COS324:
Introduction To Machine Learning

Prof. Yoram Singer

PRINCETON UNIVERSITY

Topic: Convolutional Neural Networks

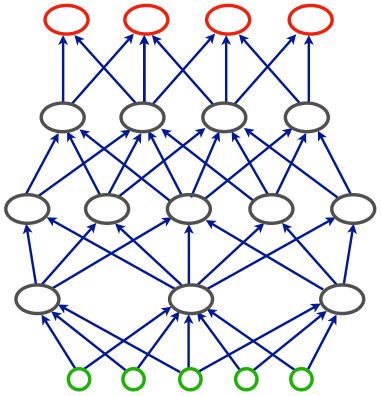
(Almost) Fully Connected Net

$\mathbf{h}^4 = \sigma(A^4 \mathbf{h}^3)$

$\mathbf{h}^3 = \sigma(A^3 \mathbf{h}^2)$

$\mathbf{h}^2 = \sigma(A^2 \mathbf{h}^1)$

$\mathbf{h}^1 = \sigma(A^1 \mathbf{h}^0)$



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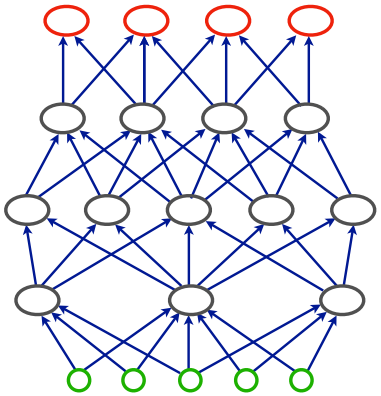

Number of neurons in layer i: $|\mathbf{h}^i| = d_i$

A^i is $d_i \times d_{i-1}$ matrix

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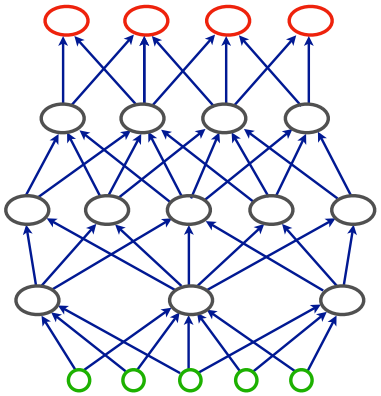

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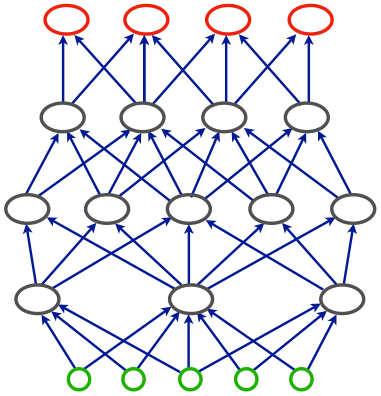

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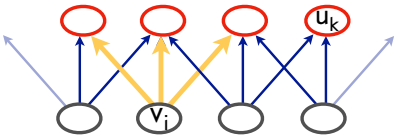
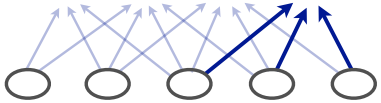
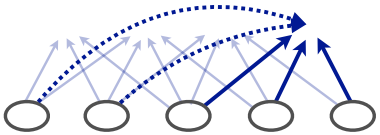
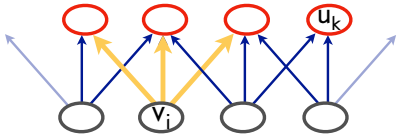
Sparsity Constraints





Sparsity Constraints





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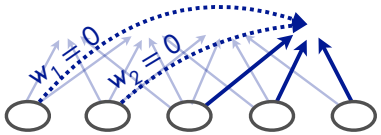
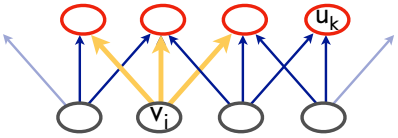
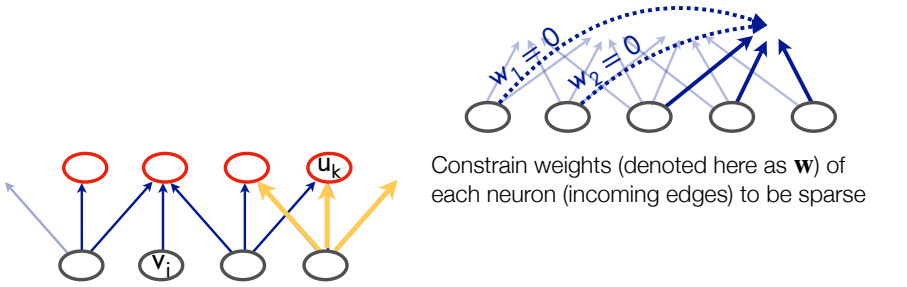




Constrain weights (denoted here as $\mathbf{w}$) of each neuron (incoming edges) to be sparse

Sparsity Constraints



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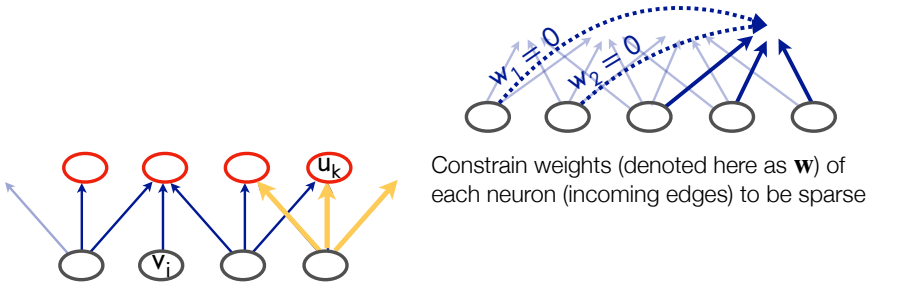


Constrain weights (denoted here as $\mathbf{w}$) of each neuron (incoming edges) to be sparse

