DARE

ARC Training Centre in
Data Analytics for Resources & Environments

DATA 6811 – Bayesian Computational Statistics

Spatiotemporal Data Analysis

Week 7

Lecturer: Nandini Ramesh

Date: 16/05/2022

Outline – Week 7, Lecture 1

- Types of Spatiotemporal Data
  - Spatial Fields
  - Trajectory Data
  - Track/Irregular Data
- Exploring Spatiotemporal Data
  - Co-ordinate Systems, Map Projections
  - Visualizing spatiotemporal data
- Dimension Reduction for Spatiotemporal Data
  - Empirical Orthogonal Functions
  - Example: El Niño-Southern Oscillation

We will be using chapters 1-2 of *Spatio-Temporal Statistics with R* by Wilke, Zammit-Magnion, Cressie.

Types of Spatiotemporal Data

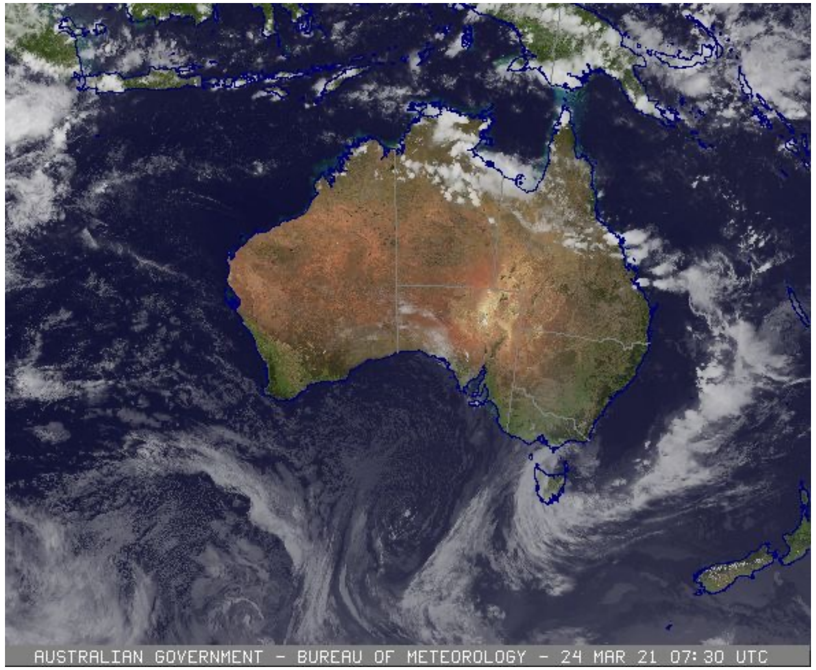
1. Spatial fields
2. Trajectory data
3. Track/irregular data

Types of Spatiotemporal Data

1. Spatial fields

- Usually an image

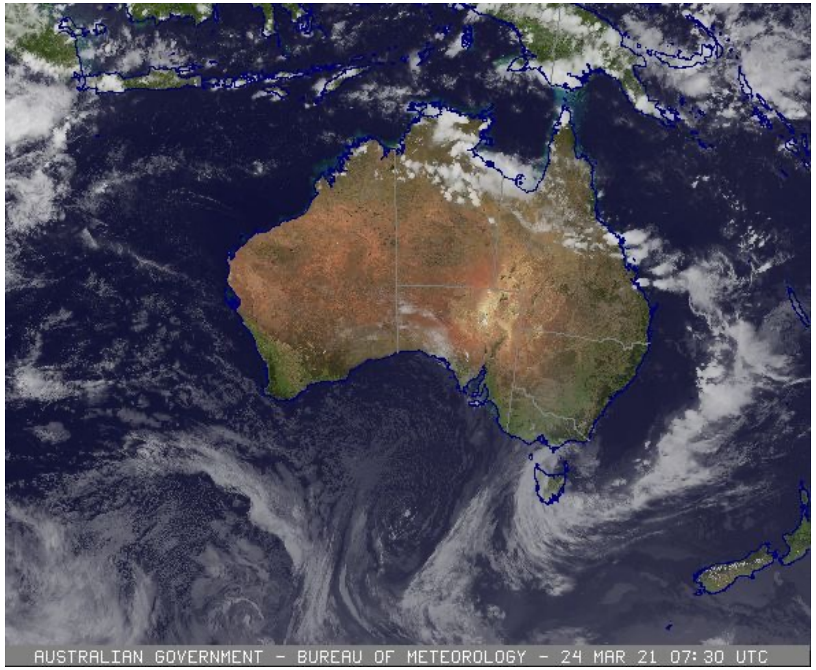
e.g., from an experiment or satellite

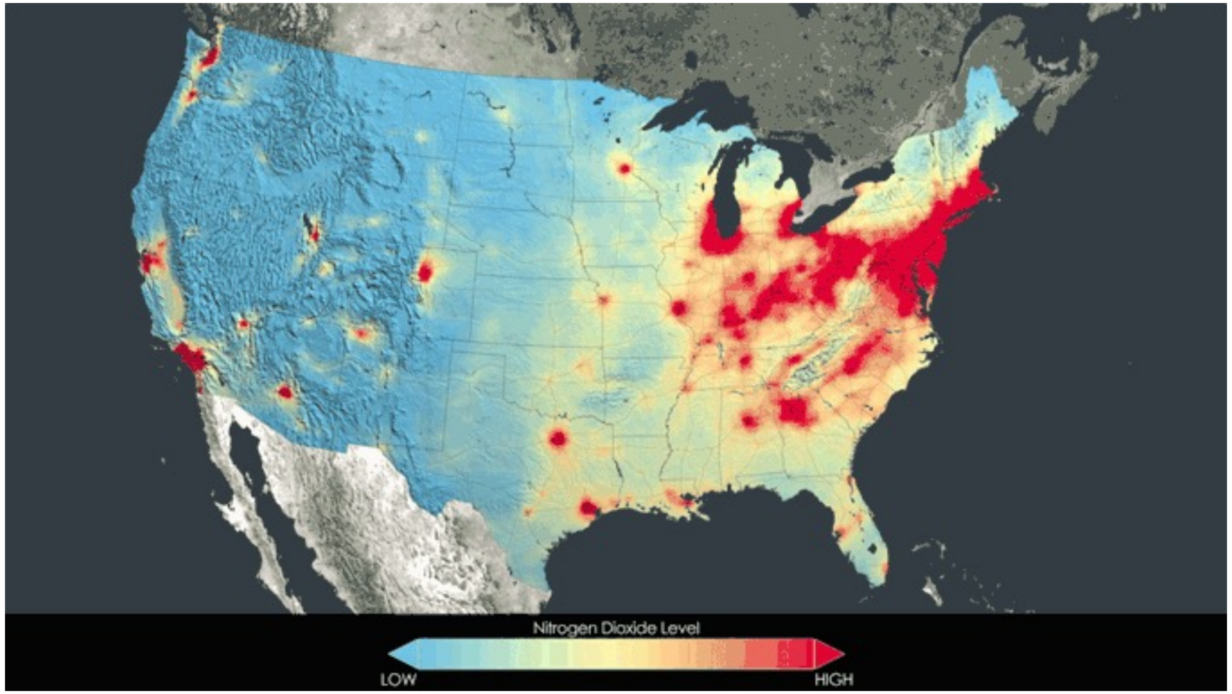


Types of Spatiotemporal Data

1. Spatial fields

- Usually an image (e.g., from an experiment or satellite)
- Not always visible photography





