

Pattern Recognition

Assignment - 1

Bayes Classifier

Hitesh Ramchandani (B15232)

Sagar Gupta (B15233)

Ankit Amrit Raj (B15107)

Group - 8

November 12, 2017

Contents

1	Objective	1
2	Procedure	1
3	Observations	2
3.1	Case 1 - $\Sigma = \sigma^2 I$	2
3.1.1	Linear Data	2
3.1.2	NonLinear Data	4
3.1.3	Real World Data	6
3.1.4	Inferences	8
3.2	Case 2 - $\Sigma_i = \Sigma$	9
3.2.1	Linear Data	9
3.2.2	NonLinear Data	11
3.2.3	Real World Data	13
3.2.4	Inferences	15
3.3	Case 3 - Σ_i is a diagonal matrix	16
3.3.1	Linear Data	16
3.3.2	NonLinear Data	18
3.3.3	RealWorld Data	20
3.3.4	Inferences	22
3.4	Case 4 - Σ_i is unique	23
3.4.1	Linear Data	23
3.4.2	NonLinear Data	25
3.4.3	RealWorld Data	28
3.4.4	Inferences	30
4	Conclusion	30

1 Objective

1. To build Bayes Classifier and classify the following datasets -
 - 2D Dataset - 1 (Artificial)
 - Linearly Separable Dataset
 - Non Linearly Separable Dataset
 - 2D Dataset - 2 (Real World)
2. Plot Decision Region for all pairs of classes.
3. Contour Region Plots for all pairs of classes.
4. Calculate Accuracy, Precision, mean recall, F-measure and Confusion Matrix.

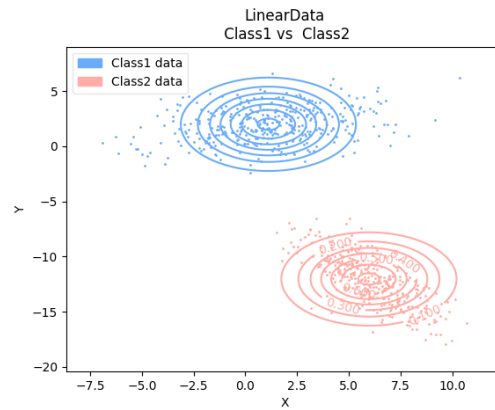
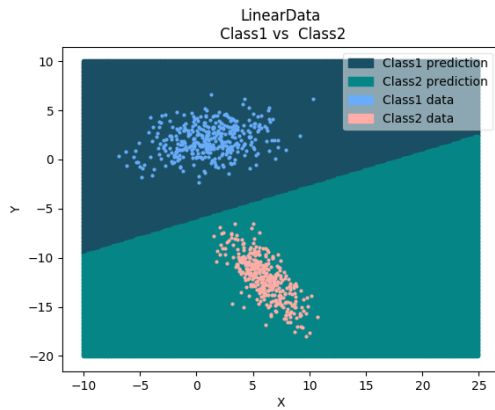
2 Procedure

1. Data for each class is partitioned into 75% for training and 25% for testing.
2. The data set for each class is assumed to come from Gaussian distribution.
3. In case 1 ($\Sigma = \sigma^2 I$), mean of covariance matrix for each class was calculated and it's off diagonal terms were assumed to be 0 for further calculations.
4. In case 2 ($\Sigma_i = \Sigma$ for every class), mean of covariance matrix for each class was calculated for further calculations.
5. In case 3 (Σ_i is diagonal matrix), covariance matrix for each class was different and it's off diagonal terms were assumed to be 0 for further calculations.
6. In case 4 (Σ_i is unique), no assumptions were made for further calculations.
7. Based of assumptions, the discriminant function ($g_i(x)$) was calculated for each class and decision region and Contour was plotted.
8. The remaining 25% data was tested for each case and analysis was made.

3 Observations

3.1 Case 1 - $\Sigma = \sigma^2 I$

3.1.1 Linear Data



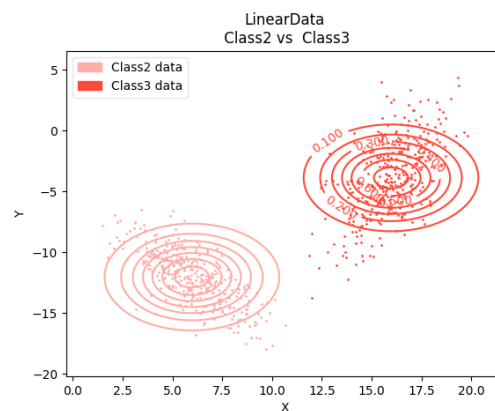
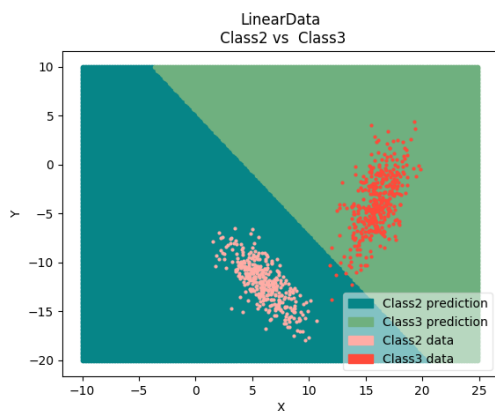
Accuracy = 99.6%

Confusion Matrix

	class1	class2
class1	125	0
class2	1	124

Analysis

	class1	class2
Precision	1.000	0.992
Recall	0.992	1.000
F-measure	0.996	0.996



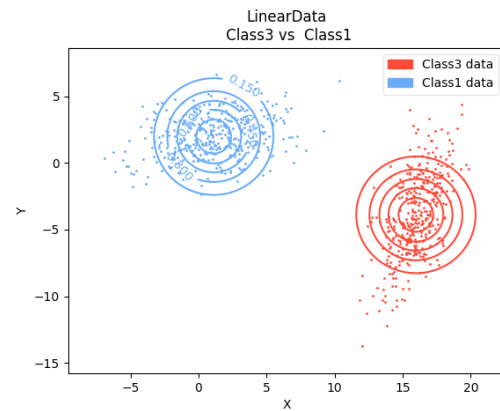
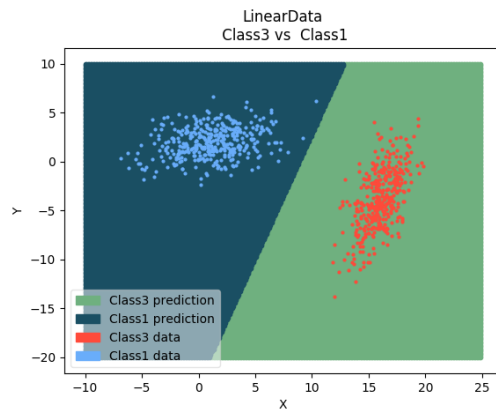
Accuracy = 99.2%

Confusion Matrix

Analysis

	class1	class2
class1	125	0
class2	2	123

	class1	class2
Precision	1.000	0.984
Recall	0.984	1.000
F-measure	0.992	0.992



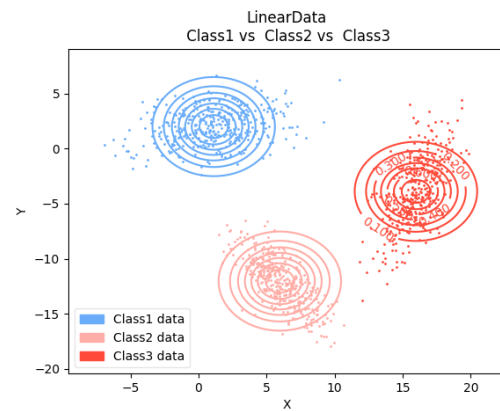
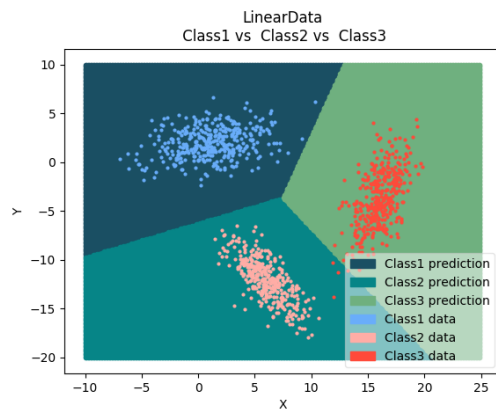
Accuracy = 99.2%

Confusion Matrix

	class1	class2
class1	125	0
class2	2	123

Analysis

	class1	class2
Precision	1.000	0.984
Recall	0.984	1.000
F-measure	0.992	0.992



Accuracy = 98.7%

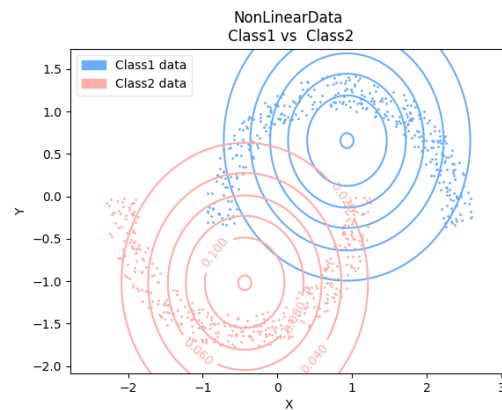
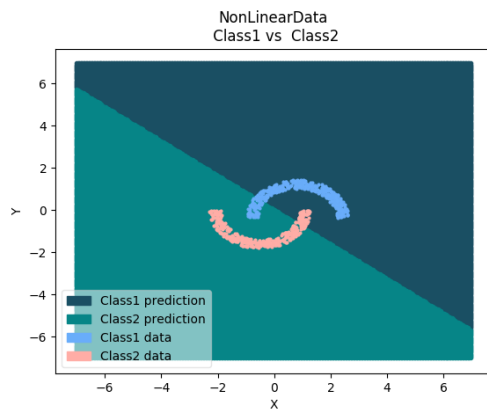
Confusion Matrix