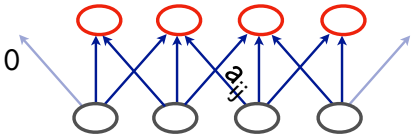
Add self-similarity constraints

$\mathbf{w}[v_i \rightarrow u_k] = 0 \;\Rightarrow\; \mathbf{w}[v_{i+1} \rightarrow u_{k+1}] = 0$

Structural Self-Similarity

$a_{ik} = \mathbf{w}[v_i \rightarrow u_k] = 0$

$\Rightarrow\; a_{i+1\,k+1} = \mathbf{w}[v_{i+1} \rightarrow u_{k+1}] = 0$



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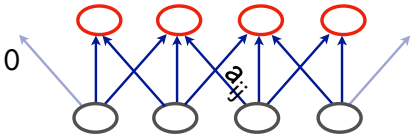

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & \dots & 0 \\ 0 & a_{22} & a_{23} & a_{24} & 0 & 0 & \dots & 0 \\ 0 & 0 & a_{33} & a_{34} & a_{35} & 0 & \dots & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} & \dots & 0 \\ & \dots & \dots & \dots & \dots & \dots & & \end{bmatrix}$$

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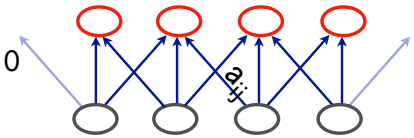

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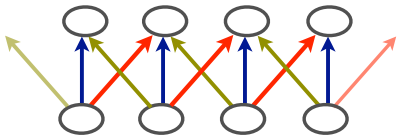
Can we further save ?

Parameter Tying



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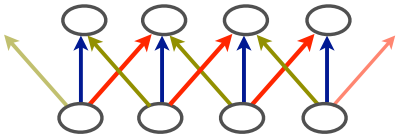

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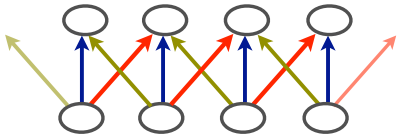

$$\begin{bmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & c_2 & c_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & c_2 & c_3 & 0 & 0 \\ & \dots & \dots & \dots & \dots & \dots & & \end{bmatrix}$$

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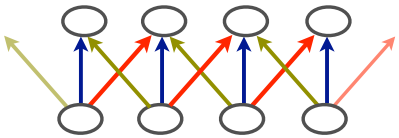

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$$h_i^{r+1} = \sigma\Big(\sum_{j=-s}^{s} c_j h_{i-j}^r \Big)$$

$$j < 0 : \; c_j \equiv c_{2s+1+j}$$

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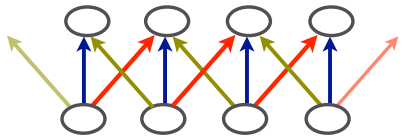
$$a_{ij} = \begin{cases} c_{j-j+1} & 0 \le j - i < s \\ 0 & o.w \end{cases}$$



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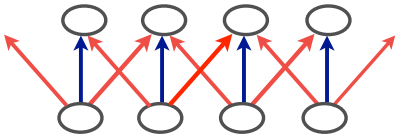
Parameter Symmetrization



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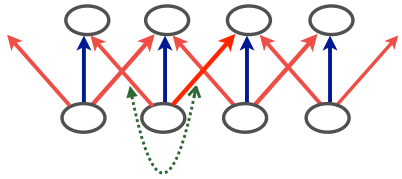

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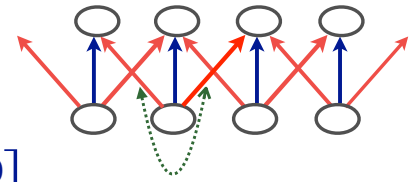
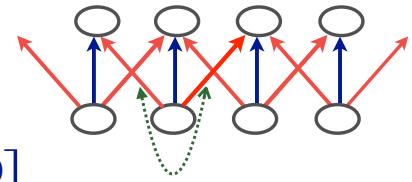


$$\begin{bmatrix} c_1 & c_2 & c_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_1 & c_2 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & c_2 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 & c_2 & c_1 & 0 & 0 \\ & \cdots & \cdots & \cdots & \cdots & \cdots & & \end{bmatrix}$$

Parameter Symmetrization

$$a_{ij} = \begin{cases} c_{|i-j|} & |j-i| < \frac{s}{2} \\ 0 & o.w \end{cases}$$



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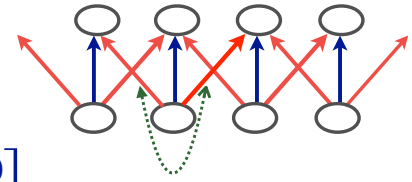

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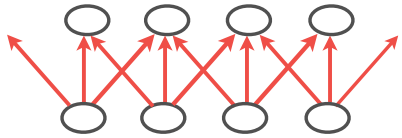
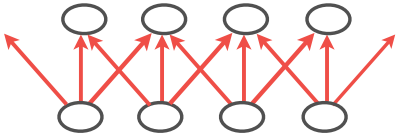
$$h_i^{r+1} = \sigma\left(\sum_{j=0}^{s} c_j \frac{h_{i-j}^r + h_{i+j}^r}{2}\right)$$

Averaging Neuron



Averaging Neuron

$$a_{ij} = \begin{cases} \frac{1}{2s+1} & |j-i| < \frac{s}{2} \\ 0 & o.w \end{cases}$$



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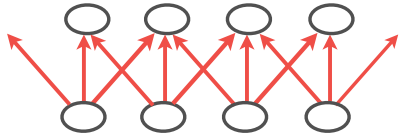

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$$h_i^{r+1} = \sigma\left(\frac{1}{2s+1}\sum_{j=-s}^{s} h_{i-j}^{r}\right)$$

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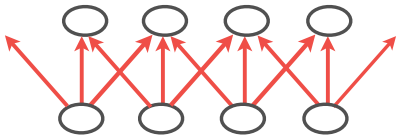
Max-Pooling (Neuron)