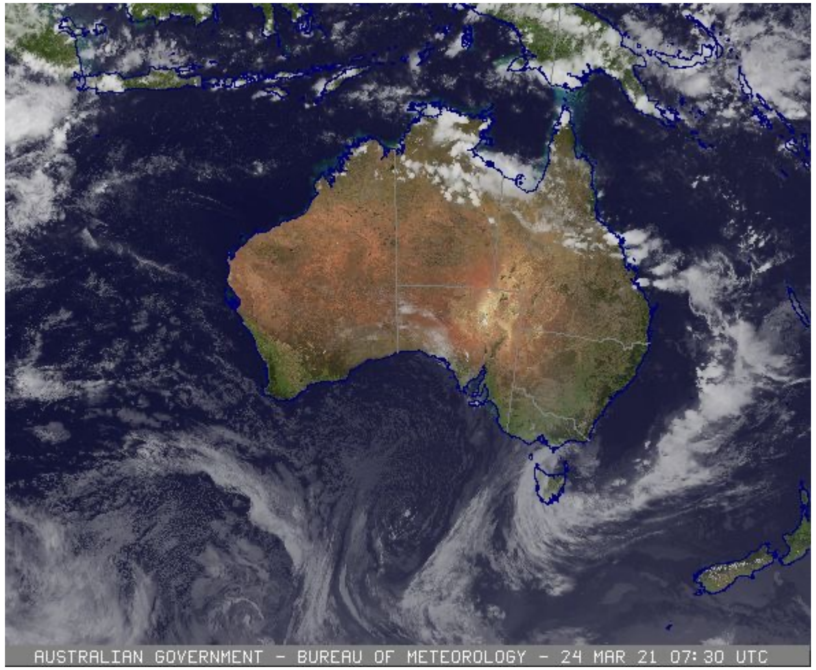
NASA Goddard's Scientific Visualization Studio/T. Schindler 2014

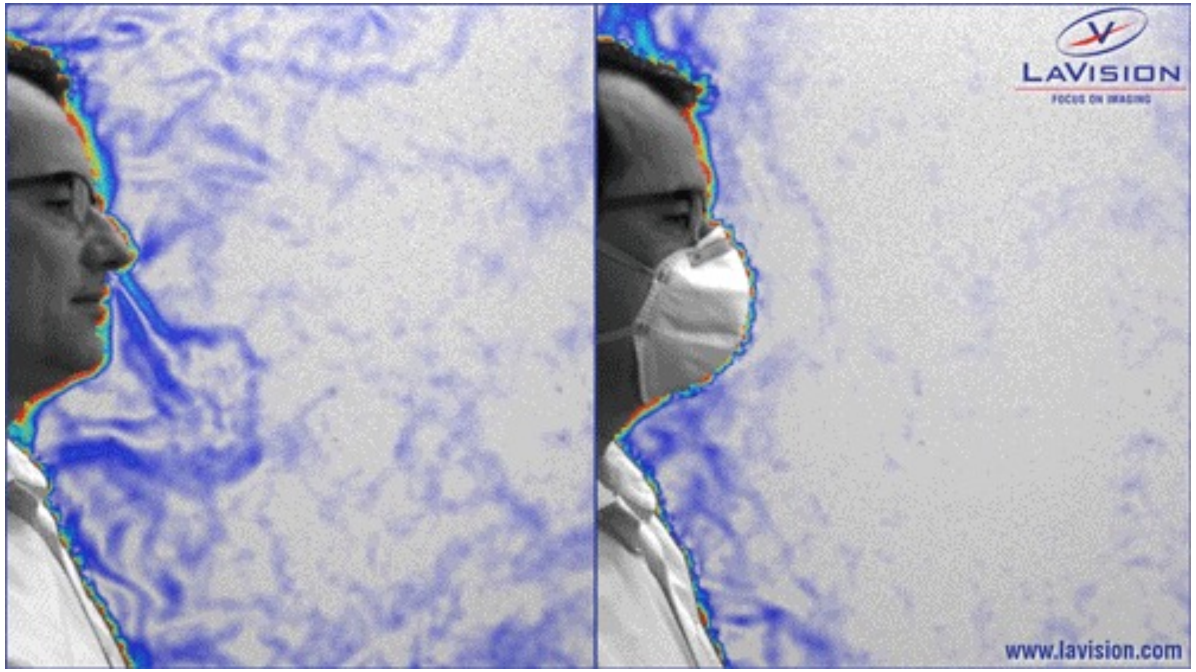
Types of Spatiotemporal Data

1. Spatial fields

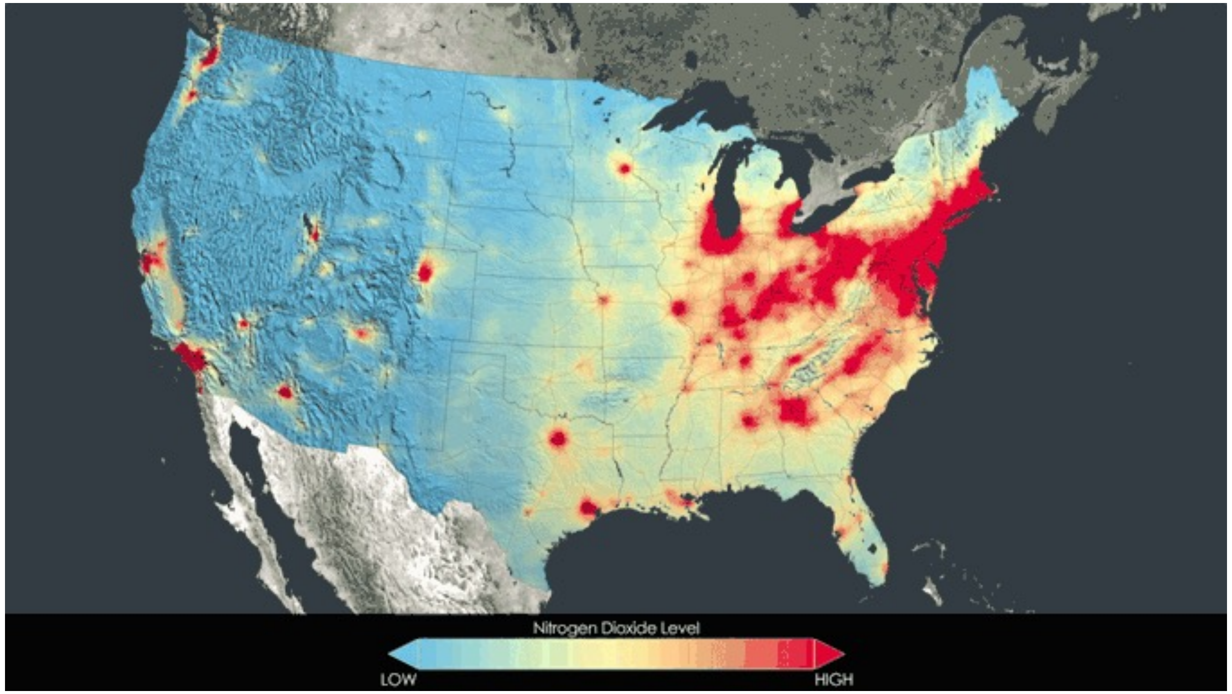
- Usually an image (e.g., from an experiment or satellite)

Can be on any scale.