Analysis

	class1	class2	class3
class1	123	0	2
class2	1	124	0
class3	0	2	123

	class1	class2	class3
Precision	0.984	0.992	0.984
Recall	0.992	0.984	0.984
F-measure	0.988	0.988	0.984

3.1.2 NonLinear Data



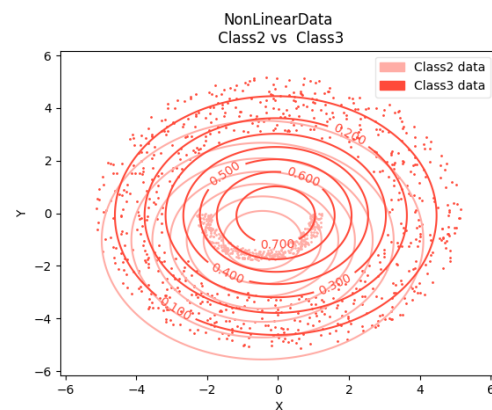
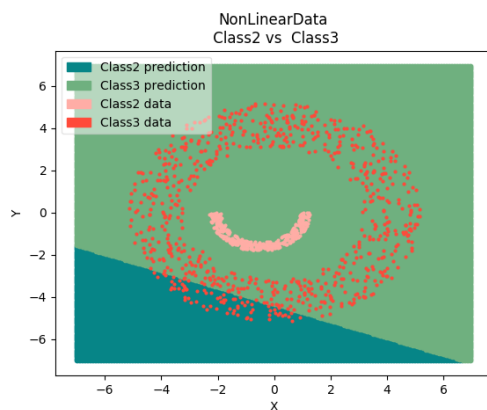
Accuracy = 85.6%

Confusion Matrix

	class1	class2
class1	110	15
class2	21	104

Analysis

	class1	class2
Precision	0.88	0.832
Recall	0.840	0.874
F-measure	0.859	0.852



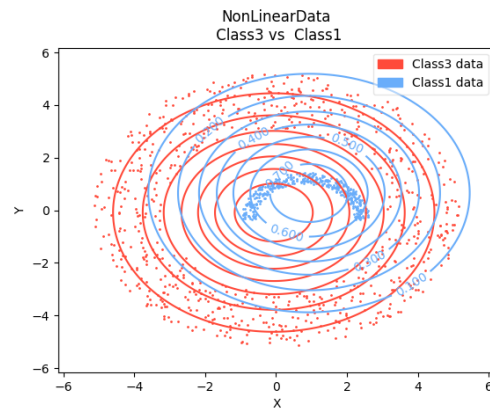
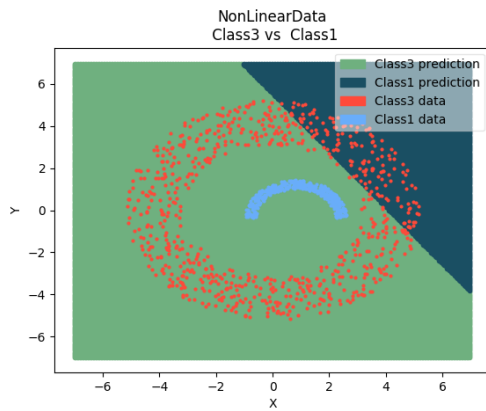
Accuracy = 61.33%

Confusion Matrix

	class1	class2
class1	0	125
class2	20	230

Analysis

	class1	class2
Precision	0.00	0.92
Recall	0.00	0.647
F-measure	-	0.76



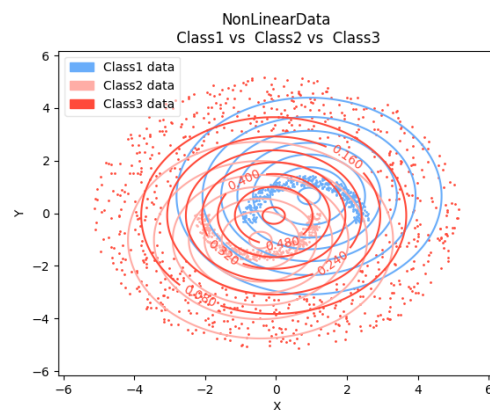
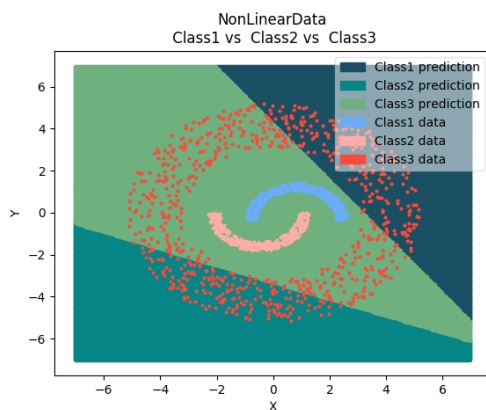
Accuracy = 49.6%

Confusion Matrix

	class1	class2
class1	186	64
class2	125	0

Analysis

	class1	class2
Precision	0.744	0.00
Recall	0.598	0.00
F-measure	0.663	-



Accuracy = 22.8%

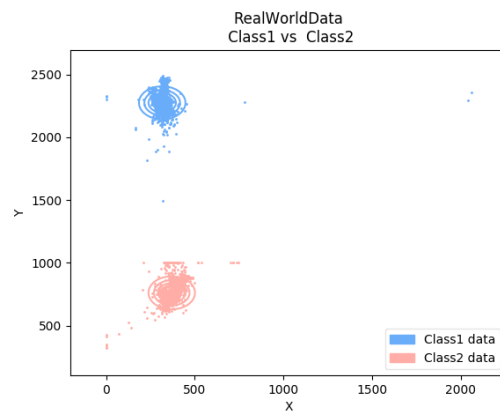
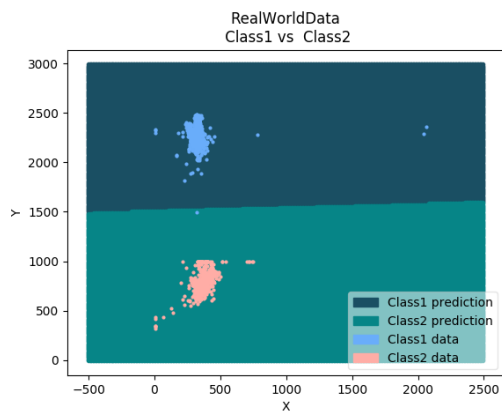
Confusion Matrix

	class1	class2	class3
class1	0	0	125
class2	0	0	125
class3	90	46	114

Analysis

	class1	class2	class3
Precision	0.00	0.00	0.456
Recall	0.00	0.00	0.313
F-measure	-	-	0.371

3.1.3 Real World Data



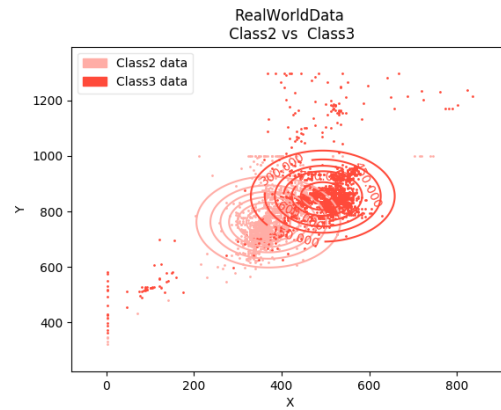
Accuracy = 99.75%

Confusion Matrix

	class1	class2
class1	594	3
class2	0	622

Analysis

	class1	class2
Precision	0.995	1.000
Recall	1.000	0.995
F-measure	0.997	0.997



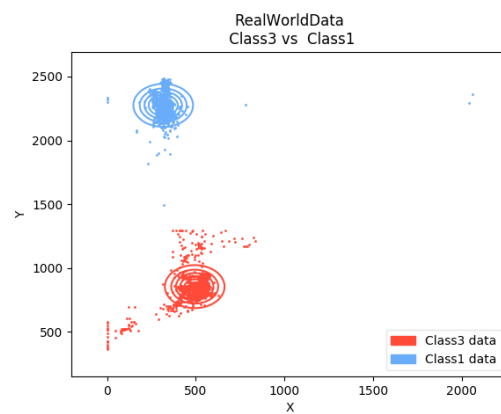
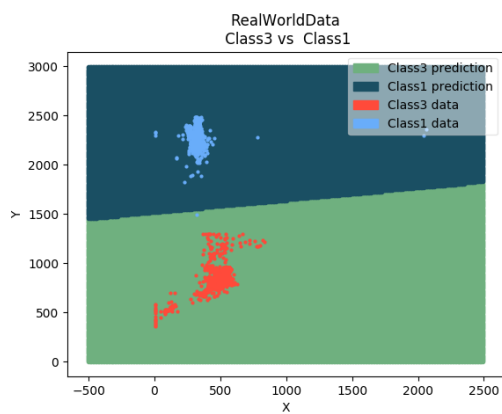
Accuracy = 81.55%

Confusion Matrix

	class1	class2
class1	572	50
class2	178	436

Analysis

	class1	class2
Precision	0.920	0.710
Recall	0.762	0.897
F-measure	0.834	0.793



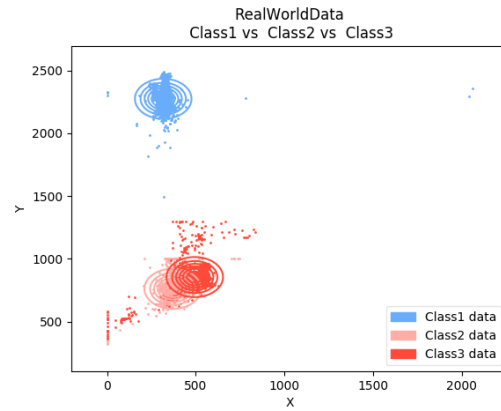
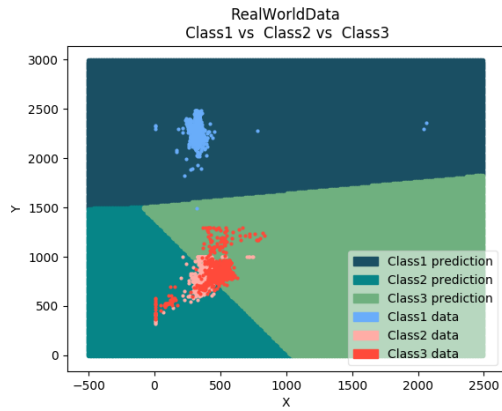
Accuracy = 99.67%