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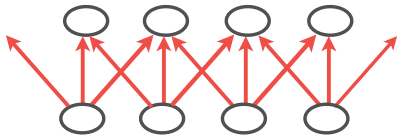
Max-Pooling (Neuron)

$$h_i^{r+1} = \sigma\left(c \max_{j=-s}^{s} h_{i-j}^{r}\right)$$



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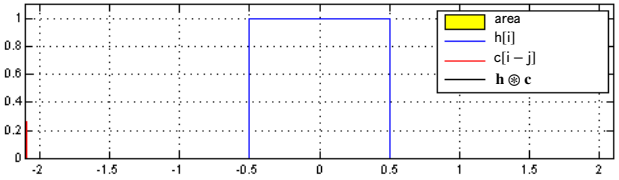
Convolution

$$h_i^{r+1} = \sigma\Big(\underbrace{\sum_{j=-s}^{s} c_j h_{i-j}^r}_{z_i} \Big) \qquad (\mathbf{c} \circledast \mathbf{h})_i = \sum_{j=-s}^{s} c_j h_{i-j}$$



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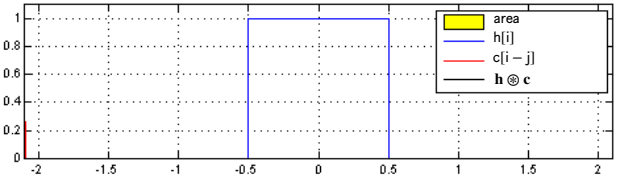

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Boundary Conditions

For $i \le s$ and $i > r - s$:

Convolution is not uniquely defined

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For $i \le s$ and $i > r - s$:

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h

0.1	7	-1	2	3

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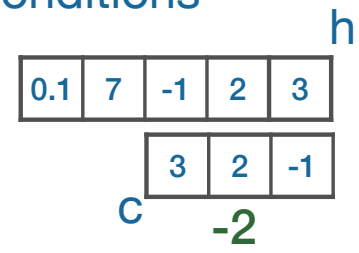
Boundary Conditions

For $i \leq s$ and $i > r - s$:

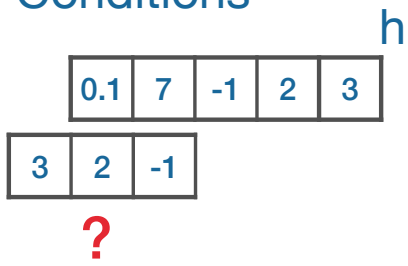
Convolution is not uniquely defined



Boundary Conditions

For $i \leq s$ and $i > r - s$:

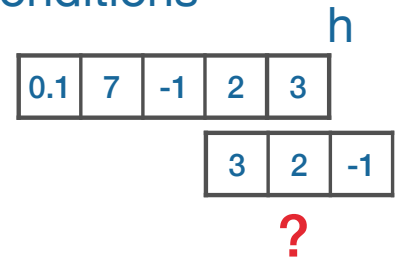
Convolution is not uniquely defined



Boundary Conditions

For $i \leq s$ and $i > r - s$:

Convolution is not uniquely defined



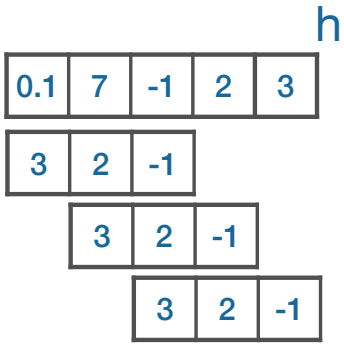
Valid Only Convolution

For $i \leq s$ and $i > r - s$:

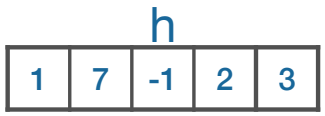
Convolution is not uniquely defined

Use only indices $s < i \leq r - s$:

Size of next layer reduced by $2s$



Padded Convolution



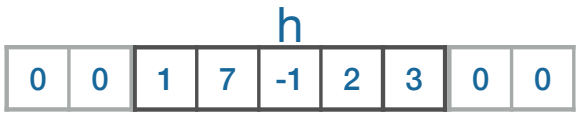
Define:

$h_i = h_0$ for $i \leq 0$

$h_i = h_{r+1}$ for $i > r$

Output defined for $1 \leq i \leq r$

Padded Convolution



Define:

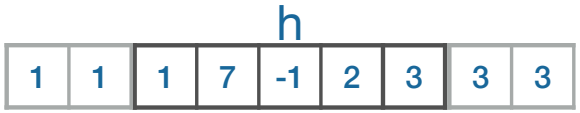
$h_i = h_0$ for $i \leq 0$

$h_i = h_{r+1}$ for $i > r$

Zero padding: $h_{r+1} = h_0 = 0$

Output defined for $1 \leq i \leq r$

Padded Convolution



Define:

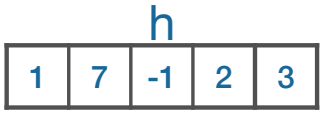
$h_i = h_0$ for $i \leq 0$

$h_i = h_{r+1}$ for $i > r$

Extension padding: $h_0 = h_1$ and $h_{r+1} = h_r$

Output defined for $1 \leq i \leq r$

Padded Convolution



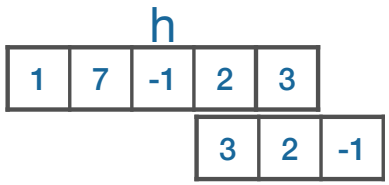
Define:

$h_i = h_0$ for $i \leq 0$

$h_i = h_{r+1}$ for $i > r$

Output defined for $1 \leq i \leq r$

Padded Convolution

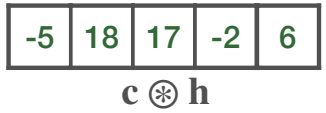


Define:

