Math 1800 Adrienne Fyeatt

Riemann Sums

① $\int_1^9 \frac{x}{1+x}\,dx$ $n=4$ left-handed

$\Delta x = \frac{9-1}{4} = 2$ $a = [1,3],[3,5],[5,7],[7,9] = b$

$f(1) = \frac{1}{2}$ $f(3) = \frac{3}{4}$ $f(5) = \frac{5}{6}$

$f(7) = \frac{7}{8}$

$2\left(\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8}\right) = 5.9167$

0.5 0.75 0.833 0.875

$\approx 8.917\ldots$

② $\int_0^3 x^2\,dx$ $n=3$ $\Delta x = \frac{3-0}{3} = 1$ $a = [0,1],[0,2],[2,3] = b$

$f\left(\frac{x_0+x_1}{2}\right) = f\left(\frac{0+1}{2}\right) = f(1/2) = \frac{1}{4}$

$f\left(\frac{x_1+x_2}{2}\right) = f\left(\frac{1+2}{2}\right) = f(3/2) = \frac{9}{4}$

$f\left(\frac{x_2+x_3}{2}\right) = f\left(\frac{2+3}{2}\right) = f\left(\frac{5}{2}\right) = 25/4$

$\Delta x = 1\,(1/4 + 9/4 + 25/4)$

0.25 2.25 6.25 $= 8.75$

③ $\int_1^3 f(x)\,dx$ $n=4$

$3-1$ $\Delta x = \frac{3-1}{4}$ $\frac{2}{4} = \frac{1}{2}$ $[1, 3/2],[3/2, 2],[2, 5/2],[5/2, 3]$

$f[1, 3/2] = 1$

$f[3/2, 2] = 1$

$f[2, 5/2] = 0$

$f[5/2, 3] = -1$

$1/2\,(1+1-1) = 1/2$