Confusion Matrix

	class1	class2
class1	614	0
class2	4	593

Analysis

	class1	class2
Precision	1.000	0.993
Recall	0.993	1.000
F-measure	0.997	0.997



Accuracy = 87.34%

Confusion Matrix

	class1	class2	class3
class1	593	3	1
class2	0	572	50
class3	0	178	436

Analysis

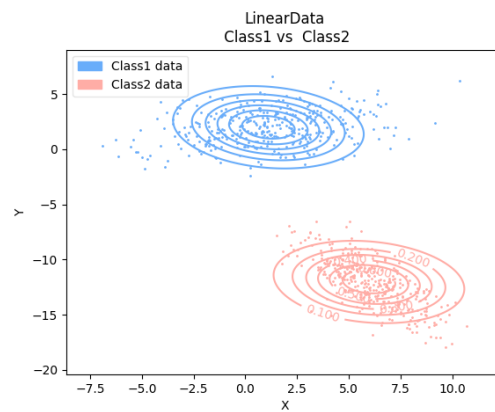
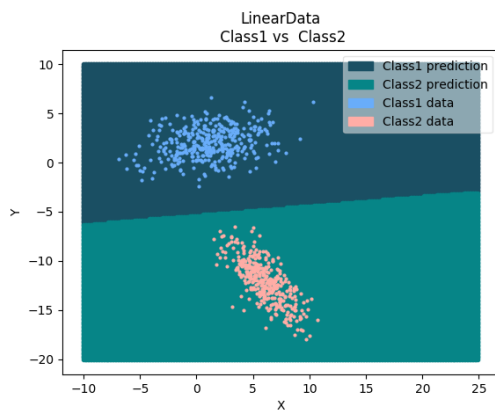
	class1	class2	class3
Precision	0.993	0.920	0.710
Recall	1.000	0.760	0.895
F-measure	0.997	0.832	0.792

3.1.4 Inferences

1. It can be seen from the plots that the decision surface is linear in nature.
2. It works well for linearly separable data but gives poor results in case of non linearly separable data and real world data.
3. The nature of contour is circular as we have taken $\Sigma_i = \sigma^2 I$

3.2 Case 2 - $\Sigma_i = \Sigma$

3.2.1 Linear Data



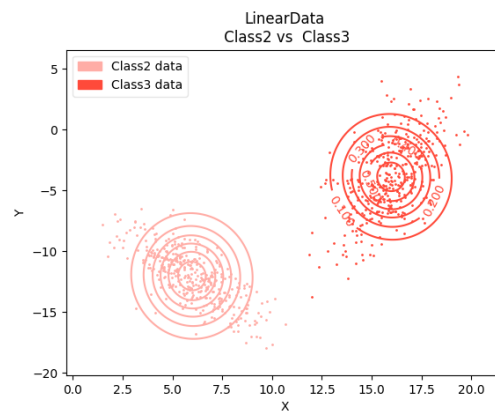
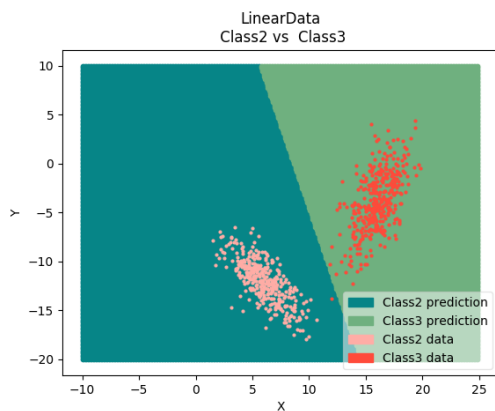
Accuracy = 100.00%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

Analysis

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



It can be seen from the plot that the decision boundary is a straight line.

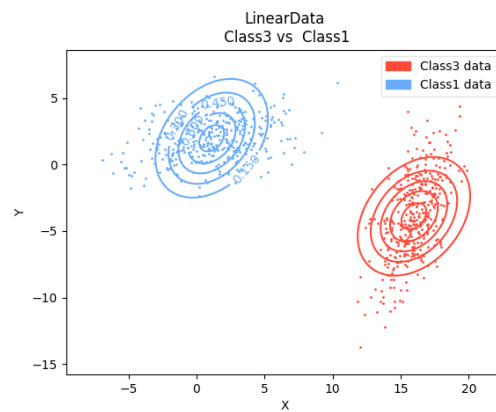
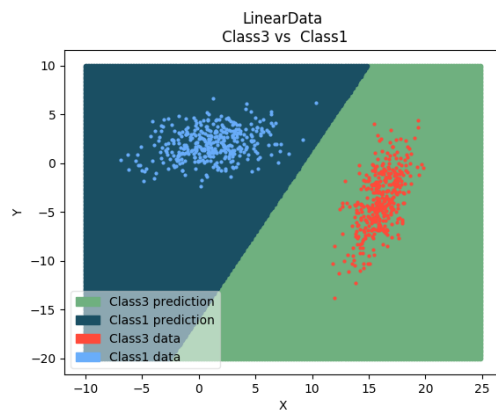
Accuracy = 100.00%

Confusion Matrix

Analysis

	class1	class2
class1	125	0
class2	0	125

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



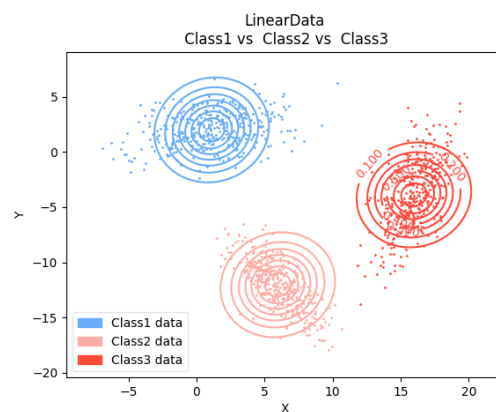
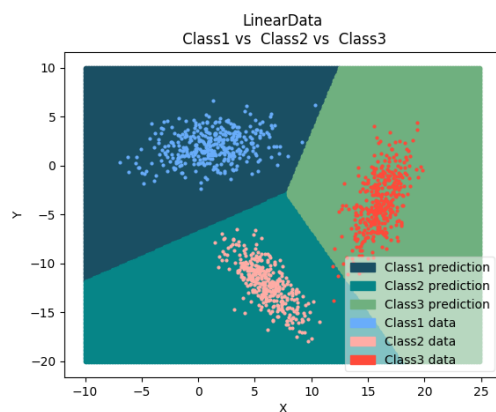
Accuracy = 99.6%

Confusion Matrix

	class1	class2
class1	125	0
class2	1	124

Analysis

	class1	class2
Precision	1.000	0.992
Recall	0.992	1.000
F-measure	0.996	0.996



Accuracy = 98.66%

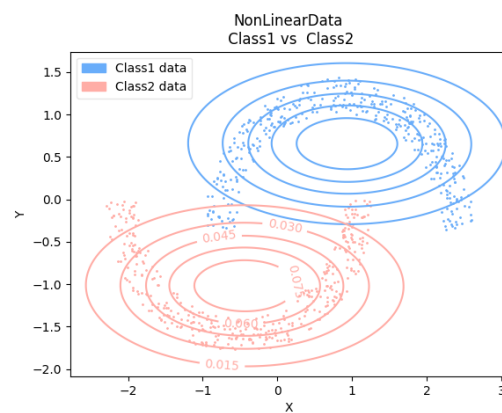
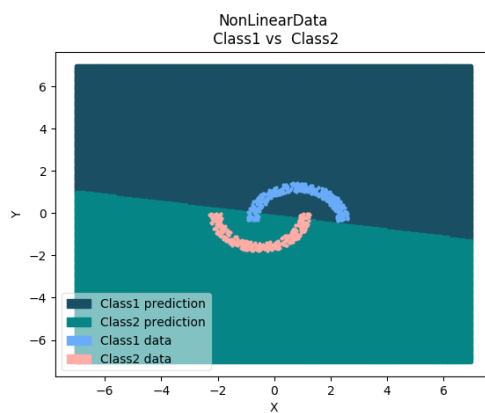
Confusion Matrix

	class1	class2	class3
class1	123	0	2
class2	1	124	0
class3	0	2	123

Analysis

	class1	class2	class3
Precision	0.984	0.992	0.984
Recall	0.991	0.984	0.984
F-measure	0.988	0.988	0.984

3.2.2 NonLinear Data



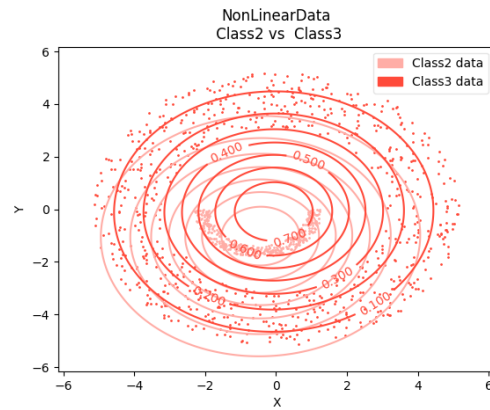
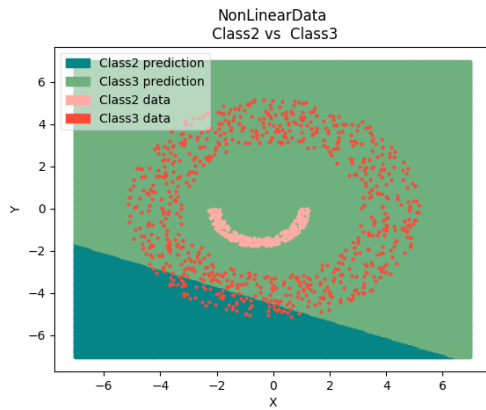
Accuracy = 95.6%