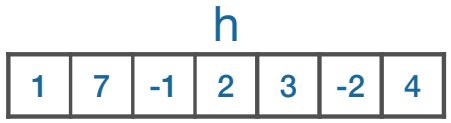
$h_i = h_0$ for $i \le 0$

$h_i = h_{r+1}$ for $i > r$

Output defined for $1 \le i \le r$



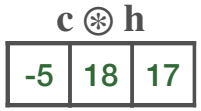
Striding

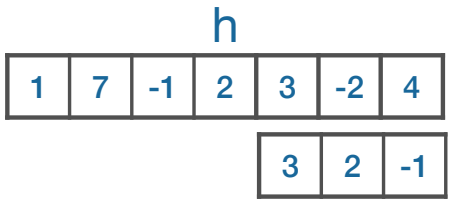


Reduce size of next layer through sub-sampling

$$h_i^{r+1} = \sigma\left(\sum_{j=-s}^{s} c_j \, h^r_{\mu(i)-j} \right) \text{ where } \mu(i) = b(i-1)+1$$

Striding

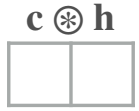


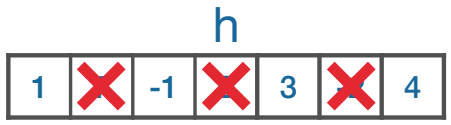


Reduce size of next layer through sub-sampling

$$h_i^{r+1} = \sigma\left(\sum_{j=-s}^{s} c_j \, h^r_{\mu(i)-j} \right) \text{ where } \mu(i) = b(i-1)+1$$

Sub-Sampling (Dilation)

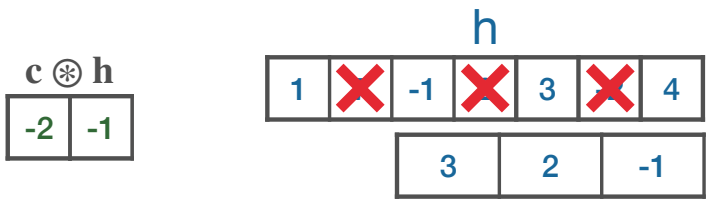




Reduce size of next layer through sub-sampling

$$h_i^{r+1} = \sigma\left(\sum_{j=-s}^{s} c_j \, h^r_{i-\mu(j)} \right) \text{ where } \mu(i) = b\,j$$

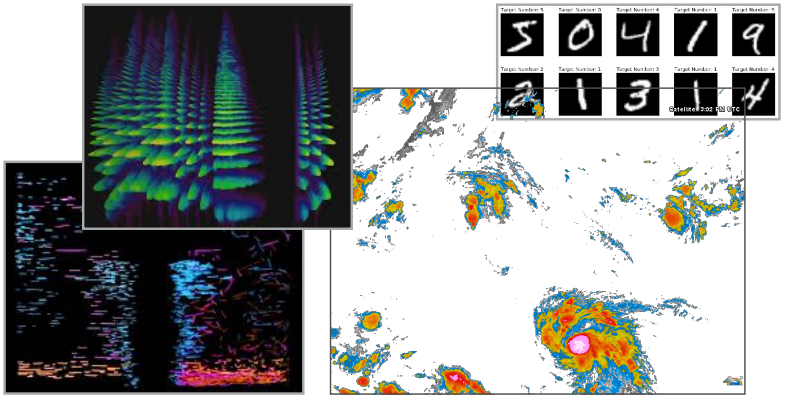
Sub-Sampling (Dilation)



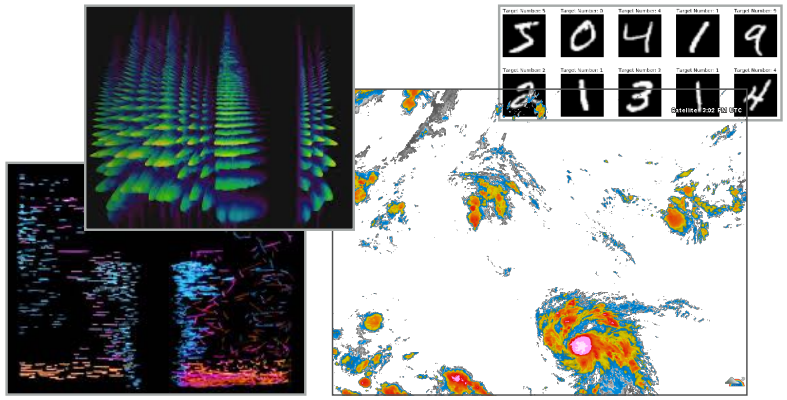
Reduce size of next layer through sub-sampling

$$h_i^{r+1} = \sigma\left(\sum_{j=-s}^{s} c_j \, h^r_{i-\mu(j)} \right) \text{ where } \mu(i) = b\,j$$

