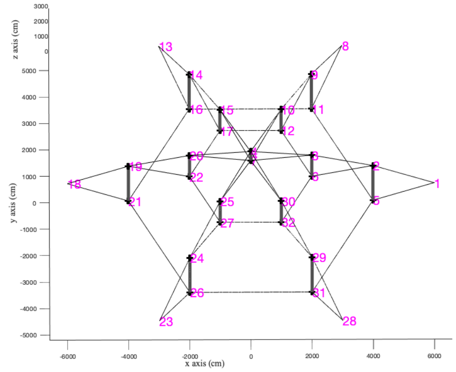
The whole process is somewhat iterative since parameters that influence the analysis results, such as the self-weight of the structure, can only be determined after the analysis is complete. This report will only present the final design results, after this iterative process has resulted in a reasonably optimized structure.

2 Geometry of the Geiger Dome

Isometric views of the structure are shown in Figure 1 and 2d views in Figure 2. The nodes are numbered as they are defined in the analysis model.

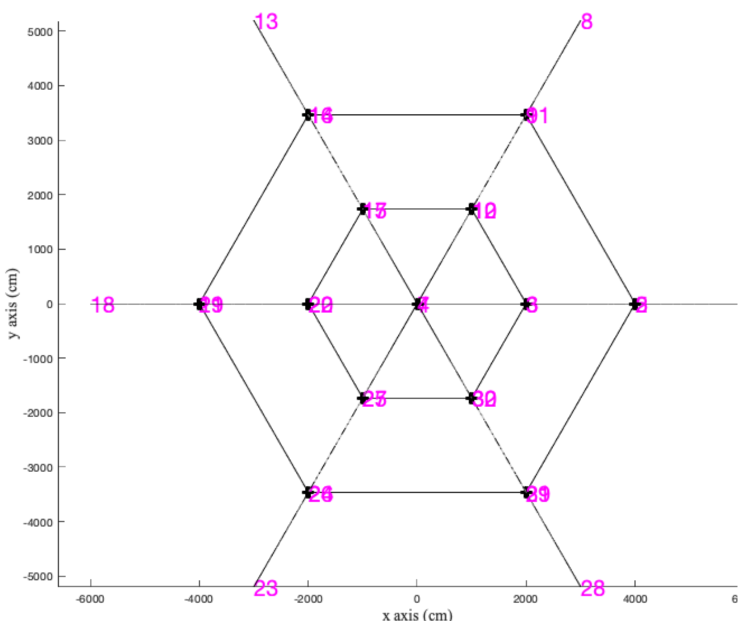


(a) Isometric view sketch

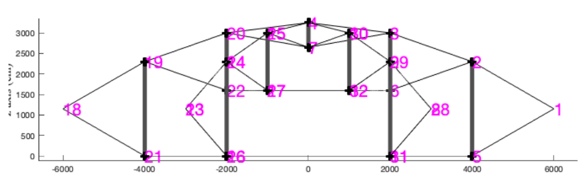


(b) Isometric view from Matlab

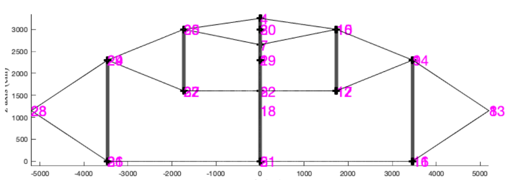
Figure 1: 3D Views of Geiger Dome.



(a) Plan view



(b) XZ view



(c) YZ view

Figure 2: 2D Views of Geiger Dome.

3 Structural Design Assumptions

3.1 Materials

All the elements are assumed to be made with CSA G40.21 - 350 grade structural steel. Equivalent grades of steel exist in the United States and Europe, but the precise grade is not relevant to this assignment. All values are shown in units of $\frac{kN}{cm^2}$, as the Matlab script is formatted in this manner, according to European preference for these units in design. All members are assumed to have the following properties:

1. Yield Strength: 350 MPa $= 35\ \frac{kN}{cm^2}$
2. Ultimate Strength: 450-620 MPa $=$ 45-62 $\frac{kN}{cm^2}$
3. Young's Modulus: 200,000 MPa $=$ 20,000 $\frac{kN}{cm^2}$
4. Density: $8000\ \frac{kg}{m^3} = 80\ \frac{kN}{m^3} = 0.000080\ \frac{kN}{cm^3}$

Note that the steel used for tension cables is typically different (i.e. lower modulus) than that used for fabricated steel sections. In lieu of more information on the material used in the structure, the stiffness values for cables shown in Figure 3 will be used.

Typical Mechanical Properties		Elastic Modulus (E)		Ultimate Strength	
		ksi (x10³)	GPa	ksi	MPa
SOLID RODS	Steel Rod, Grade 460	30.0	207	90	621
	High Strength Stainless Rod	26.0	179	200	1379
	Medium Strength Stainless Rod	26.0	179	140	965
	LCW Stainless Rod	26.0	179	100	690
CABLES	Galvanized Strand	18.0	124	153	1055
	Full Locked Galvanized	19.5	134	176	1214
	Stainless Steel Strand	15.0	103	132	910
	Galvanized Wire Rope	12.0	83	117	807
	Stainless Wire Rope	7.0	48	117	807

Figure 3: Steel rods vs. cables

3.2 Loading

The loading on the structure is assumed to be primarily vertical; no unbalanced horizontal wind load cases will be examined. In lieu of information regarding the actual design provisions at the time of construction, the design loads and combinations used in this assignment are based on the standard values from modern North American design codes.