Confusion Matrix

	class1	class2
class1	121	4
class2	7	118

Analysis

	class1	class2
Precision	0.968	0.944
Recall	0.945	0.967
F-measure	0.955	0.955



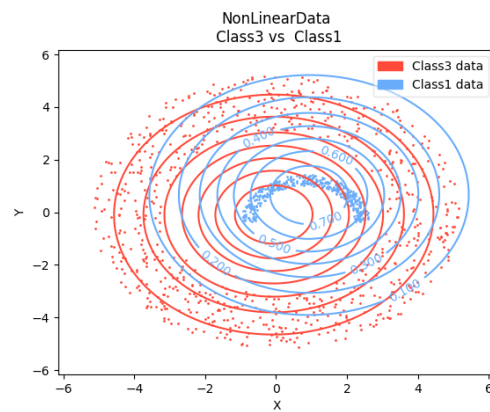
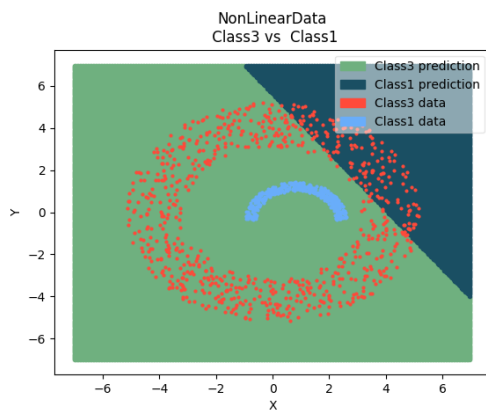
Accuracy = 61.33%

Confusion Matrix

	class1	class2
class1	0	124
class2	20	230

Analysis

	class1	class2
Precision	0.000	0.920
Recall	0.000	0.647
F-measure	-	0.760



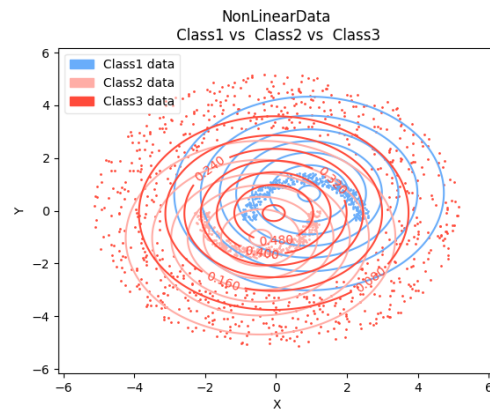
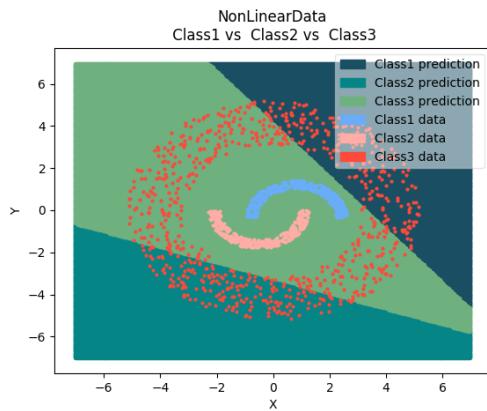
Accuracy = 49.33%

Confusion Matrix

	class1	class2
class1	185	65
class2	125	0

Analysis

	class1	class2
Precision	0.740	0.000
Recall	0.596	0.000
F-measure	0.661	0.996



Accuracy = 22.40%

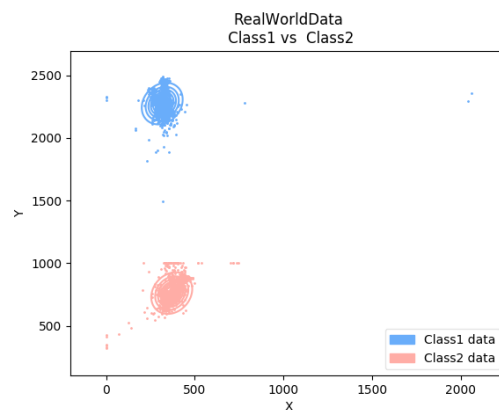
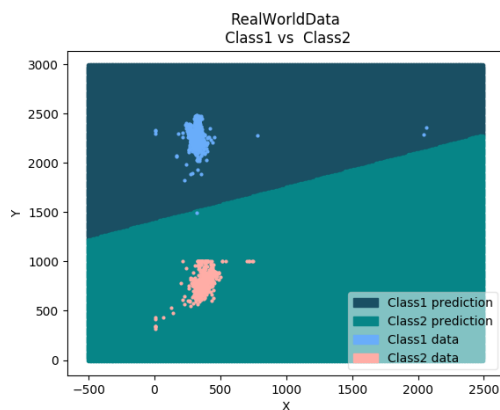
Confusion Matrix

	class1	class2	class3
class1	0	0	125
class2	0	0	125
class3	89	49	112

Analysis

	class1	class2	class3
Precision	0.000	0.000	0.448
Recall	0.000	0.000	0.309
F-measure	-	-	0.366

3.2.3 Real World Data



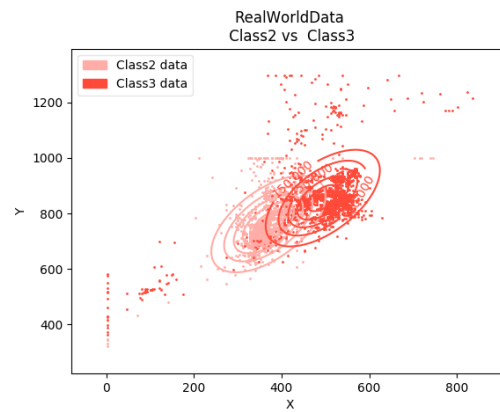
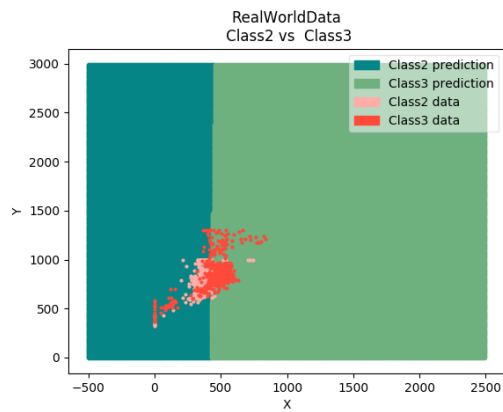
Accuracy = 99.67%

Confusion Matrix

Analysis

	class1	class2
class1	593	4
class2	0	622

	class1	class2
Precision	0.993	1.000
Recall	1.000	0.993
F-measure	0.996	0.996



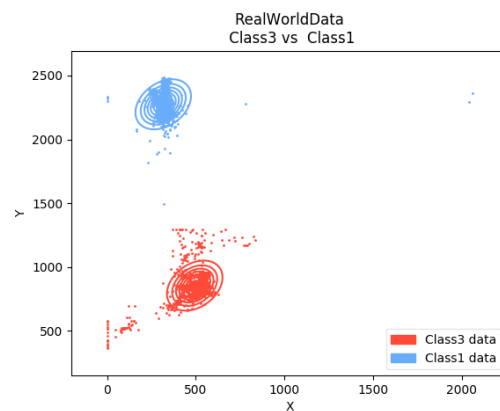
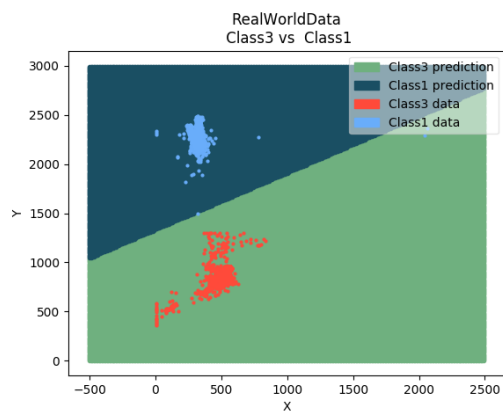
Accuracy = 81.60%

Confusion Matrix

	class1	class2
class1	601	21
class2	206	408

Analysis

	class1	class2
Precision	0.966	0.664
Recall	0.744	0.951
F-measure	0.841	0.782



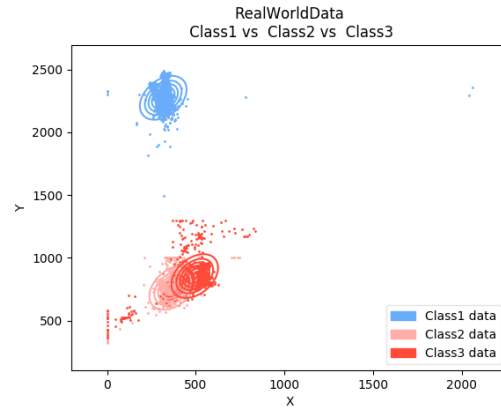
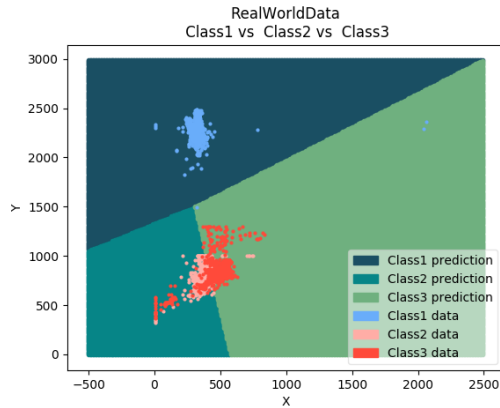
Accuracy = 98.80%