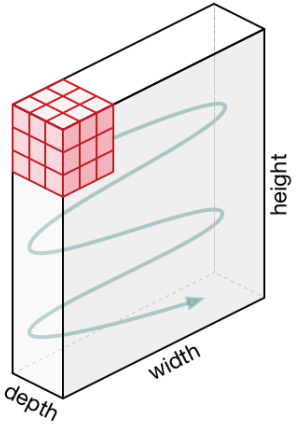
Multidimensional Convolutions

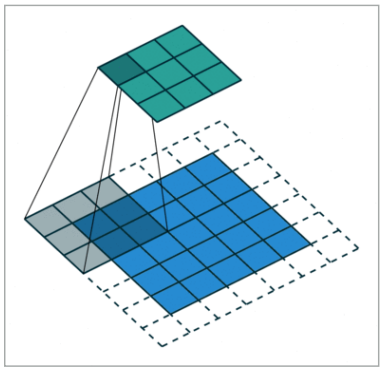


Multidimensional Convolutions

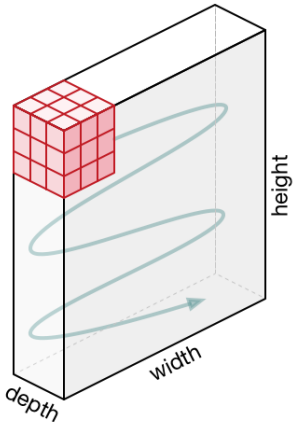


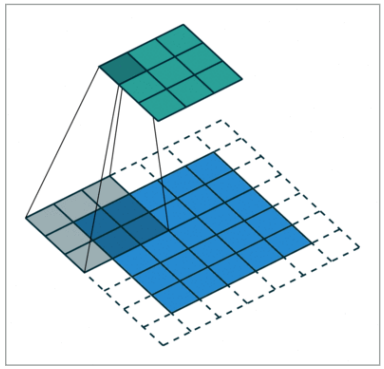
2D & 3D Convulutions

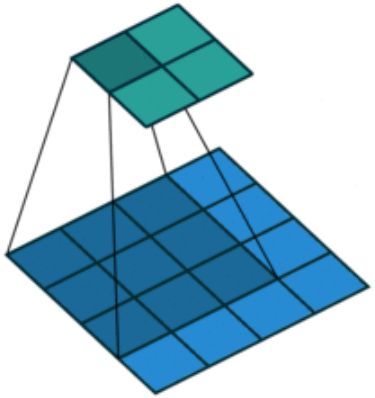




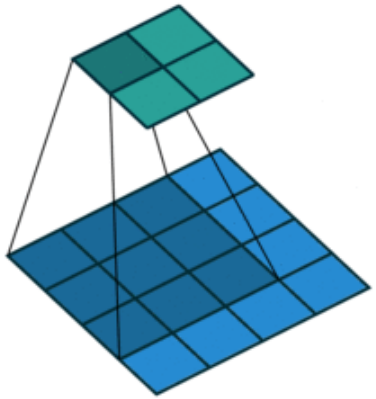
2D & 3D Convulutions



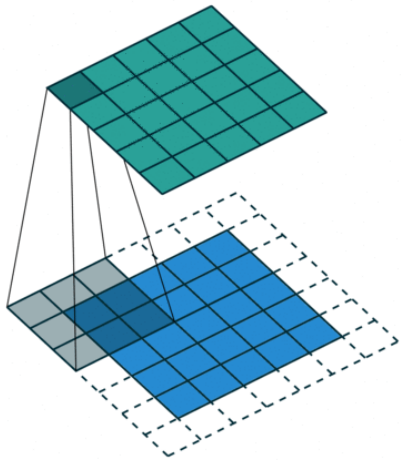




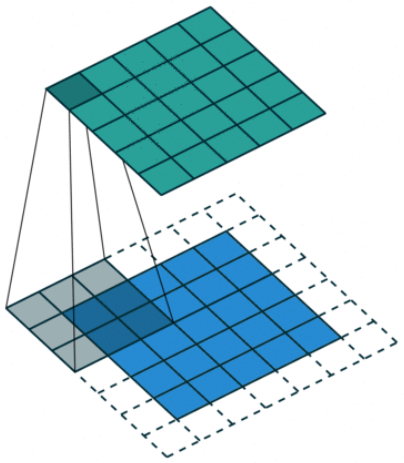
No Padding
No Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



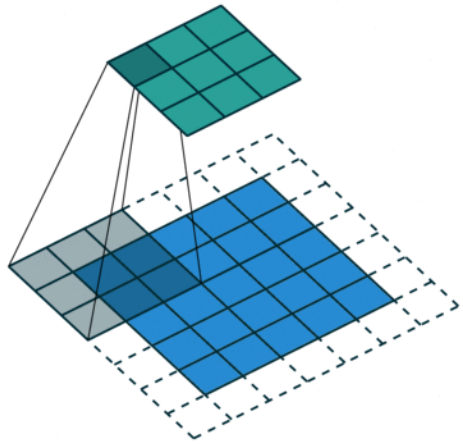
No Padding
No Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



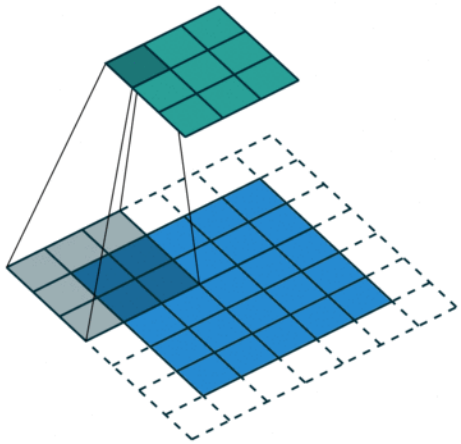
Padding
No Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



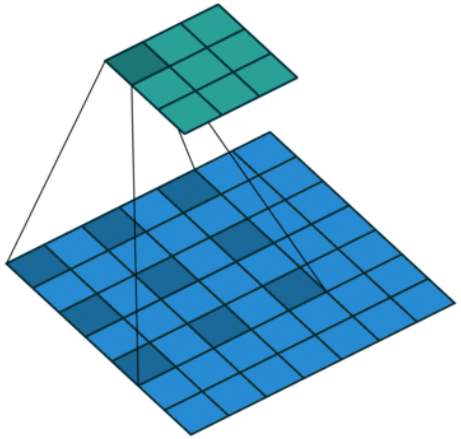
Padding
No Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



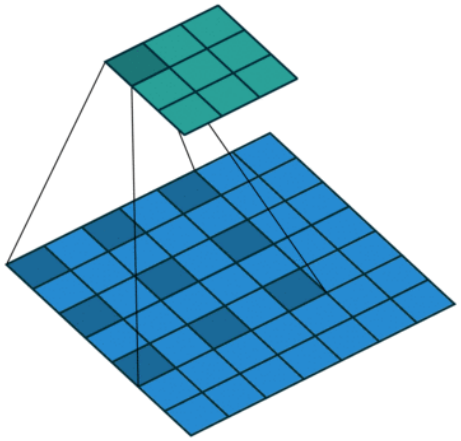
Padding
Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