Cai, Berg, Karniadakis 2020: Quantifying the airflow of coughing



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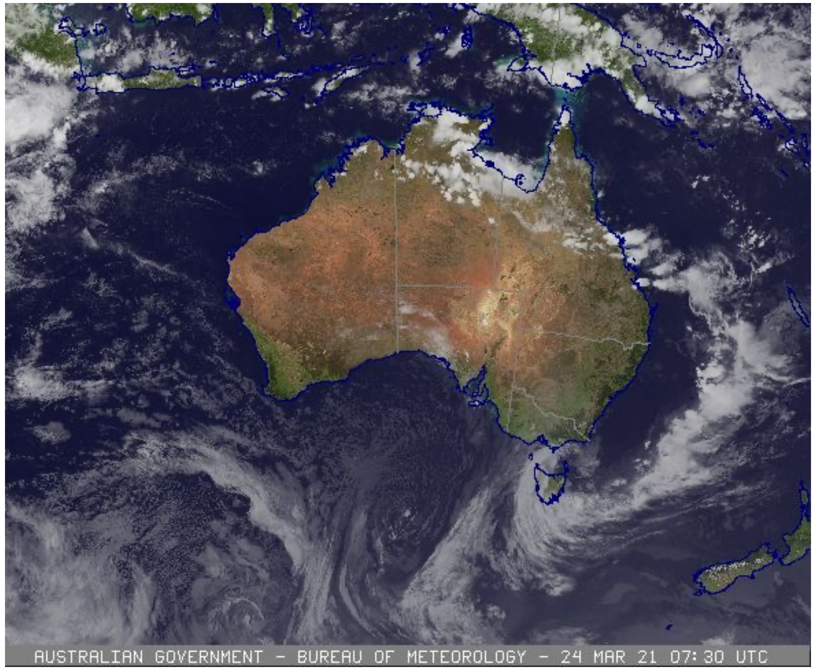
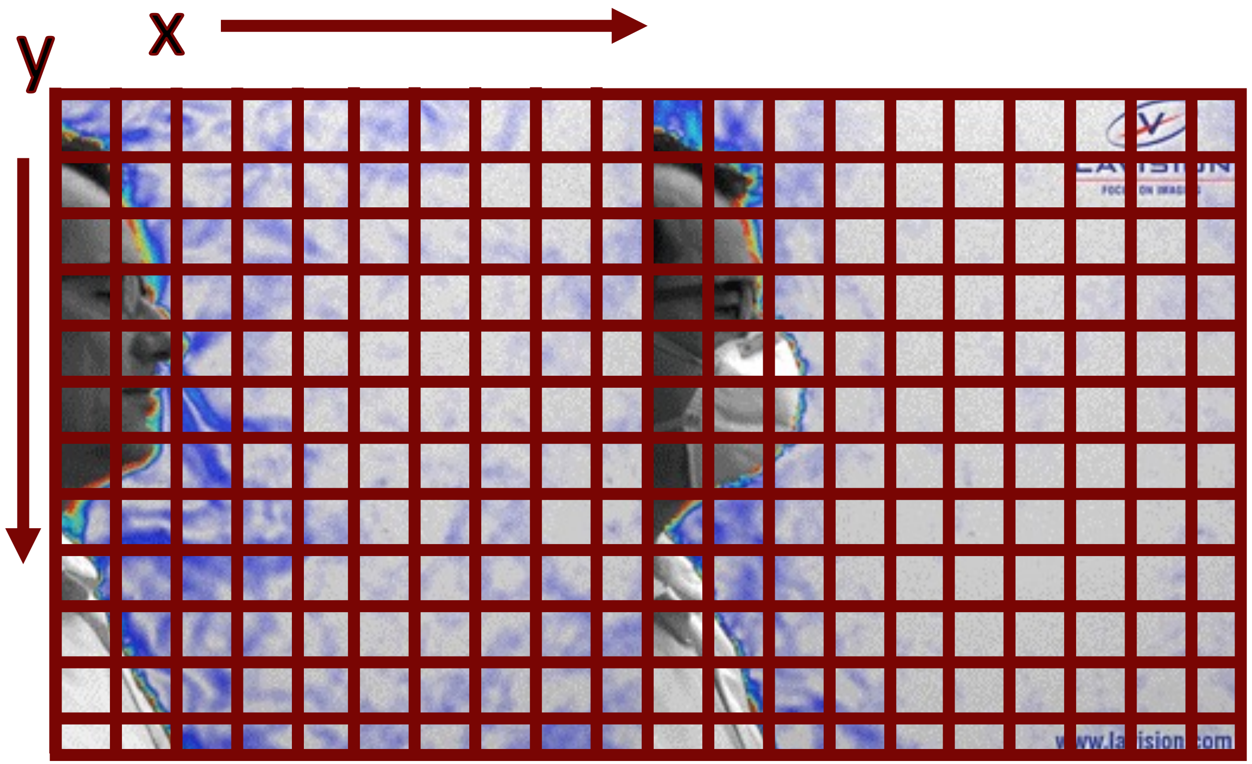
Types of Spatiotemporal Data

1. Spatial fields

- Usually an image
- Indexed in some co-ordinate system





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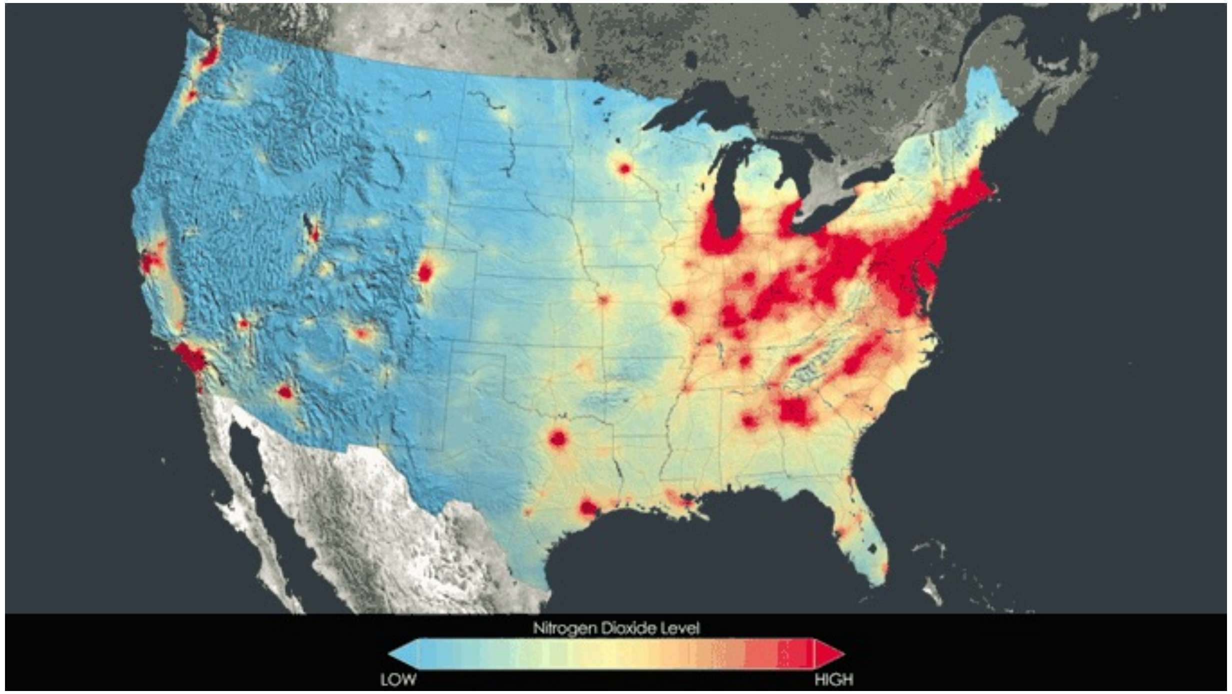


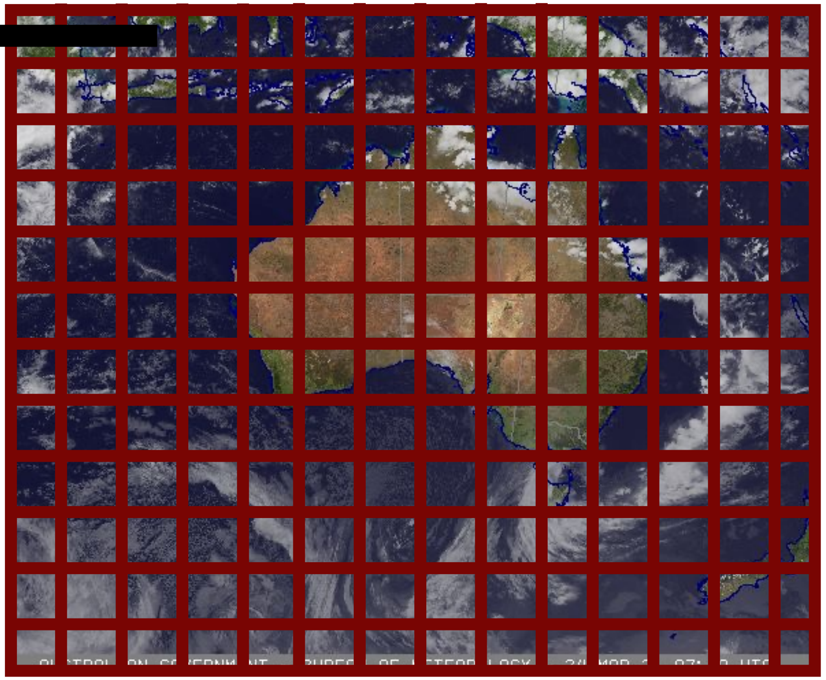
NASA Goddard's Scientific Visualization Studio/T. Schindler 2014