Confusion Matrix

	class1	class2
class1	614	0
class2	14	583

Analysis

	class1	class2
Precision	1.000	0.976
Recall	0.977	1.000
F-measure	0.988	0.988



Accuracy = 87.10%

Confusion Matrix

	class1	class2	class3
class1	581	3	13
class2	0	592	30
class3	0	191	423

Analysis

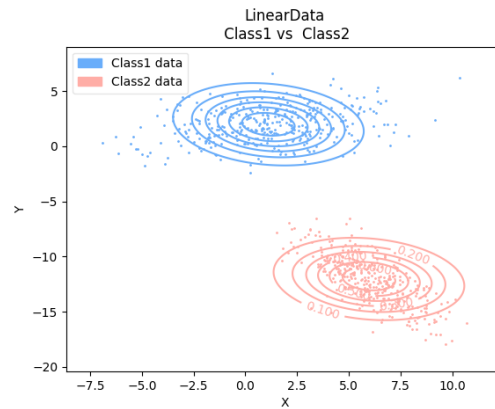
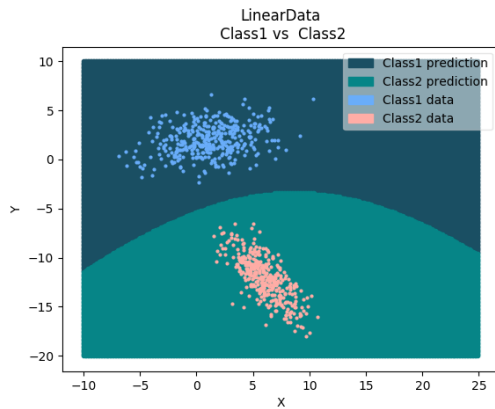
	class1	class2	class3
Precision	0.973	0.951	0.688
Recall	1.000	0.753	0.907
F-measure	0.986	0.841	0.783

3.2.4 Inferences

1. It can be seen from the plots that the decision surface is linear in nature.
2. It works well for linearly separable data but gives poor results in case of non linearly separable data and real world data.
3. The nature of contour is elliptical as we have taken $\Sigma_i = \Sigma$.
4. To find same Σ_i , we took mean of the Σ'_i s of the three classes.

3.3 Case 3 - Σ_i is a diagonal matrix

3.3.1 Linear Data



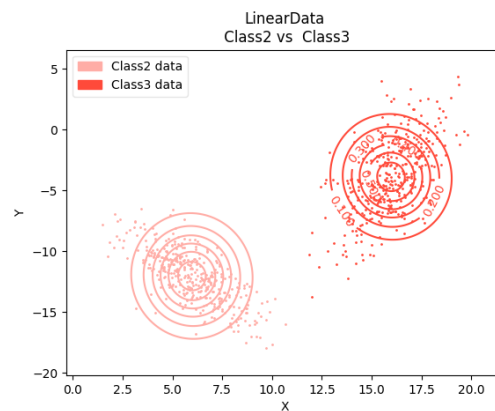
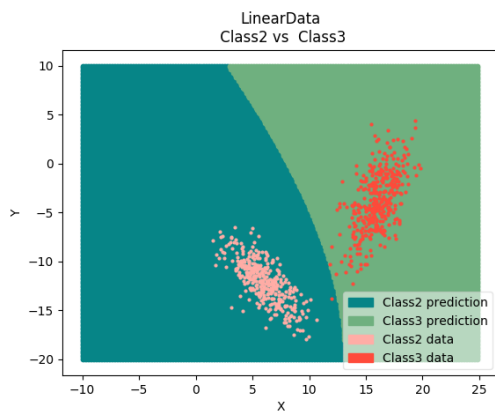
Accuracy = 100.00%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

Analysis

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



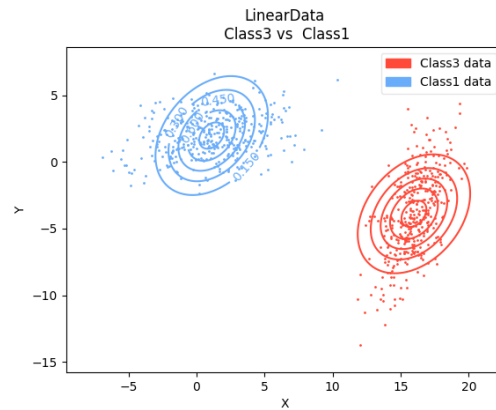
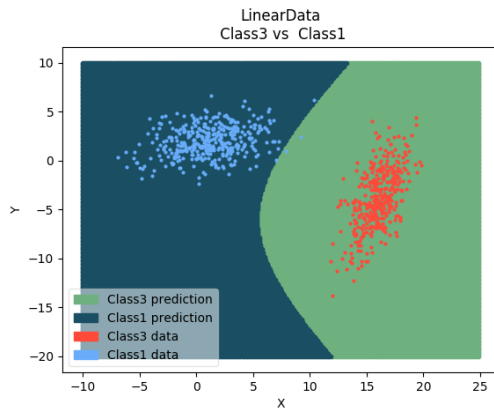
Accuracy = 100.00%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

Analysis

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



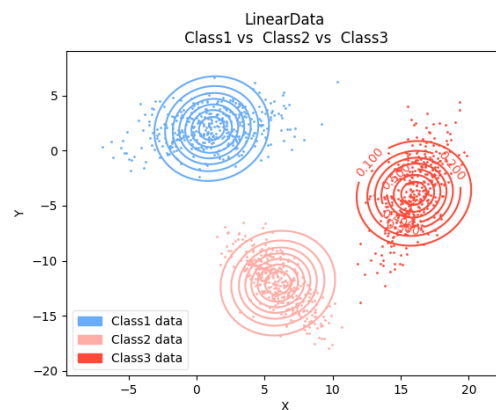
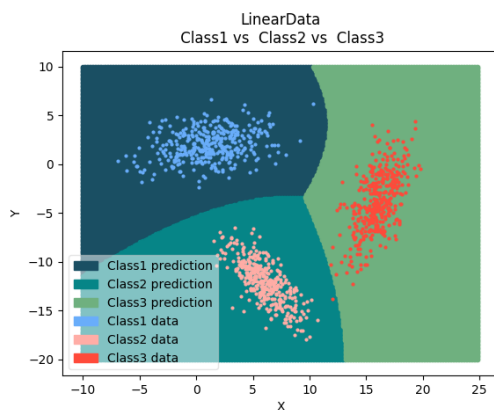
Accuracy = 99.2%

Confusion Matrix

	class1	class2
class1	125	0
class2	2	123

Analysis

	class1	class2
Precision	1.000	0.984
Recall	0.984	1.000
F-measure	0.992	0.992



Accuracy = 100.00%

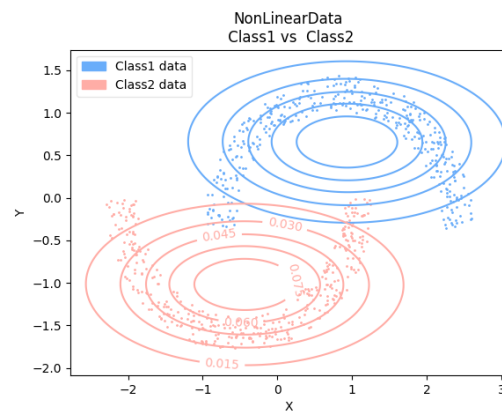
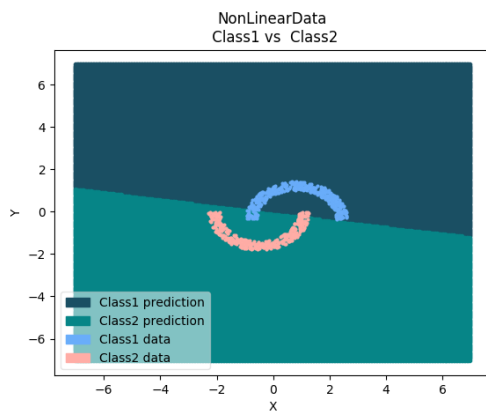
Confusion Matrix

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

Analysis

	class1	class2	class3
Precision	1.000	1.000	1.000
Recall	1.000	1.000	1.000
F-measure	1.000	1.000	1.000

3.3.2 NonLinear Data



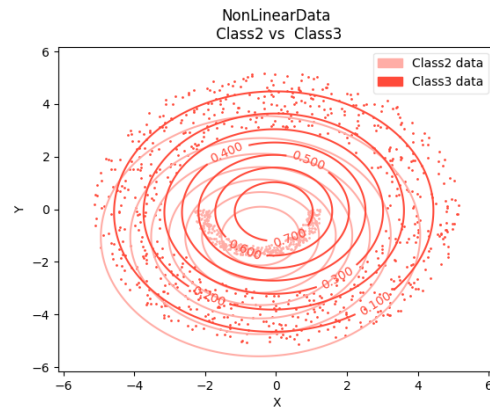
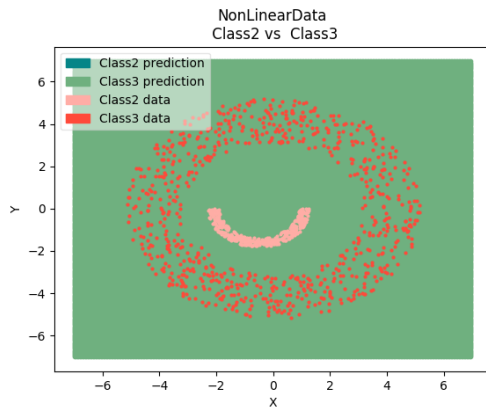
Accuracy = 96.4%

