Mechanics

Homework 5

Problem 1 From total mechanical energy preservation we get $K_a + U_a = K_b + U_b$

1) $x_0 = 1$, $x_2 = 10$

$K_a - K_b = U_{b=10} - U_{a=1} = 10 - 2 = 8$

$\frac{mv_0^2}{2} - \frac{mv_2^2}{2} = 8 \Rightarrow v_0^2 - v_2^2 = 16$

assume the body stops at $x_2 = 10 \Rightarrow v_2 = 0 \Rightarrow v_0^2 = 16$

minimal initial speed $v_0 = 4$ (not -4, because the body would move to the left)

2) $t - ?$ $x_1 = 6 \rightarrow x_2 = 10$

$v_1^2 - v_2^2 = 8$, assume the body stops at $x_2 = 10 \Rightarrow v_2 = 0$

$v_1 = 2\sqrt{2}$

$U(x) \approx x$, in the region $[6, 10]$ the function of the potential energy behaves like a linear function.

$$\begin{cases} F = -U(x)' = -1 \\ F = ma \end{cases} \Rightarrow \quad ma = -1, \; m = 1 \quad a = -1$$

$x_2 = x_1 + v_1 \cdot t + \frac{1}{2} a t^2$

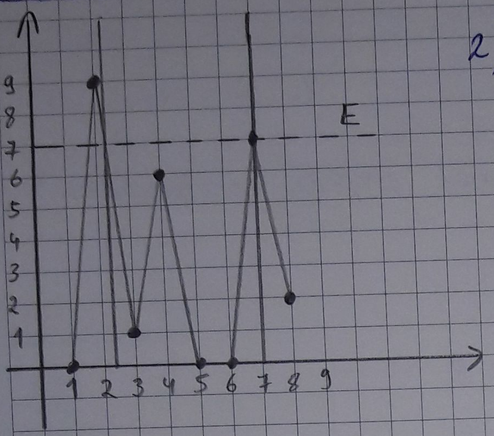
$10 = 6 + 2\sqrt{2}\, t - \frac{1}{2} t^2 \Leftrightarrow t^2 - 4\sqrt{2}\, t + 8 = 0$

$(t - 2\sqrt{2})^2 = 0$

$t = 2\sqrt{2}$ seconds

Problem 2 AUA ID = 09160072

1)



2) for $E = 7$ the body performs a bounded motion with largest amplitude

$x_R = 7$

$\frac{x-2}{3-2} = \frac{y-9}{1-9} \Rightarrow y = 25 - 8x$ (line passing through points (2,9) and (3,1))

when $y = 7$, $x_L = 2,25$

$x_R - x_L = 7 - 2,25 = 4,75$

3) minimize the function

$$\sum_{i=1}^{n}\left(y_i - (a + b x_i + c x_i^2)^{a}\right)^2$$

$$\begin{cases} 14 = 5a + 25b + 135c \\ 76 = 25a + 135b + 775c \\ 448 = 135a + 775b + 4659c \end{cases} \Rightarrow \quad a = \frac{568}{35} \quad b = -\frac{229}{35} \quad c = \frac{5}{7}$$

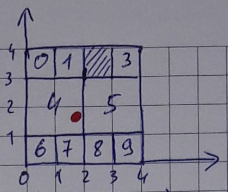
$y(x) = \frac{5}{7}x^2 - \frac{229}{35}x + \frac{568}{35}$ ← illustration is in figure 1.

$F = -U(x)' = \frac{10}{7}x - \frac{229}{35} = 0$ $\quad x_0 = \frac{7 \cdot 229}{10 \cdot 35} \approx 4,6$ equilibrium point

$k = U(x)'' = \frac{10}{7}$

$\omega = \sqrt{\frac{10/7}{m}} = \sqrt{\frac{10}{7m}}$ ← frequency of small oscillations

Problem 3 | AUA ID = 09160072

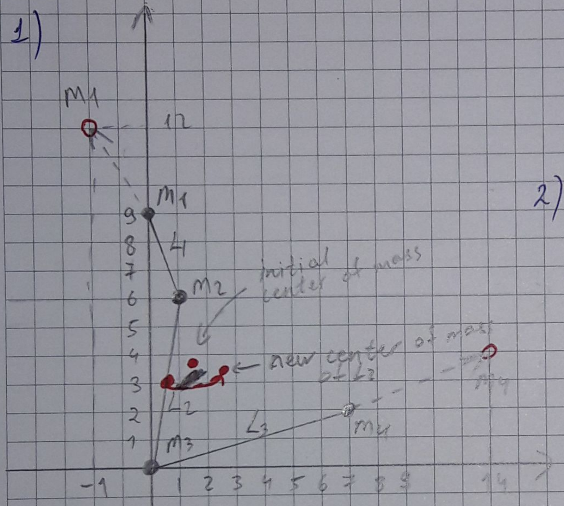


new mass $= \frac{15}{16} M$

$$\vec{R} = \frac{16}{15M}\left(\frac{1}{16}M\begin{pmatrix}0,5\\0,5\end{pmatrix} + \frac{1}{16}M\begin{pmatrix}1,5\\0,5\end{pmatrix} \cdots\cdots\right) = \begin{pmatrix}1,967\\1,9\end{pmatrix}$$ shown in red.

Problem 4

1)



$m_1 = m_4 = 1\,kg$

$m_2 = m_3 = 2\,kg$

no external forces

$\vec{F} = 0 \Rightarrow \vec{P} = const$

2) $$\vec{R} = \frac{1}{6}\left(1\cdot\begin{pmatrix}0\\9\end{pmatrix} + 2\begin{pmatrix}1\\6\end{pmatrix} + 2\begin{pmatrix}0\\0\end{pmatrix} + 1\begin{pmatrix}7\\2\end{pmatrix}\right) =$$

$= \begin{pmatrix}1,5\\3,833\end{pmatrix}$ shown in red.

3) initial cener of mass of L_2 is $(0,5, 3)$

L_2 rod moves away from m_1, $m_1 v_1 = -M v_{Rod}$

$1\begin{pmatrix}-1\\3\end{pmatrix} = -4\, v_{Rod}$ $\quad v_{Rod1} = \begin{pmatrix}1/4\\-3/4\end{pmatrix}$

L_2 rod moves away from m_4, $m_4 v_4 = -M v_{Rod}$

$1\begin{pmatrix}7\\2\end{pmatrix} = -4\cdot v_{Rod2}$ $\quad v_{Rod2} = \begin{pmatrix}-7/4\\-1/2\end{pmatrix}$

$$\vec{R}_{new} = \vec{R} + v_{Rod1} + v_{Rod2} = \begin{pmatrix}1,5\\3,833\end{pmatrix} + \begin{pmatrix}1/4\\-3/4\end{pmatrix} + \begin{pmatrix}7/4\\-1/2\end{pmatrix} = \begin{pmatrix}3,5\\2,583\end{pmatrix}$$

new center of mass of L_2 is $(3,5, 2,583)$