Types of Spatiotemporal Data

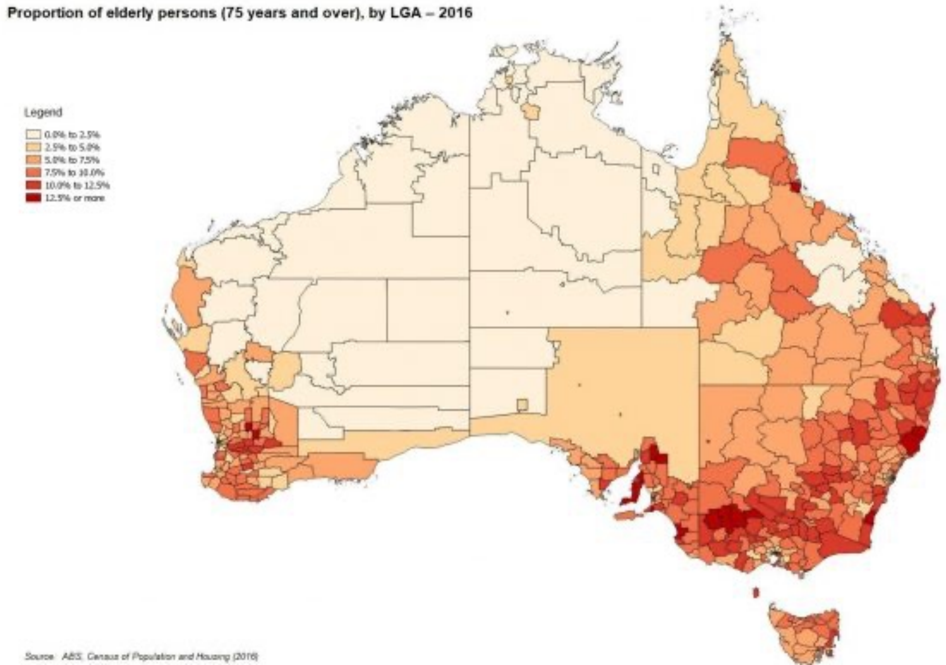
1. Spatial fields

Generally stored as a matrix, where indices correspond to coordinates.

Element *i,j*
is at
longitude φ
latitude θ



Data may be aggregated into irregular shapes instead (e.g., states in a country). A single index, whose ordering doesn't matter, is used to denote these shapes.

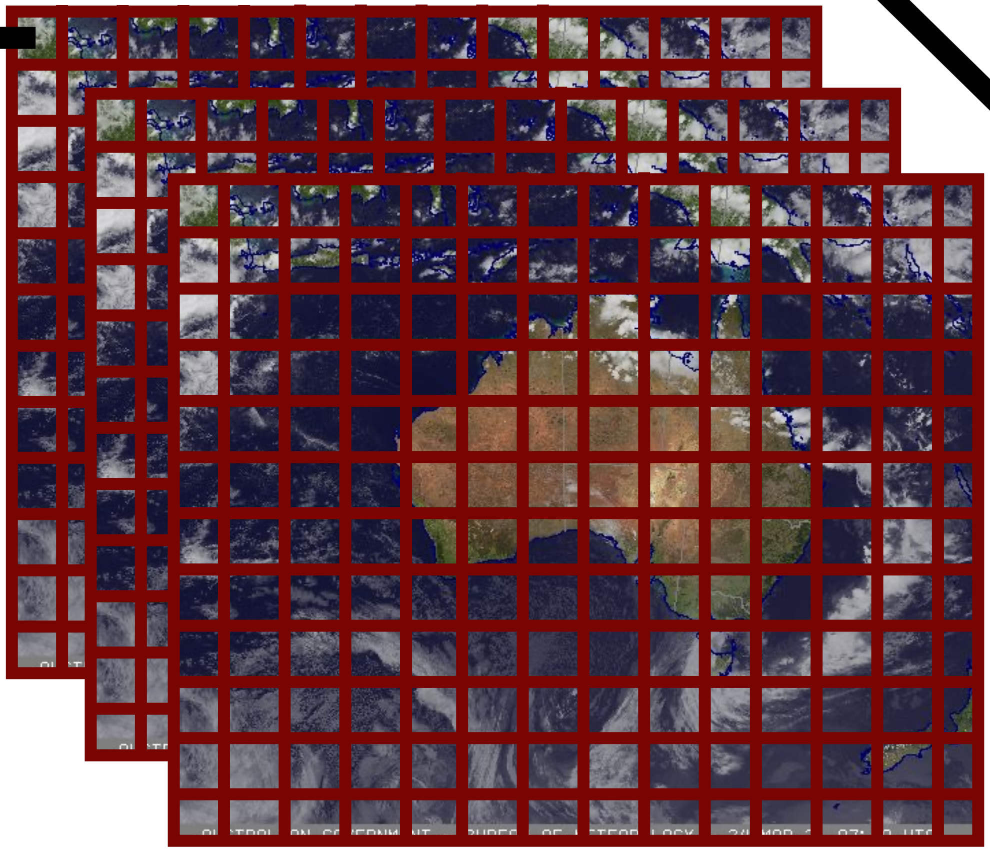


Types of Spatiotemporal Data

1. Spatial fields

Generally stored as a matrix, where indices correspond to coordinates.

Element
i,j,k



Time

In general, up to 3 space dimensions + 1 time dimension

E.g., ocean temperature

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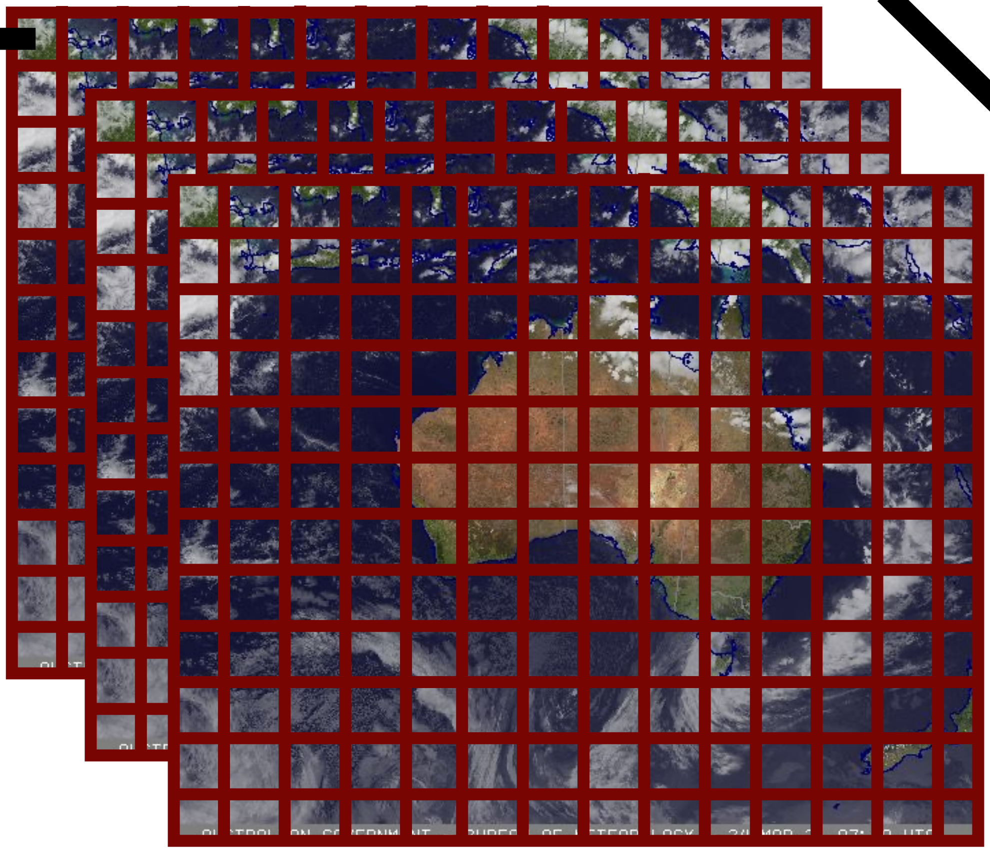