Confusion Matrix

	class1	class2
class1	120	5
class2	4	121

Analysis

	class1	class2
Precision	0.960	0.968
Recall	0.967	0.960
F-measure	0.963	0.964



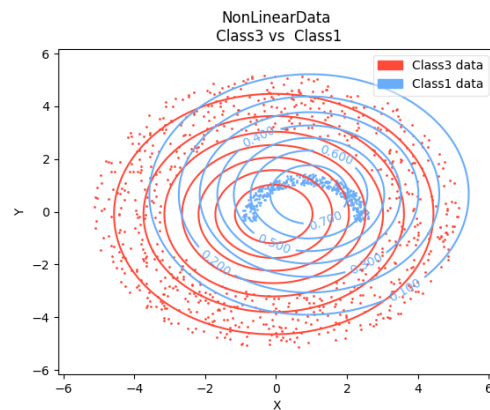
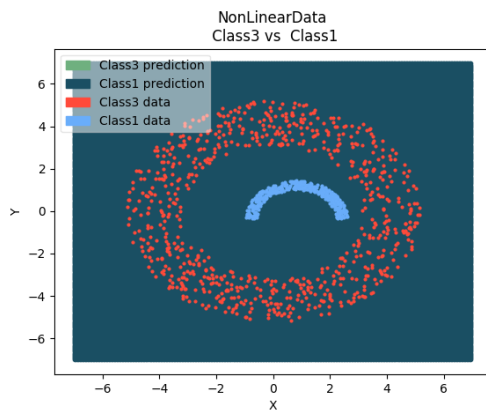
Accuracy = 66.66%

Confusion Matrix

	class1	class2
class1	0	125
class2	0	250

Analysis

	class1	class2
Precision	0.000	1.000
Recall	-	0.666
F-measure	-	0.800



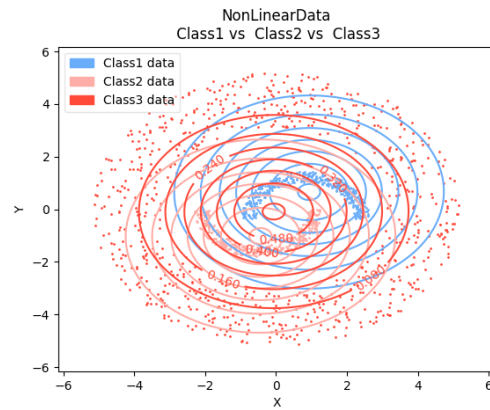
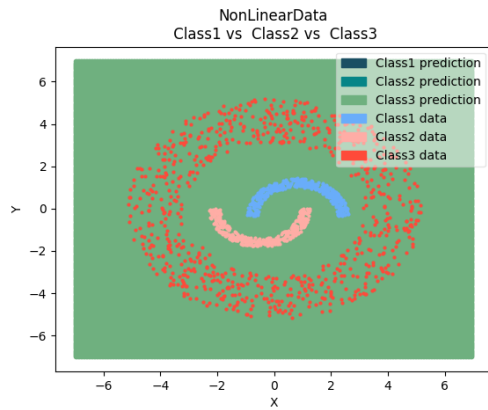
Accuracy = 33.33%

Confusion Matrix

	class1	class2
class1	0	250
class2	0	125

Analysis

	class1	class2
Precision	0.000	1.000
Recall	-	0.333
F-measure	-	0.500



Accuracy = 50.00%

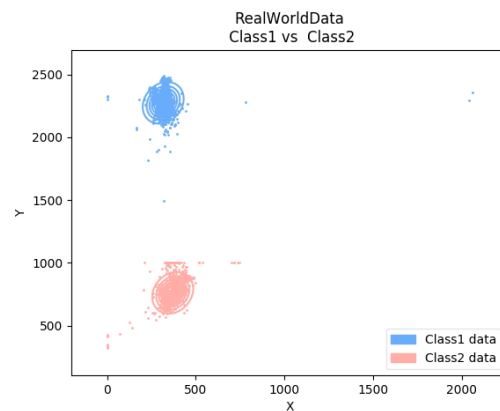
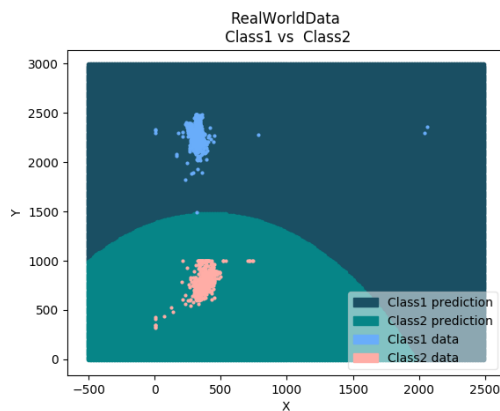
Confusion Matrix

	class1	class2	class3
class1	0	0	125
class2	0	0	125
class3	0	0	250

Analysis

	class1	class2	class3
Precision	0.000	0.000	1
Recall	-	-	0.500
F-measure	-	-	0.666

3.3.3 RealWorld Data



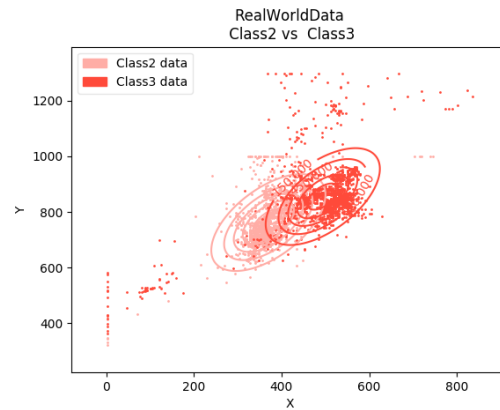
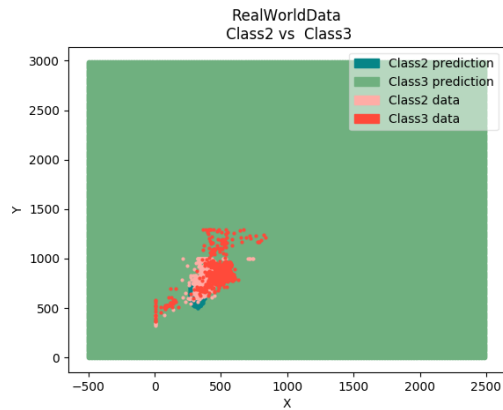
Accuracy = 99.75%

Confusion Matrix

	class1	class2
class1	594	3
class2	0	622

Analysis

	class1	class2
Precision	0.994	1.000
Recall	1.000	0.994
F-measure	0.997	0.997



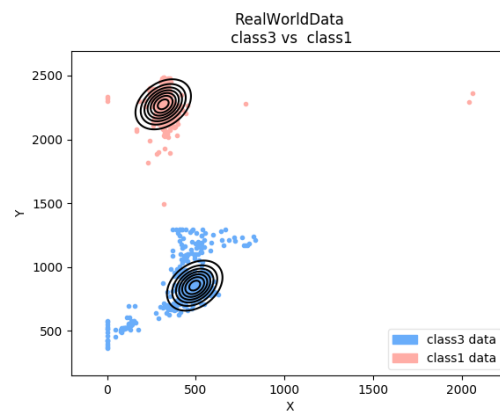
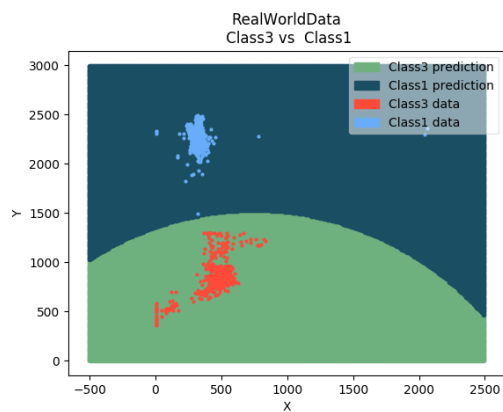
Accuracy = 87.70%

Confusion Matrix

	class1	class2
class1	500	122
class2	30	584

Analysis

	class1	class2
Precision	0.804	0.951
Recall	0.943	0.827
F-measure	0.868	0.884



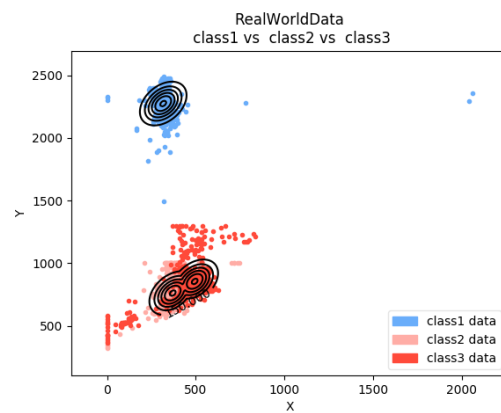
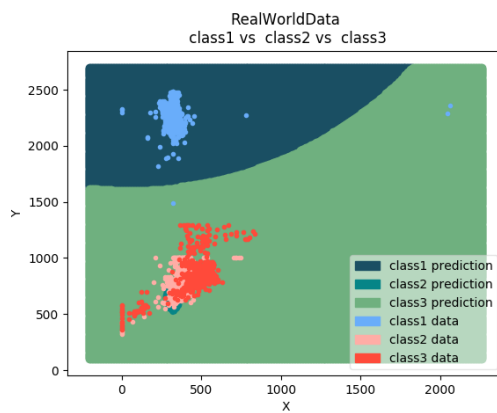
Accuracy = 99.75%