Padding
Striding
$\mathbf{c} \in \mathbf{R}^{3\times3}$



No Padding
Dilation
$\mathbf{c} \in \mathbf{R}^{3\times3}$



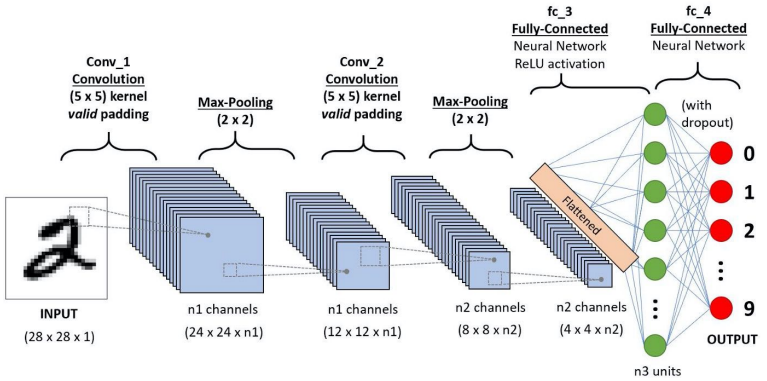
No Padding

Dilation

$\mathbf{c} \in \mathbf{R}^{3\times3}$

LeNet

LeCun, Boser, Denker, Henderson, Howard, Hubbard, Jackel
Backpropagation applied to handwritten zip code recognition.
Neural Computation, 1989



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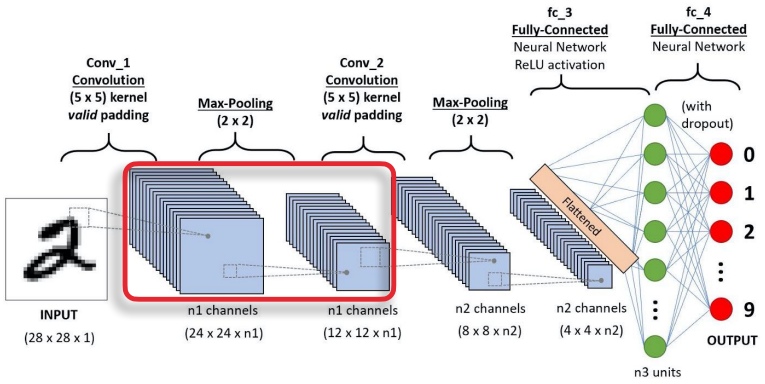
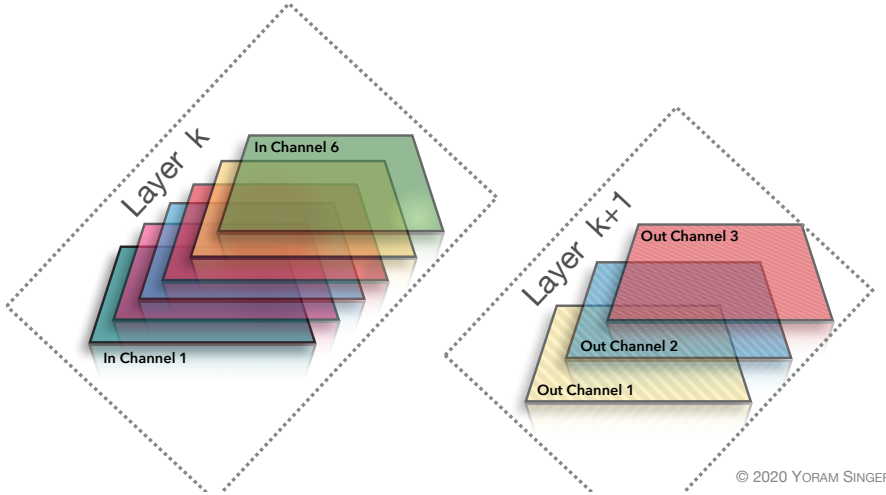
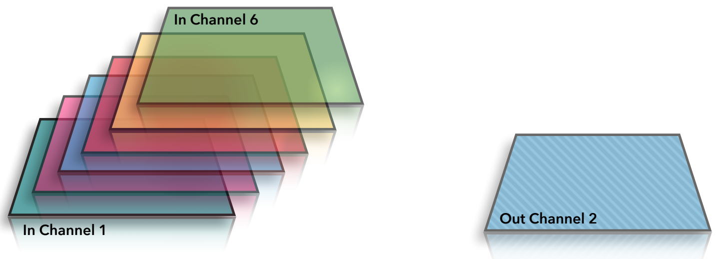
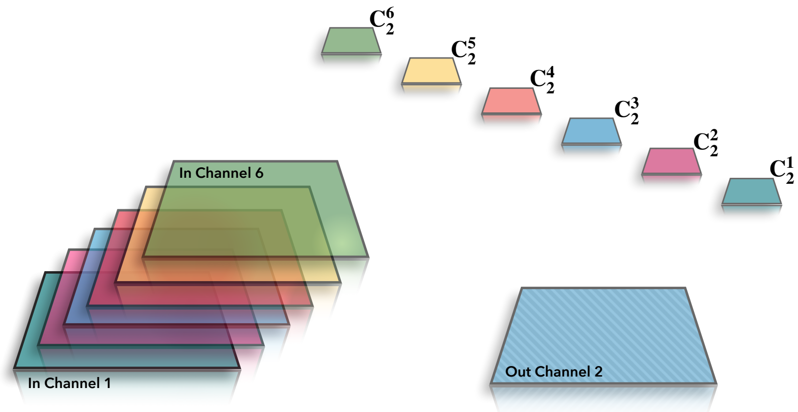
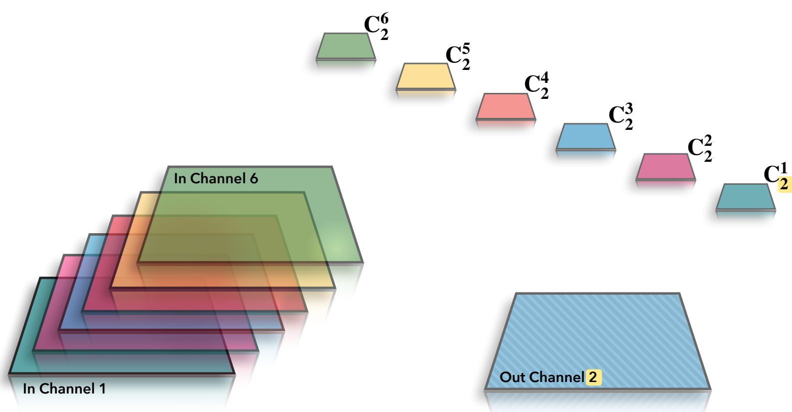
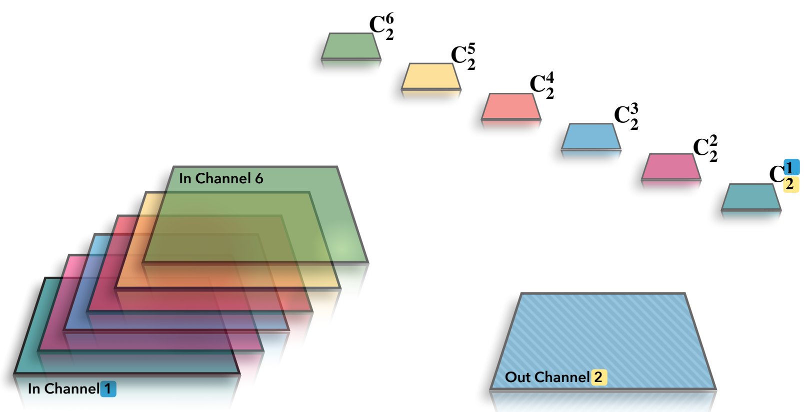