Time

In general, up to 3 space dimensions + 1 time dimension

E.g., ocean temperature

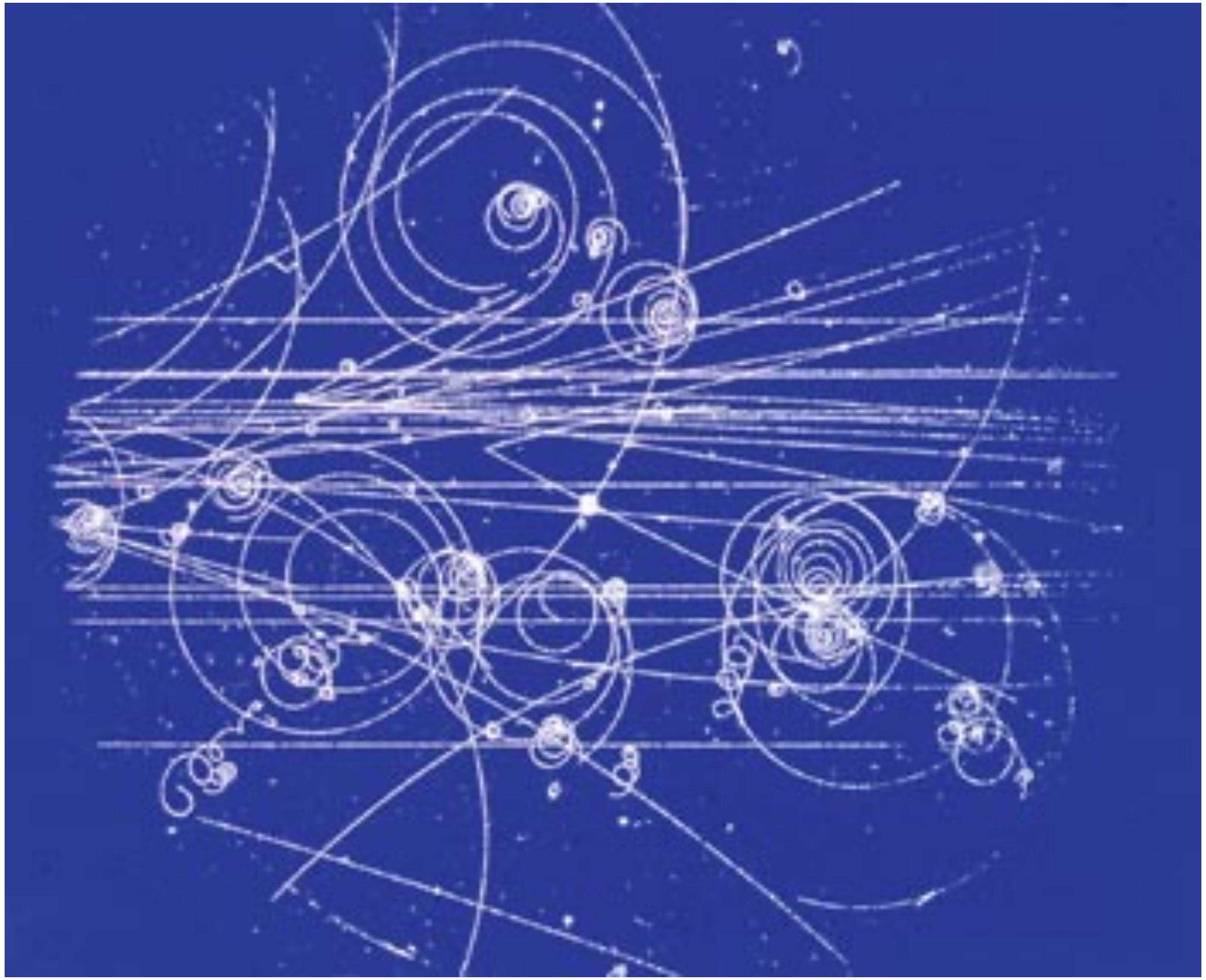
Today's lecture will mainly focus on analysis methods for data structured in this manner; but first, let's look at some other types of spatiotemporal data.

Types of Spatiotemporal Data

2. Trajectory data

The spatiotemporal co-ordinates **are** the data.

This is typically the case when you want to know about the location of an object as it moves.



[CERN courier]

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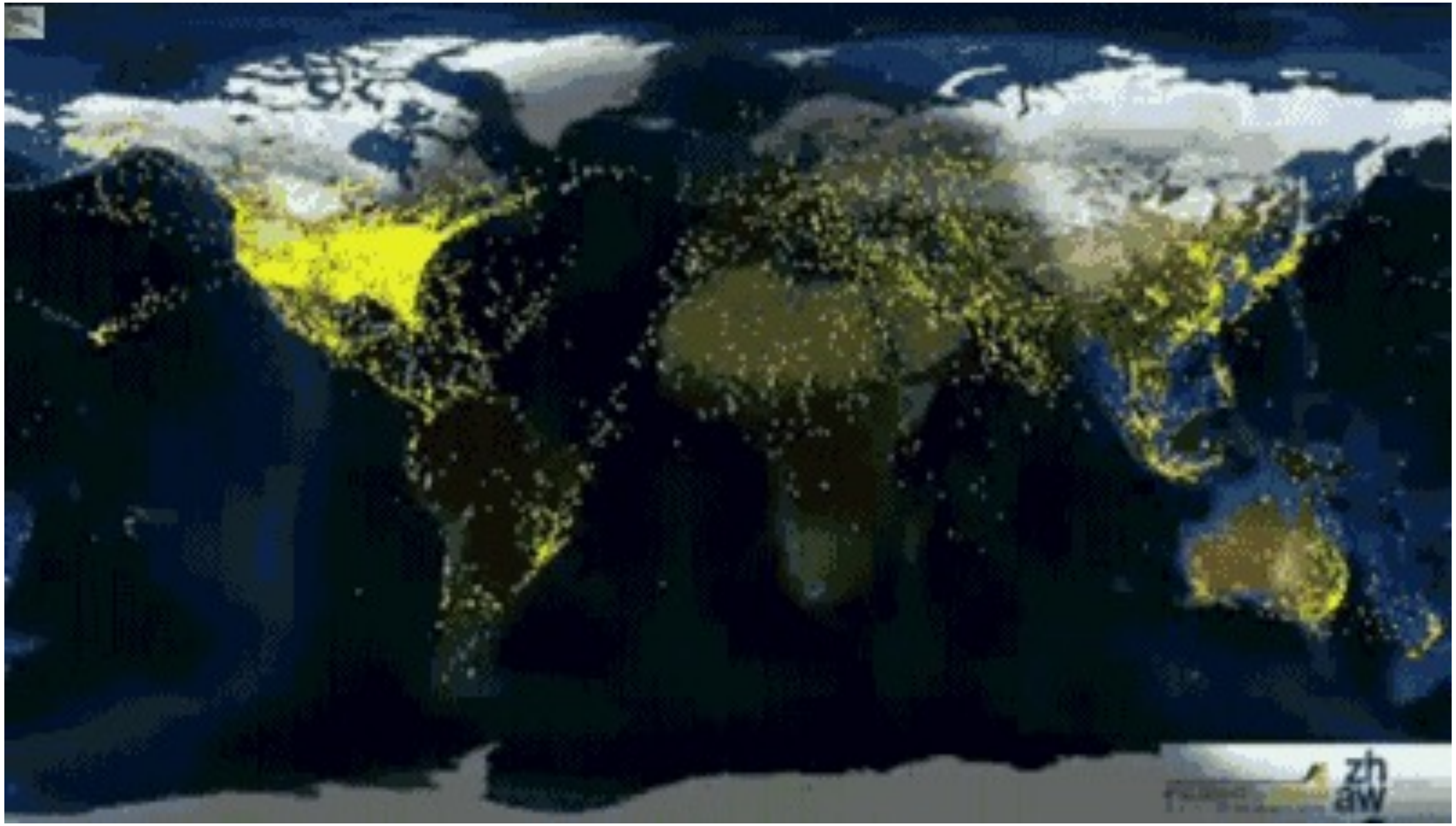