The types of vertical loads fall under the following categories:

- Member self-weight: $\gamma \cdot A \cdot L$ for each element
- Suspended fixtures: 0.5 kPa
- Snow or Rain: 0.4 kPa
- Wind: ±0.5 kPa
- Live: 1.0 kPa

Although the roof is not accessible, a minimum provision of 1 kPa live load is typically provided in the design of roofs to account for maintenance activities. The snow/rain load is also low since the expected snow fall in Seoul is not large, and the curved design of the roof eliminates any drift or ponding accumulation.

3.3 Design Load Cases

The design of the structural members will be carried out based on the limit states design principle. Most importantly, the structure must satisfy the Ultimate Limit State (ULS) of life-safety, where all members must resist at least the highest load calculated from several different factored load combinations. The structure must also satisfy the Serviceability Limit State (SLS) of deflection, to limit excessive movement under unfactored loads.

Out of the many possible loading conditions, the two shown in Equation 1 represent the worst case "downward" and "upward" (in the direction of gravity) ULS load cases. The factors used in these load cases are taken from the National Building Code of Canada (NBCC), but are similar across all major design codes. The deflections are checked under the SLS load case, which consists of the unfactored wind load.

$$\begin{aligned} w_{ULS1} &= 1.25 \cdot DL + 1.5 \cdot LL + 0.5 \cdot WL = 1.25 \cdot 0.5 + 1.5 \cdot 1.0 + 0.5 \cdot 0.5 = 2.125\, kPa \\ w_{ULS2} &= 0.9 \cdot DL - 1.5 \cdot WL = 0.9 \cdot 0.5 - 1.5 \cdot 0.5 = -0.3\, kPa \end{aligned} \tag{1}$$

Table 1 shows the loading on the roof nodes (located in the top of the XY-view) based on their respective tributary areas multiplied by the calculated area load for the particular load case. Note that downwards loads are positive here, while in the analysis model this convention is flipped. The nodes in the last row (1, 8, 13, 18, 23, 28) correspond to the pin supports for the structure, so although tributary load is applied here, it is transferred directly into the foundation and does not have any influence on axial member design.

Table 1: Nodal loading

Nodes	Area (m^2)	P_{ULS1} (kN)	P_{ULS2} (kN)	P_{SLS} (kN)
4	260	550	-80	130
3,10,15,20,25,30	346	740	-100	175
2,9,14,19,24,29	693	1470	-210	345
1,8,13,18,23,28	476	1010	-140	225

3.4 Compression Strut Design

There are a total of 13 compression struts in the structure, which can be split into three different types based on their location in the structure (i.e. MID = compressive struts in the middle ring of the structure).

Each compressive member must be designed to have adequate factored sectional and buckling strength (calculated based on Equations 2). The resulting design is summarized in Table 2, where the size of the square section that is necessary to resist the applied load is shown. The design is safe when the utilization ratio is below 1. Since the struts are long and unbraced (i.e. K = 1), it means that the buckling resistance will be the governing factor, which is reflected in the high utilization ratio.

$$\begin{aligned} P_{axial} &= \phi \cdot F_y \cdot A = 0.9 \cdot 350 \cdot A_{strut} \\ P_{buckling} &= \frac{\phi \cdot \pi^2 \cdot E \cdot I}{(KL)^2} = \frac{0.75 \cdot \pi^2 \cdot 200,000 \cdot I_{strut}}{(1.0 \cdot L_{strut})^2} \end{aligned} \tag{2}$$

Table 2: Summary of Compression Member Design

Type	Count	L (mm)	$P_{applied}$ (kN)	Section	$\frac{P_{applied}}{P_{axial}}$	$\frac{P_{applied}}{P_{buckling}}$
TOP	1	6,000	1110	150x150 mm	0.16	0.64
MID	6	14,000	1553	225x225 mm	0.10	0.96
BOT	6	23,000	4833	400x400 mm	0.10	0.81

3.5 Tension Cable Design

The tension cables were designed in a similar manner to the compression struts, except only the axial strength check is required (i.e. the additional buckling check is not required).

A cable of diameter of 250mm was required to resist the maximum ULS load of 15,500 kN, which occurs in the cables towards the supports. Some material optimization in the structure is possible, by using smaller diameter elements in locations closer to the top. For approximately half of the cables (i.e. those in the upper rings), a 150mm diameter rod/cable is adequate.

3.6 Prestress and Deflections

The amount of prestress that was added to the tension rods was equal to 0.01% strain, which resulted in approximately 5% of the yield stress - this is a reasonable value. This amount of prestress was necessary to maintain tension in all the cables during both the "downwards" and "upwards" load cases. Without this prestress, it was found that some of the cables would go into compression (i.e. resultant force of 0), which is not desirable from the perspective of global stability.

The prestress force also helped to reduce deflections in the structure. The structure with the finalized cross-sectional values listed above, experienced a maximum deflection of 0.89 m at its center under unfactored wind loads. Maximum deflection values differ across design codes and types of structures, but in this case the maximum deflection was taken as the Span/120, which in this case would result in an allowable deflection of 120m/120 = 1 m.