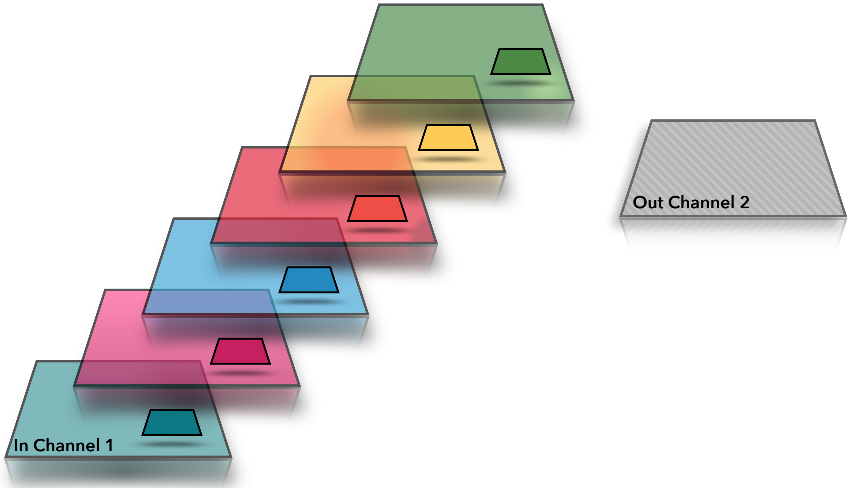


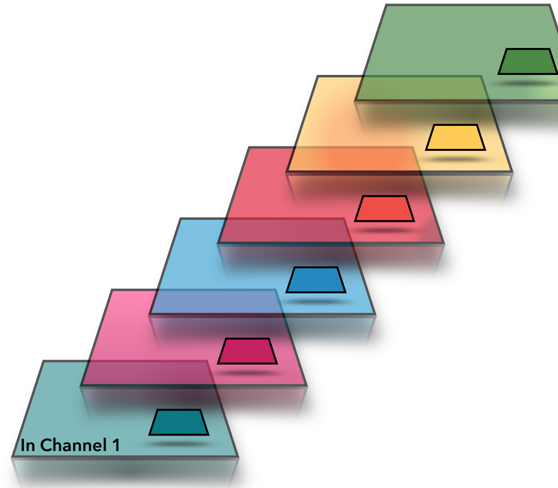

In Channel 6
In Channel 1
Out Channel 2


C_2^6
C_2^5
C_2^4
C_2^3
C_2^2
C_2^1
In Channel 6
In Channel 1
Out Channel 2

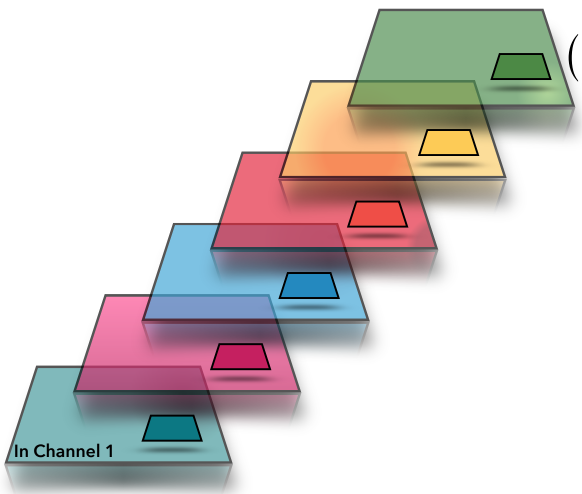

C_2^6
C_2^5
C_2^4
C_2^3
C_2^2
C_2^1
In Channel 6
In Channel 1
Out Channel 2


C_2^6
C_2^5
C_2^4
C_2^3
C_2^2
C_2^1
In Channel 6
In Channel 1
Out Channel 2

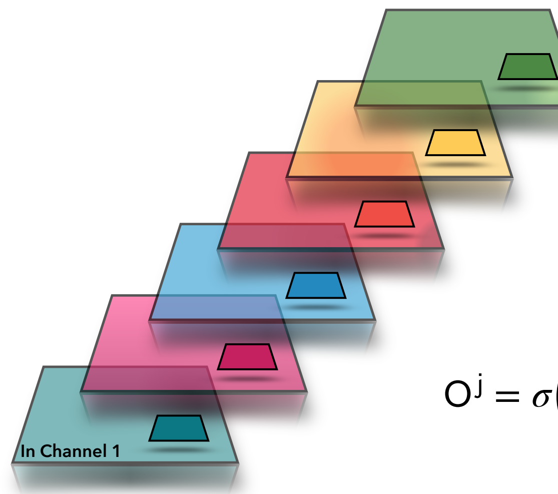




$$\left(\mathrm{In}^6 \circledast \mathbf{c}_2^6\right)[x, y]$$

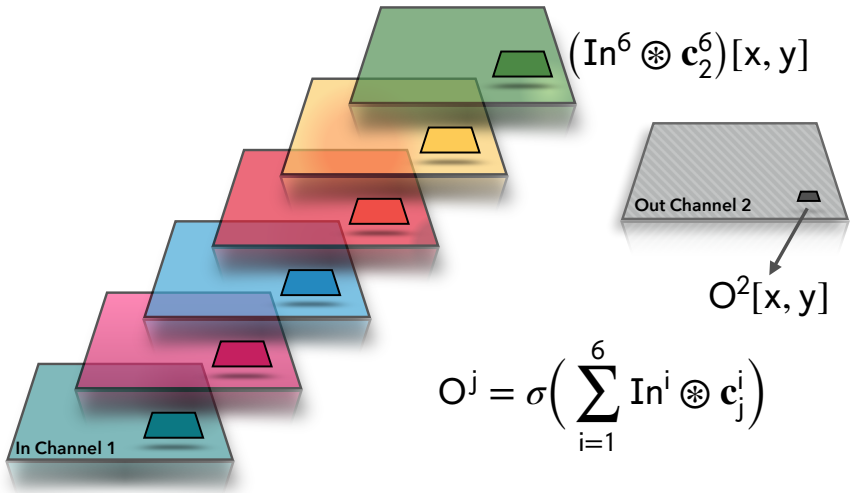


$$\left(\mathrm{In}^6 \circledast \mathbf{c}_2^6\right)[x, y]$$

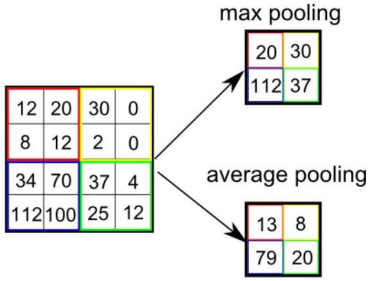


$$\left(\mathrm{In}^6 \circledast \mathbf{c}_2^6\right)[x, y]$$

$$\mathrm{O}^j = \sigma\left(\sum_{i=1}^{6} \mathrm{In}^i \circledast \mathbf{c}_j^i\right)$$



2D Pooling & Averaging



2xx2 Max-Pooling reduces #neurons by 4

Tool for building a pyramid of layers

Channels flattened layer d-2 or d-3

Two-three fully connected layers