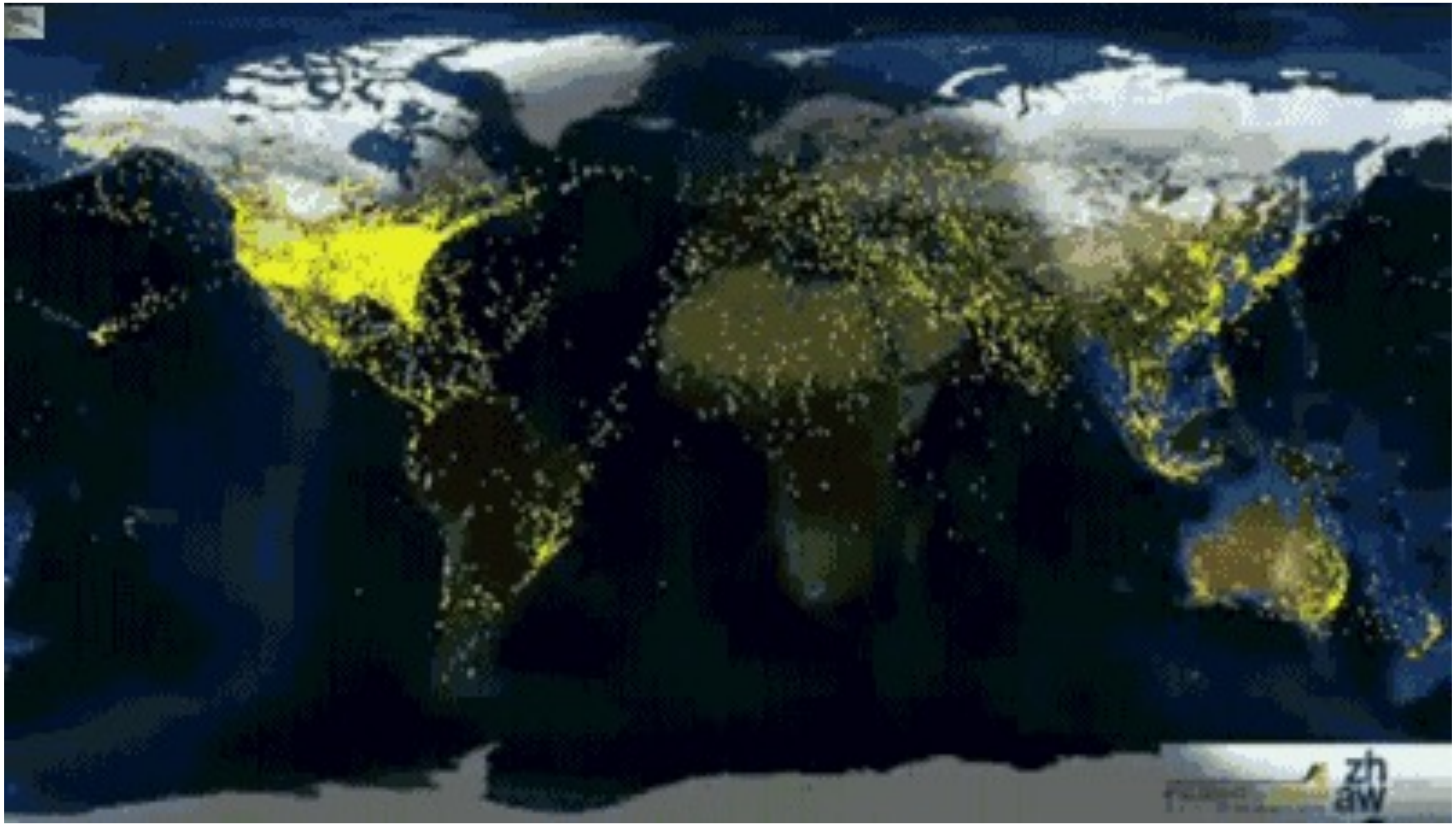
Commercial flight paths over 24 hours from FlightStats (Rege, 2008).

Types of Spatiotemporal Data

2. Trajectory data

The spatiotemporal co-ordinates **are** the data.

Flight #	t_1	t_2	t_3
AF123	lon(t_1), lat(t_1)	lon(t_2), lat(t_2)	lon(t_3), lat(t_3)
…	…	…	…
…	…	…	…



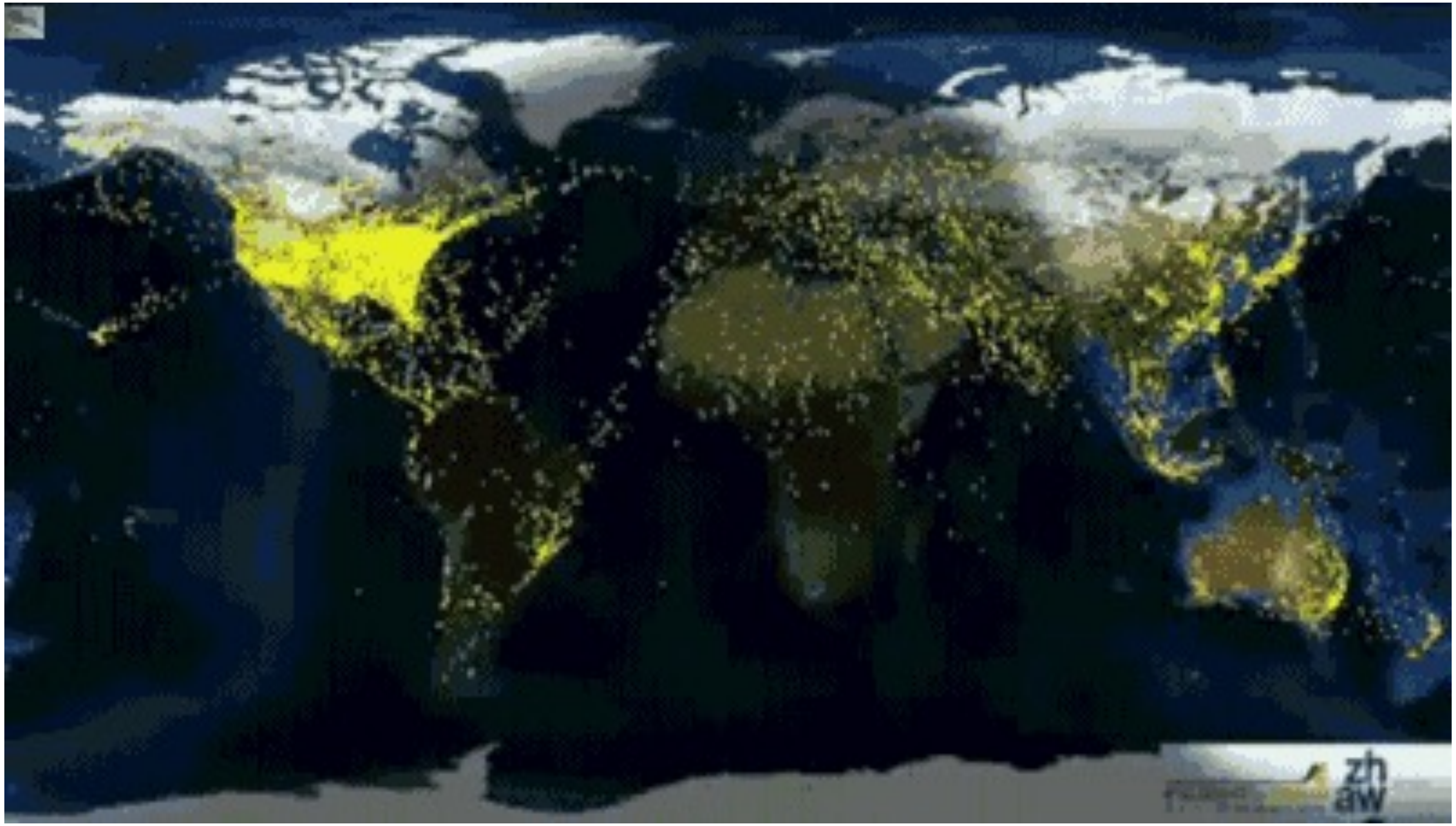
Commercial flight paths over 24 hours from FlightStats (Rege, 2008).

Types of Spatiotemporal Data

2. Trajectory data

One way to organize this data: as a **graph**, with sources/destinations as nodes.

Flight #	Source	Destination
AF123	lon, lat, t_S	lon, lat, t_D
…	…	…
…	…	…



Commercial flight paths over 24 hours from FlightStats (Rege, 2008).