Confusion Matrix

	class1	class2
class1	614	0
class2	3	594

Analysis

	class1	class2
Precision	1.000	0.995
Recall	0.995	1.000
F-measure	0.997	0.997



Accuracy = 90.80%

Confusion Matrix

	class1	class2	class3
class1	580	2	15
class2	0	500	122
class3	0	30	584

Analysis

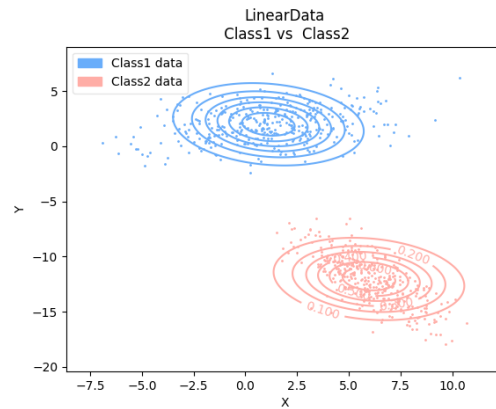
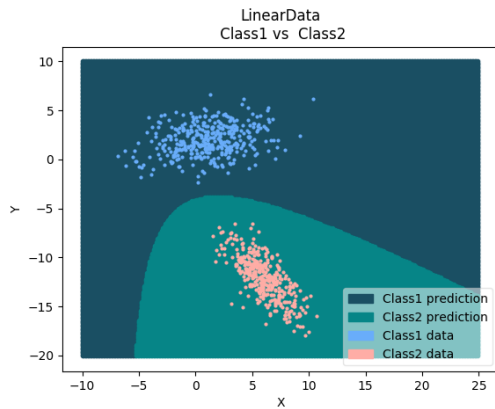
	class1	class2	class3
Precision	0.971	0.804	0.951
Recall	1.000	0.939	0.809
F-measure	0.985	0.866	0.875

3.3.4 Inferences

1. It can be seen from the plots that the decision surface is non - linear in nature.
2. It works well for linearly separable data as well as non-linearly separable data.
3. The nature of contour is elliptical and is different for each class but oriented horizontally.

3.4 Case 4 - Σ_i is unique

3.4.1 Linear Data



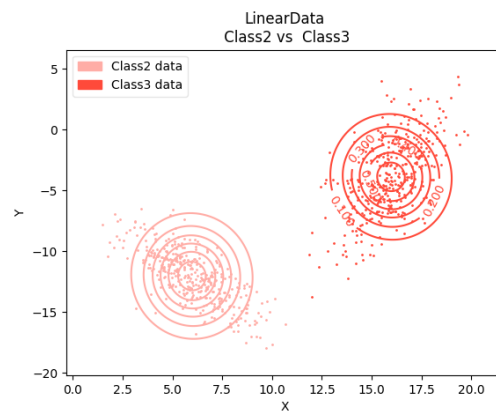
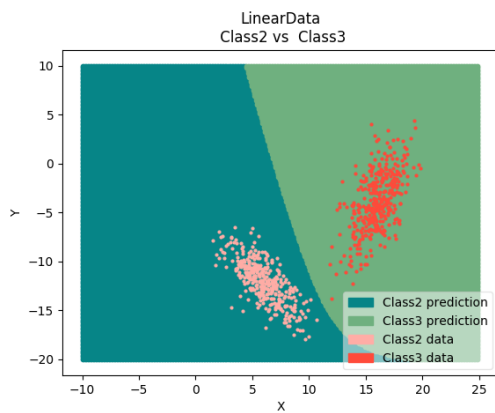
Accuracy = 100.00%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

Analysis

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



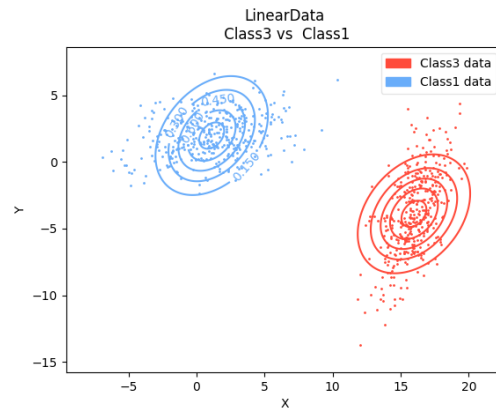
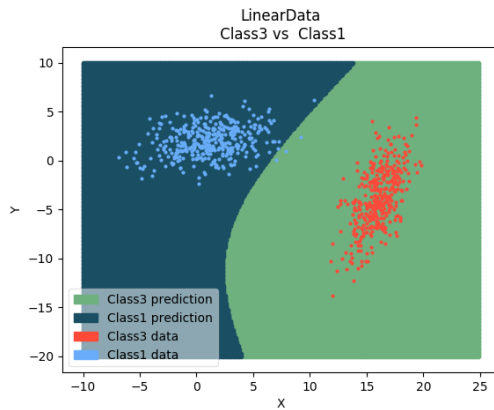
Accuracy = 100.00%

Confusion Matrix

	class1	class2
class1	125	0
class2	0	125

Analysis

	class1	class2
Precision	1.000	1.000
Recall	1.000	1.000
F-measure	1.000	1.000



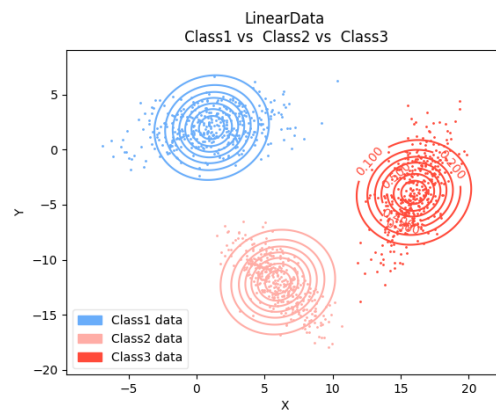
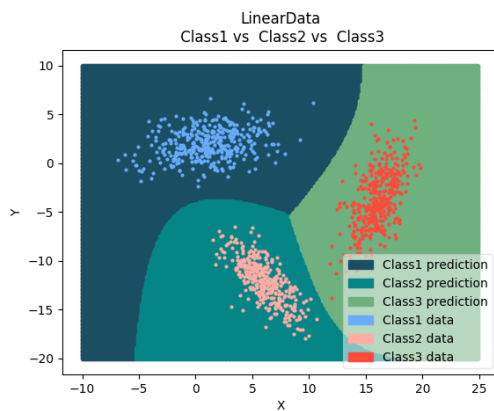
Accuracy = 99.2%

Confusion Matrix

	class1	class2
class1	125	0
class2	2	123

Analysis

	class1	class2
Precision	1.000	0.984
Recall	0.984	1.000
F-measure	0.992	0.992



Accuracy = 100.00%

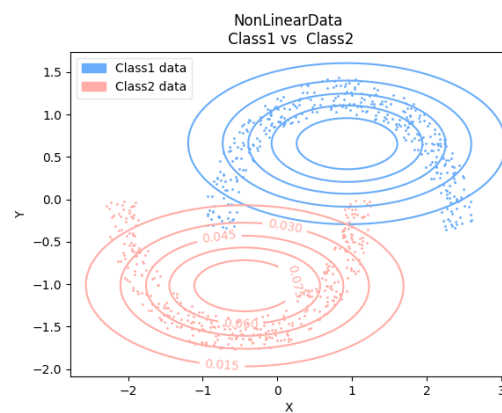
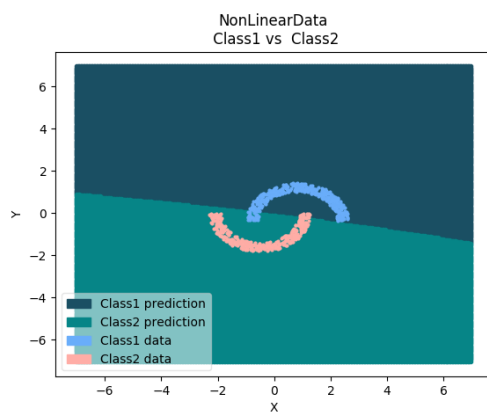
Confusion Matrix

	class1	class2	class3
class1	125	0	0
class2	0	125	0
class3	0	0	125

Analysis

	class1	class2	class3
Precision	1.000	1.000	1.000
Recall	1.000	1.000	1.000
F-measure	1.000	1.000	1.000

3.4.2 NonLinear Data



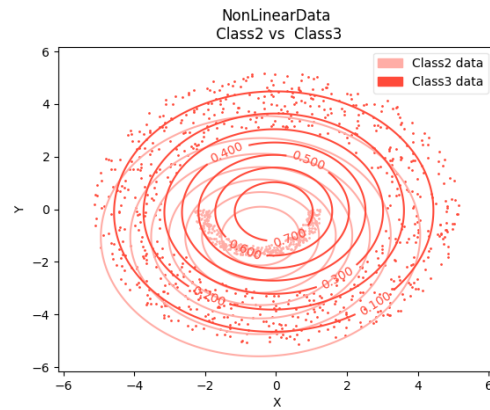
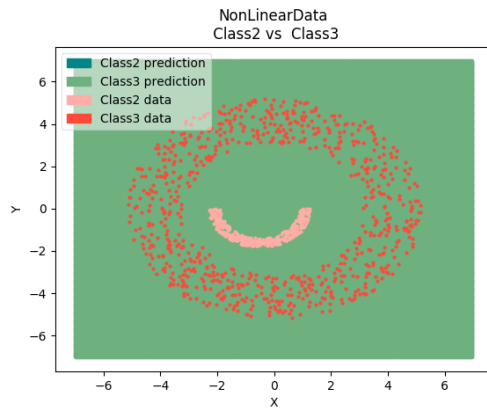
Accuracy = 96.4%