2D Max & Avg Pooling

-2	5	-2	0
2	3	1	1
4	4	-2	1
4	4	3	2

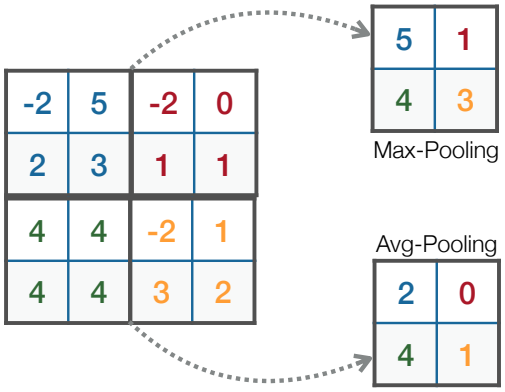
2D Max & Avg Pooling

-2	5	-2	0
2	3	1	1
4	4	-2	1
4	4	3	2

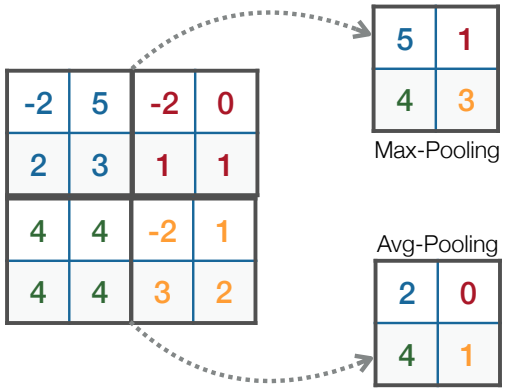
5	1
4	3

Max-Pooling

2D Max & Avg Pooling

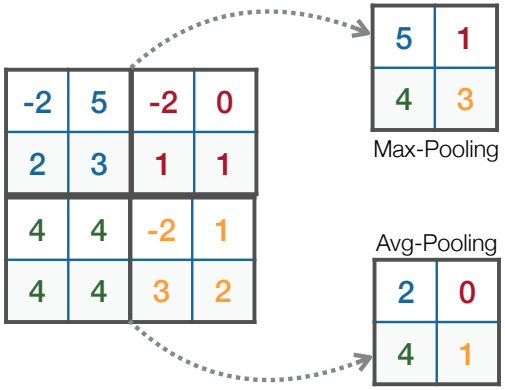


2D Max & Avg Pooling



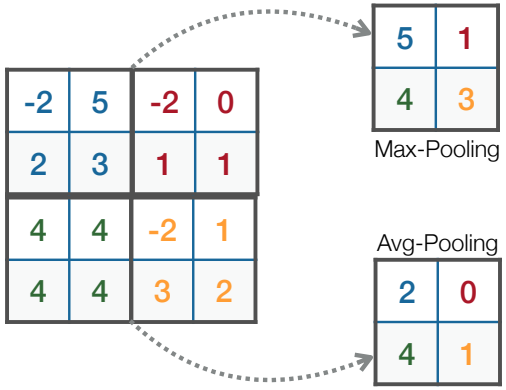
- Pooling:
  - Reduces #neurons
  - Forms pyramid of layers

2D Max & Avg Pooling



- Pooling:
  - Reduces #neurons
  - Forms pyramid of layers
- Channels flattened to vector before top layers

2D Max & Avg Pooling



- Pooling:
  - Reduces #neurons
  - Forms pyramid of layers
- Channels flattened to vector before top layers
- Top layers fully connected

Code

1. Dataset: CIFAR10 for image classification, more difficult than MNIST for OCR
2. Notebooks: cifar10_mlp & cifar10_conv
3. Uses PyTorch with autograd: no need to calculate gradients by hand
4. Main loop written explicitly — examine carefully
5. Load and read notebook once before running cell by cell