Types of Spatiotemporal Data

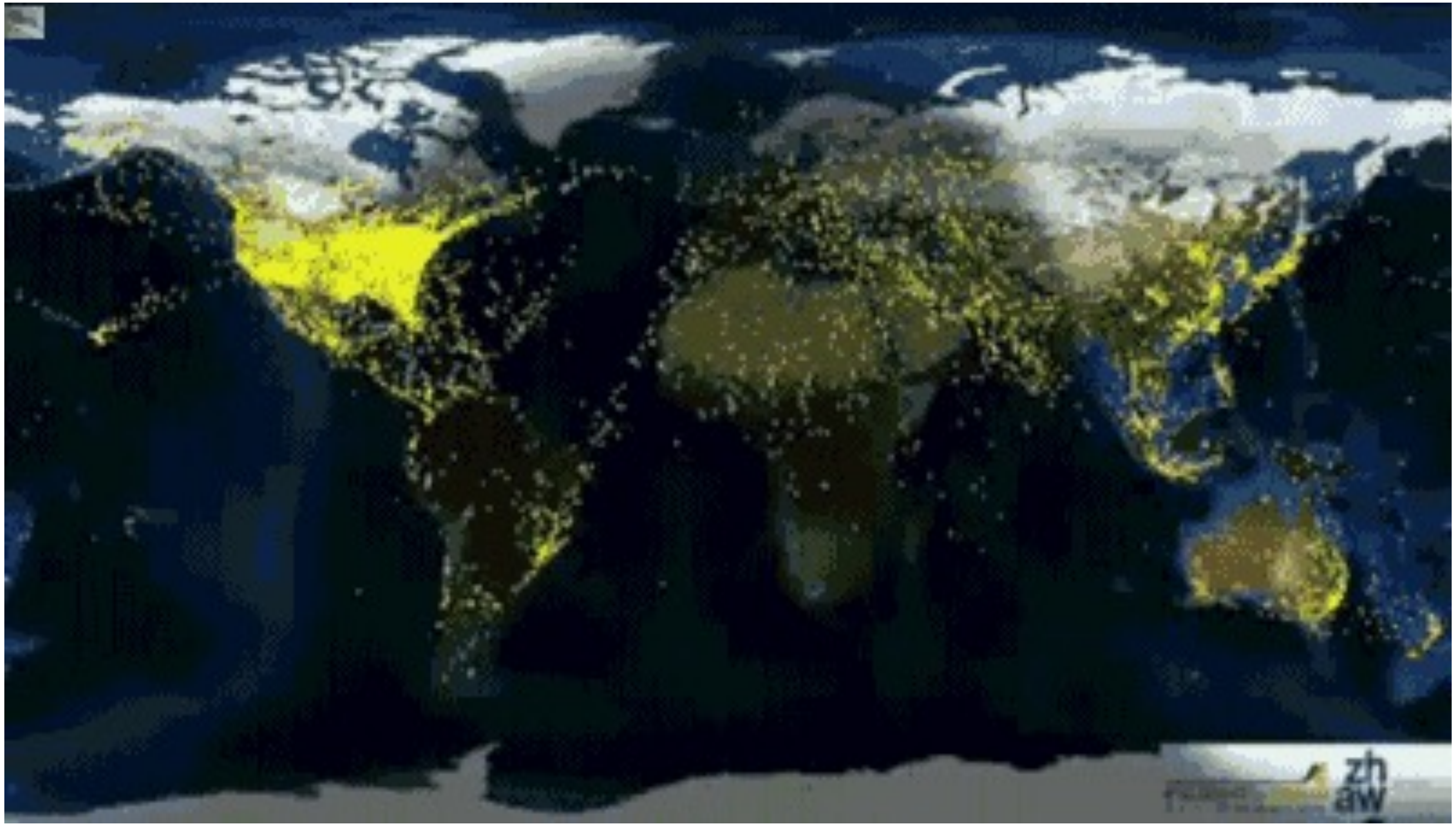
Entries: number of flights per day connecting source *S* to destination *D*, divided by total number of flights leaving *S*.

What is the empirical probability that a flight is headed to London, given that it is leaving from Paris?

Transition Probability Matrix:

	D_1	D_2	D_3	
S_1				
S_2				
S_3				

Each row should sum to 1



Commercial flight paths over 24 hours from FlightStats (Rege, 2008).

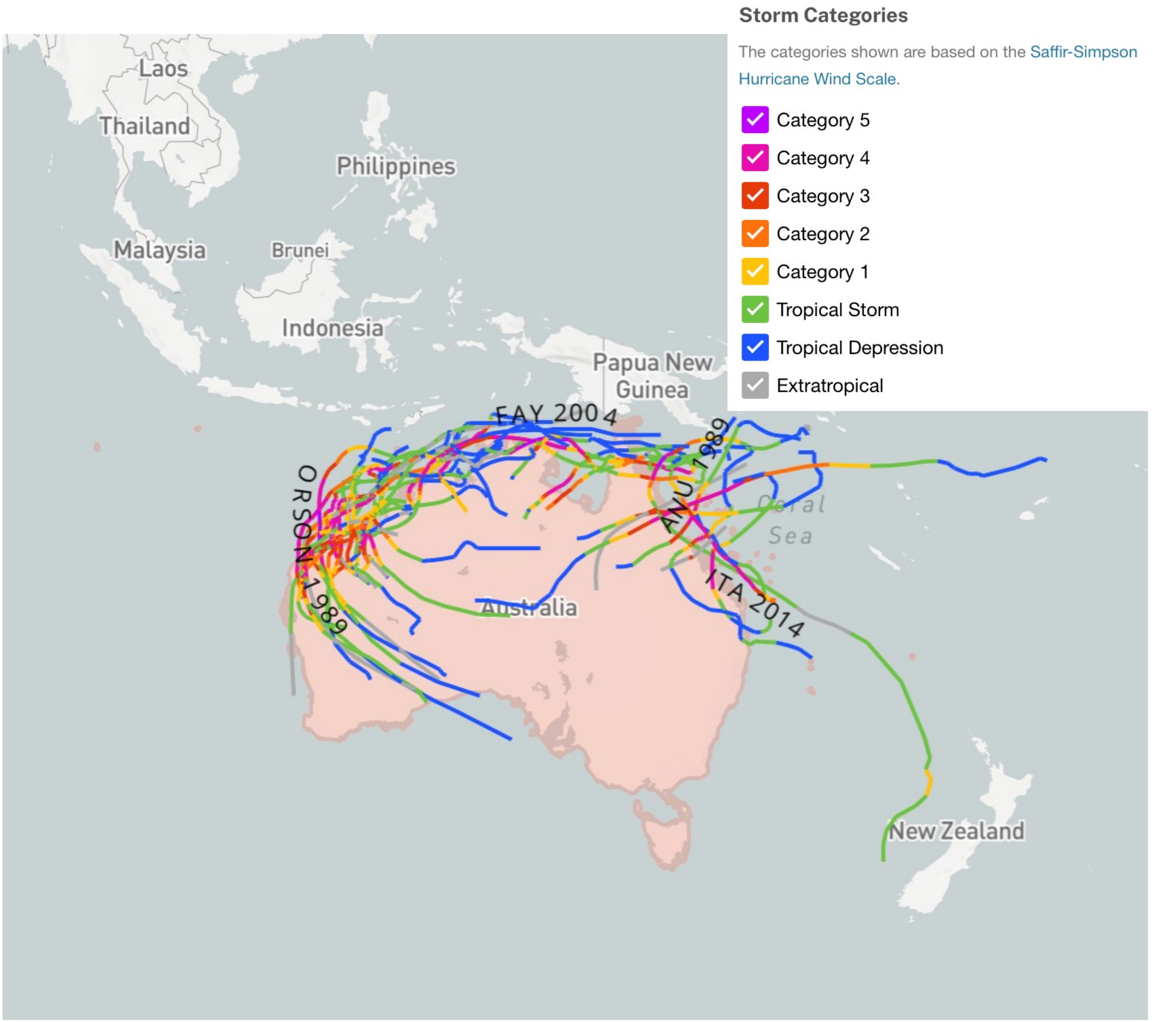
Types of Spatiotemporal Data

2. Trajectory data

Can also include variables of interest (here tropical cyclone strength) at each point along the trajectory.

Just the source and destination are not enough to describe this.

What are some ways to organize this data?



IBTraCS: historical tropical cyclone paths of all tropical cyclones >category 3 that made landfall in Australia.

