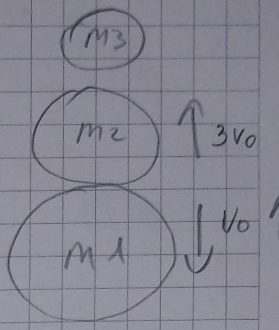
Mechanics
Homework 6

Problem 1 • first m_1 hits the ground.

$E_{before} = E_{after} \Rightarrow \frac{m_1 v_0^2}{2} = \frac{m_1 v_{after}^2}{2} \Rightarrow v_{after} = v_0$



• m_1 hits m_2

$P_{before} = m_1 v_0 - m_2 v_0 \qquad P_{after} = m_1 v_1 + m_2 v_2$

$E_{before} = \frac{m_1 v_0^2}{2} + \frac{m_2 v_0^2}{2} \qquad E_{after} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$

$m_1 (v_0^2 - v_1^2) = m_1 (v_0 - v_1)(v_0 + v_1) = m_2 (v_2^2 - v_0^2)$

$m_1 (v_0 - v_1) = m_2 (v_0 + v_2)$

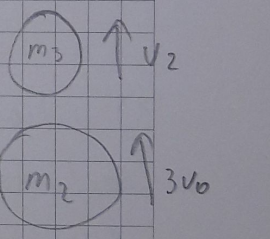
$m_1 >> m_2 \Rightarrow$ the change of the speed of m_1 is neglected

$v_0 \approx v_1$, $2v_0 \approx v_0 + v_1$

$m_1 (v_0 - v_1)(v_0 + v_1) \approx 2v_0 \cdot m_1 (v_0 - v_1) \approx m_2 (v_2^2 - v_0^2) = m_2 (v_2 - v_0)(v_2 + v_0)$

$2 m_2 v_0 (v_0 + v_2) = m_2 (v_2 - v_0)(v_2 + v_0) \Rightarrow 2v_0 = v_2 - v_0 \Rightarrow v_2 = 3v_0$

m_2 moves upward and hits m_3 with speed $3v_0$



$P_{before} = m_2 \cdot 3v_0 - m_3 v_0 \qquad P_{after} = m_2 v_1 + m_3 v_2$

$E_{before} = \frac{m_2 \cdot 9 v_0^2}{2} + \frac{m_3 v_0^2}{2} \qquad E_{after} = \frac{m v_1^2}{2} + \frac{m_3 v_2^2}{2}$

$3 m_2 v_0 - m_3 v_0 = m_2 v_1 + m_3 v_2 \Leftrightarrow m_2 (3v_0 - v_1) = m_3 (v_0 + v_2)$

$m_2 (9 v_0^2 - v_1^2) = m_2 (3v_0 - v_1)(3v_0 + v_1) = m_3 (v_2^2 - v_0^2) = m_3 (v_2 - v_0)(v_2 + v_0)$

$m_2 >> m_3 \Rightarrow$ the change of the speed of m_2 is neglected.

$v_1 \approx 3v_0$, $v_1 + 3v_0 \approx 6v_0$

$m_2 (9 v_0^2 - v_1^2) = m_2 (3v_0 - v_1)(3v_0 + v_2) \approx 6v_0 m_2 (3v_0 - v_1) \approx m_3 (v_2^2 - v_0^2) =$

$= m_3 (v_2 - v_0)(v_2 + v_0)$

$$m_2(3v_0 - v_1) = \frac{m_3 (v_2 - v_0)(v_2 + v_0)}{6v_0}$$

$$\cancel{m_3}(v_0 + v_2) = \frac{\cancel{m_3}(v_2 - v_0)\cancel{(v_2 + v_0)}}{6v_0}$$

$$6v_0 = v_2 - v_0 \Rightarrow v_2 = 7v_0$$

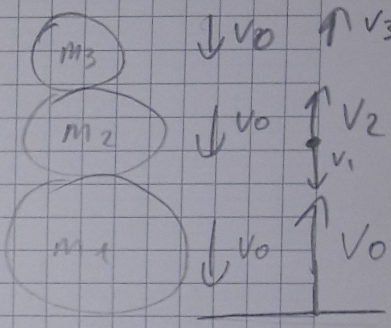
$$m_3 g h_{fall} = \frac{m_3 v_0^2}{2} \Rightarrow h_{fall} = \frac{v_0^2}{2g} \qquad h_{rise} = \frac{(7v_0)^2}{2g} = \frac{49 v_0^2}{2g} = 49 \cdot h_{fall}$$

Problem 2 | AUA ID = 09160072

$m_1 = 7$, $m_2 = 5$, $m_3 = 2$.

in this case the masses are not significantly different => change of speed after hitting cannot be neglected

assume $v_0 = 1$.



- m_1 hits m_2

$$7v_0 - 5v_0 = 7v_1 + 5v_2 \Leftrightarrow 2 = 7v_1 + 5v_2 \qquad v_1 = 1$$

$$7(v_0^2 - v_1^2) = 5(v_2^2 - v_0^2) \Leftrightarrow 12 = 7v_1^2 + 5v_2^2 \qquad v_2 = -1$$

$$16 = 2 \cdot (1 + v_3) \qquad v_3 = 7.$$

$$h_{fall} = \frac{1}{2g} \qquad h_{rise} = \frac{49}{2g} = 49 \cdot h_{fall}$$