Confusion Matrix

	class1	class2
class1	120	5
class2	4	121

Analysis

	class1	class2
Precision	0.960	0.968
Recall	0.967	0.960
F-measure	0.963	0.964



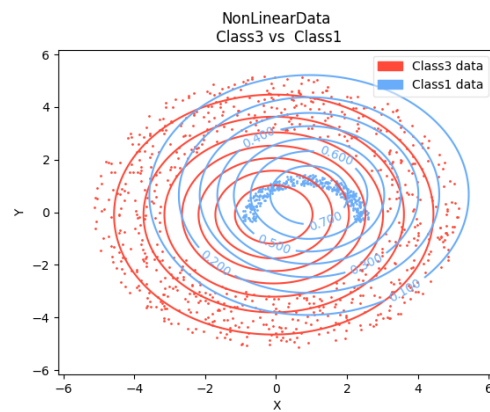
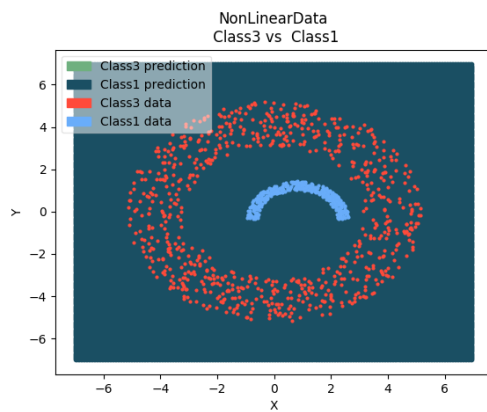
Accuracy = 66.66%

Confusion Matrix

	class1	class2
class1	0	125
class2	0	250

Analysis

	class1	class2
Precision	0.000	1.000
Recall	-	0.666
F-measure	-	0.800



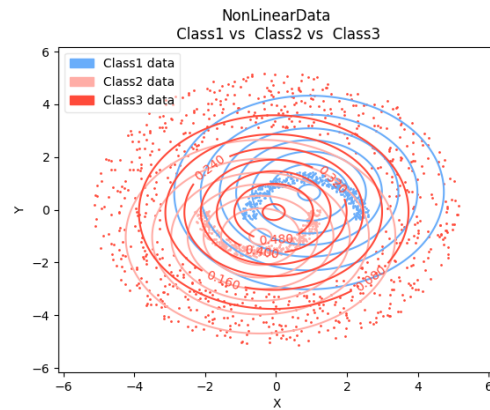
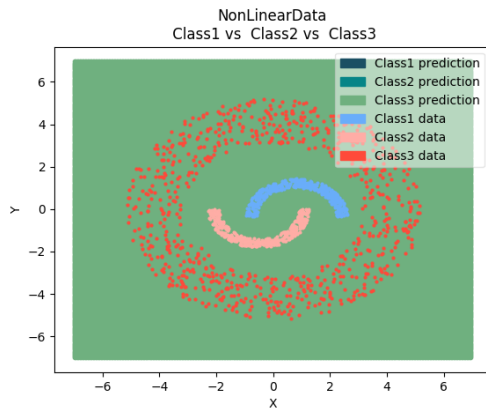
Accuracy = 33.33%

Confusion Matrix

	class1	class2
class1	0	250
class2	0	125

Analysis

	class1	class2
Precision	0.000	1.000
Recall	-	0.333
F-measure	-	0.500



Accuracy = 50.00%

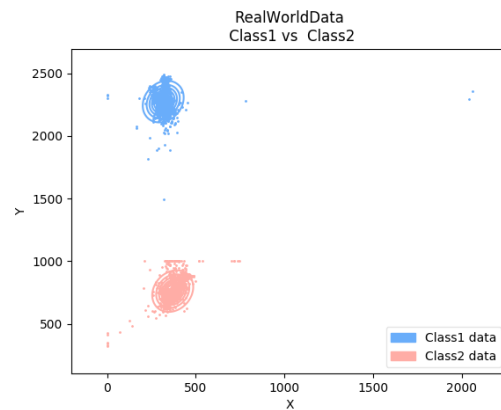
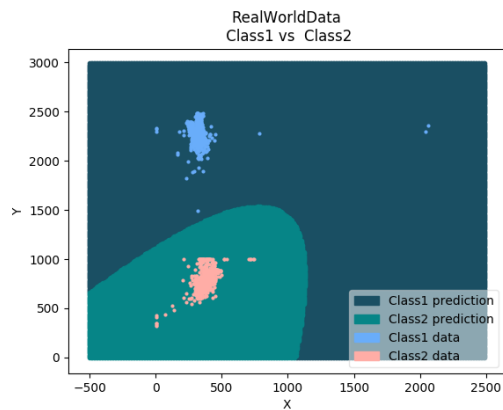
Confusion Matrix

	class1	class2	class3
class1	0	0	125
class2	0	0	125
class3	0	0	250

Analysis

	class1	class2	class3
Precision	0.000	0.000	1
Recall	-	-	0.500
F-measure	-	-	0.666

3.4.3 RealWorld Data



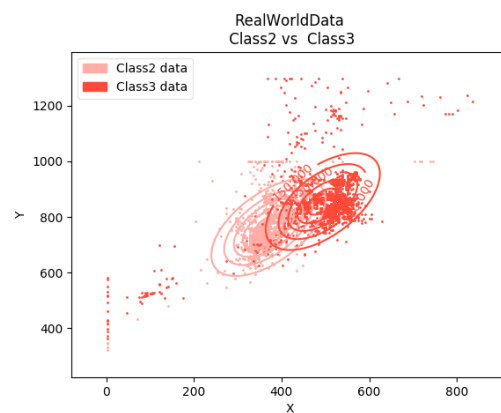
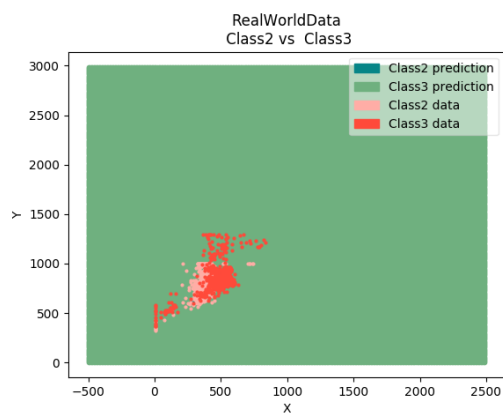
Accuracy = 99.75%

Confusion Matrix

	class1	class2
class1	594	3
class2	0	622

Analysis

	class1	class2
Precision	0.994	1.000
Recall	1.000	0.994
F-measure	0.997	0.997



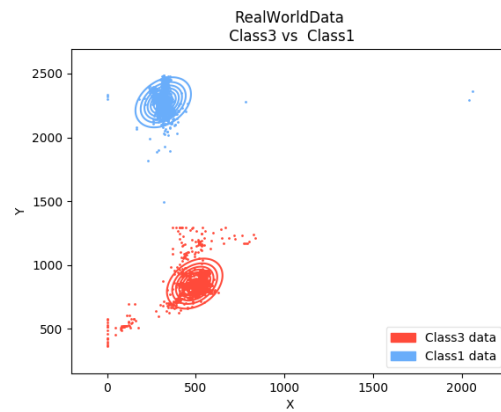
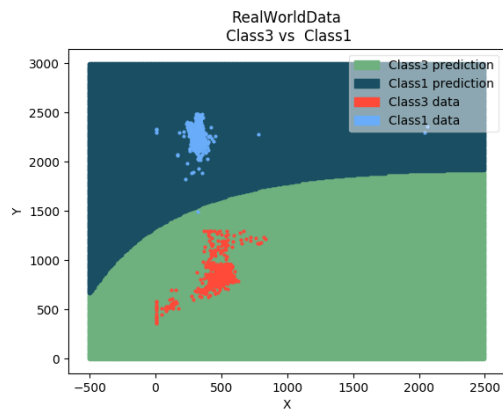
Accuracy = 77.10%

Confusion Matrix

Analysis

	class1	class2
class1	350	272
class2	11	603

	class1	class2
Precision	0.562	0.982
Recall	0.969	0.689
F-measure	0.712	0.809



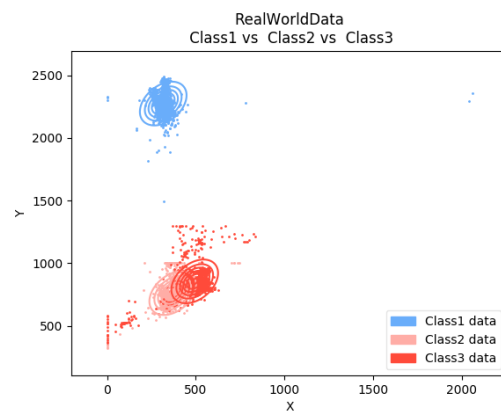
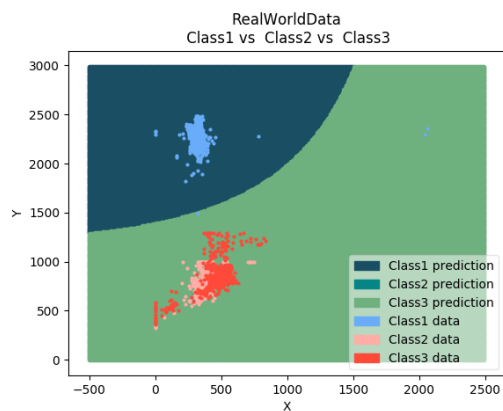
Accuracy = 99.66%

Confusion Matrix

	class1	class2
class1	614	0
class2	4	593

Analysis

	class1	class2
Precision	1.000	0.993
Recall	0.993	1.000
F-measure	0.996	0.996



Accuracy = 83.68%

Confusion Matrix

	class1	class2	class3
class1	581	2	14
class2	0	350	272
class3	0	11	603

Analysis

	class1	class2	class3
Precision	0.973	0.562	0.982
Recall	1.000	0.964	0.678
F-measure	0.986	0.710	0.802

3.4.4 Inferences

1. It can be seen from the plots that the decision surface is non - linear in nature.
2. It works well for linearly separable data and good (but not as good as linearly separable data) for non linearly separable data.
3. The nature of contour is elliptical and different for every class and also oriented independently in any directions.

4 Conclusion

1. Bayes Classifier works well for Linearly separable data in all cases with high accuracy but fails for Non linearly separable data with poor accuracy.
2. In case of Real World Data, the data is overlapping and thus results in lesser accuracy.
3. In first two cases, the decision boundary comes out to be straight line whereas in last two cases, the decision boundary is quadratic due to different covariance matrix chosen.