

## Riemann Sums

$$\textcircled{1} \quad \int_1^9 \frac{x}{1+x} dx \quad n=4 \quad \text{left-handed}$$

$$\Delta x = \frac{9-1}{4} = 2 \quad a = [1, 3], [3, 5], [5, 7], [7, 9] = b$$

$$f(1) = \frac{1}{2}, \quad f(3) = \frac{3}{4}, \quad f(5) = \frac{5}{6}$$

$$f(7) = \frac{7}{8}$$

$$2\left(\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8}\right) = 5.9167$$

0.5 0.75 0.833 0.875  $\approx 5.9167\dots$

$$\textcircled{2} \quad \int_0^3 x^2 dx \quad n=3 \quad \Delta x = \frac{3-0}{3} = 1 \quad a = [0, 1], [1, 2], [2, 3] = b$$

$$f\left(\frac{x_0+x_1}{2}\right) = f\left(\frac{0+1}{2}\right) = f\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$f\left(\frac{x_1+x_2}{2}\right) = f\left(\frac{1+2}{2}\right) = f(3/2) = \frac{9}{4}$$

$$f\left(\frac{x_2+x_3}{2}\right) = f\left(\frac{2+3}{2}\right) = f(5/2) = \frac{25}{4}$$

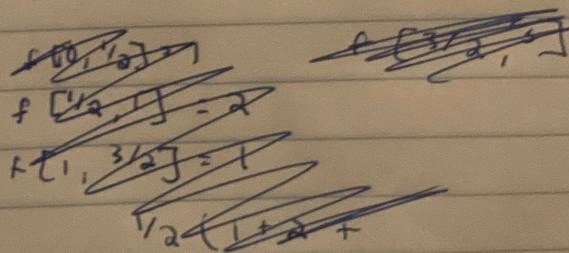
$$\Delta x = 1 \left( \frac{1}{4} + \frac{7}{4} + \frac{25}{4} \right)$$

0.25 2.25 6.25 =  $\textcircled{8.75}$

$$\textcircled{3} \quad \int_1^3 f(x) dx \quad n=4$$

$$3-1 \quad \Delta x = \frac{3-1}{4} = \frac{1}{2} \quad [1, \frac{3}{2}], [\frac{3}{2}, 2], [2, \frac{5}{2}], [\frac{5}{2}, 3]$$

graph



$$f[1, \frac{3}{2}] = 1$$

$$f[\frac{3}{2}, 2] = 1$$

$$f[2, \frac{5}{2}] = 0$$

$$f[\frac{5}{2}, 3] = -1$$

$$\frac{1}{2}(1+1-1) = \textcircled{1/2}$$