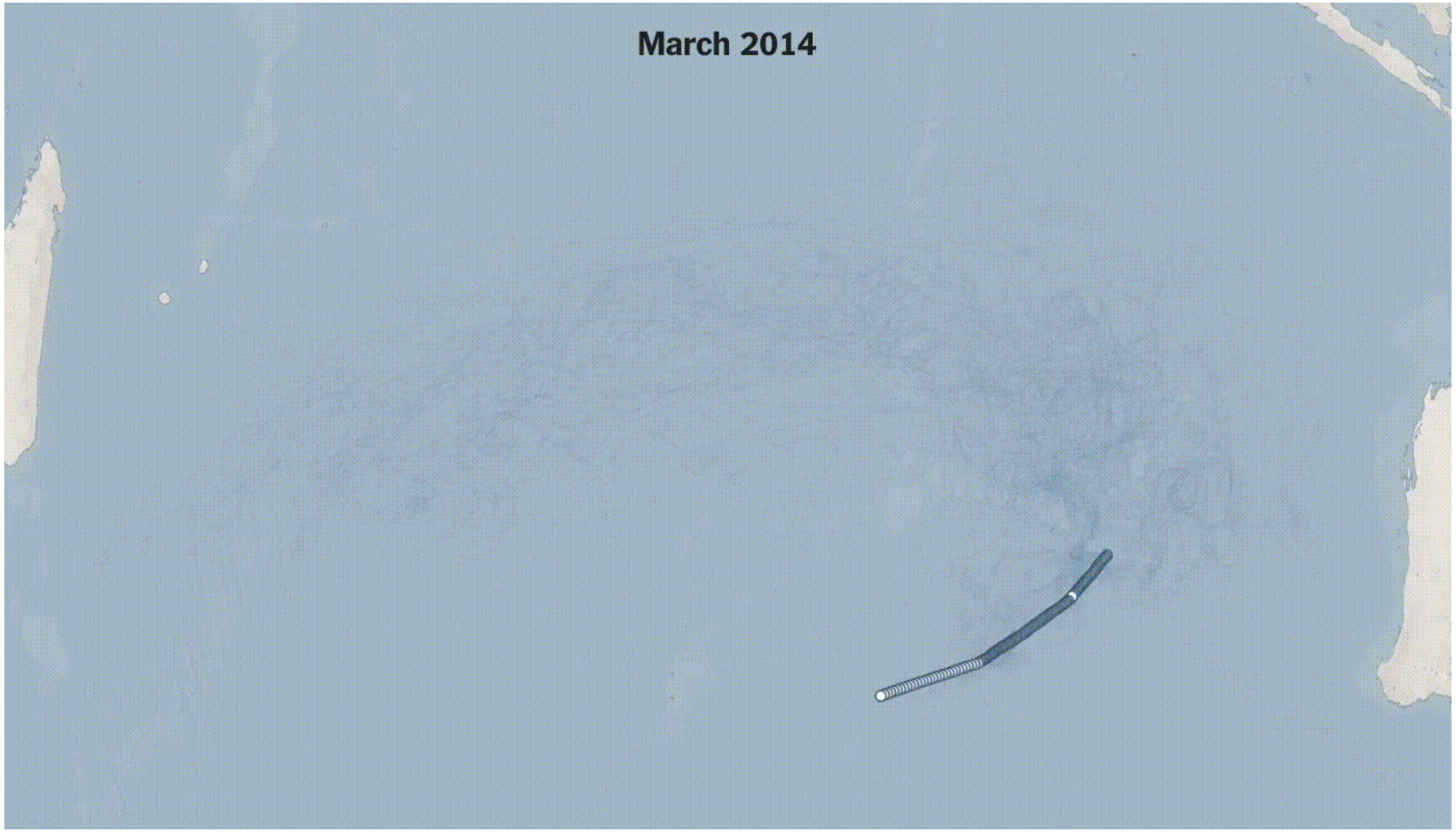
Types of Spatiotemporal Data

2. Trajectory data

Rate of spreading, velocity, distances between trajectories may also be of interest.

Simulated trajectory data is often used in estimating the location of marine debris.

Including biological material, plastic pollution, oil spills.
In this case:
debris from MH370.



[New York Times, 2015]

Types of Spatiotemporal Data

2. Trajectory data

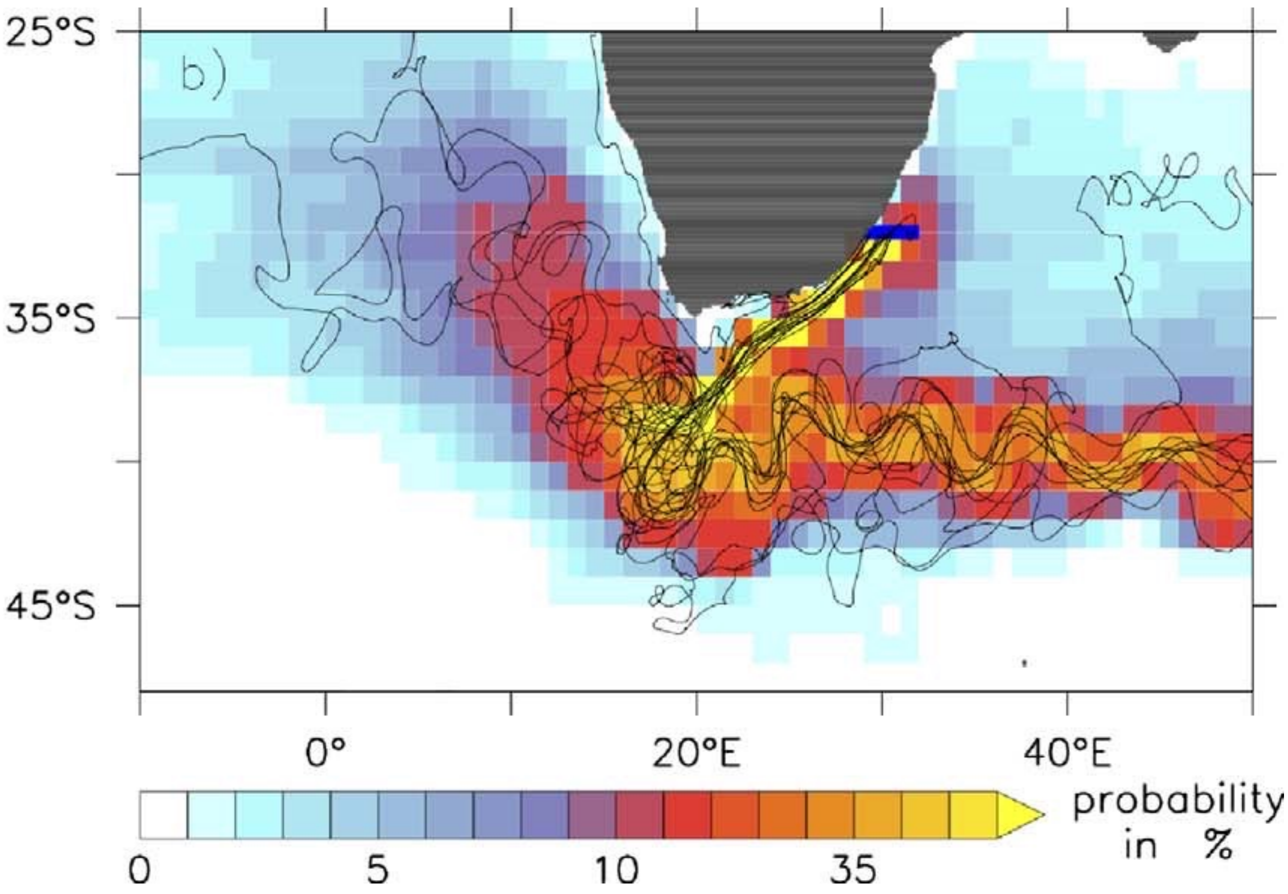
Rate of spreading, velocity, distances between trajectories may also be of interest.

Visualizing the most likely pathway:

Discretize the space into grid boxes

Calculate

$$\frac{Number\ of\ particles\ that\ pass\ through\ grid\ box\ i,j}{Total\ number\ of\ particles}$$



vanSebille et al., 2017: A study of (simulated) particles released off the east coast of South Africa. Colors indicate the empirical probability that a particle will pass through each bin.

Types of Spatiotemporal Data

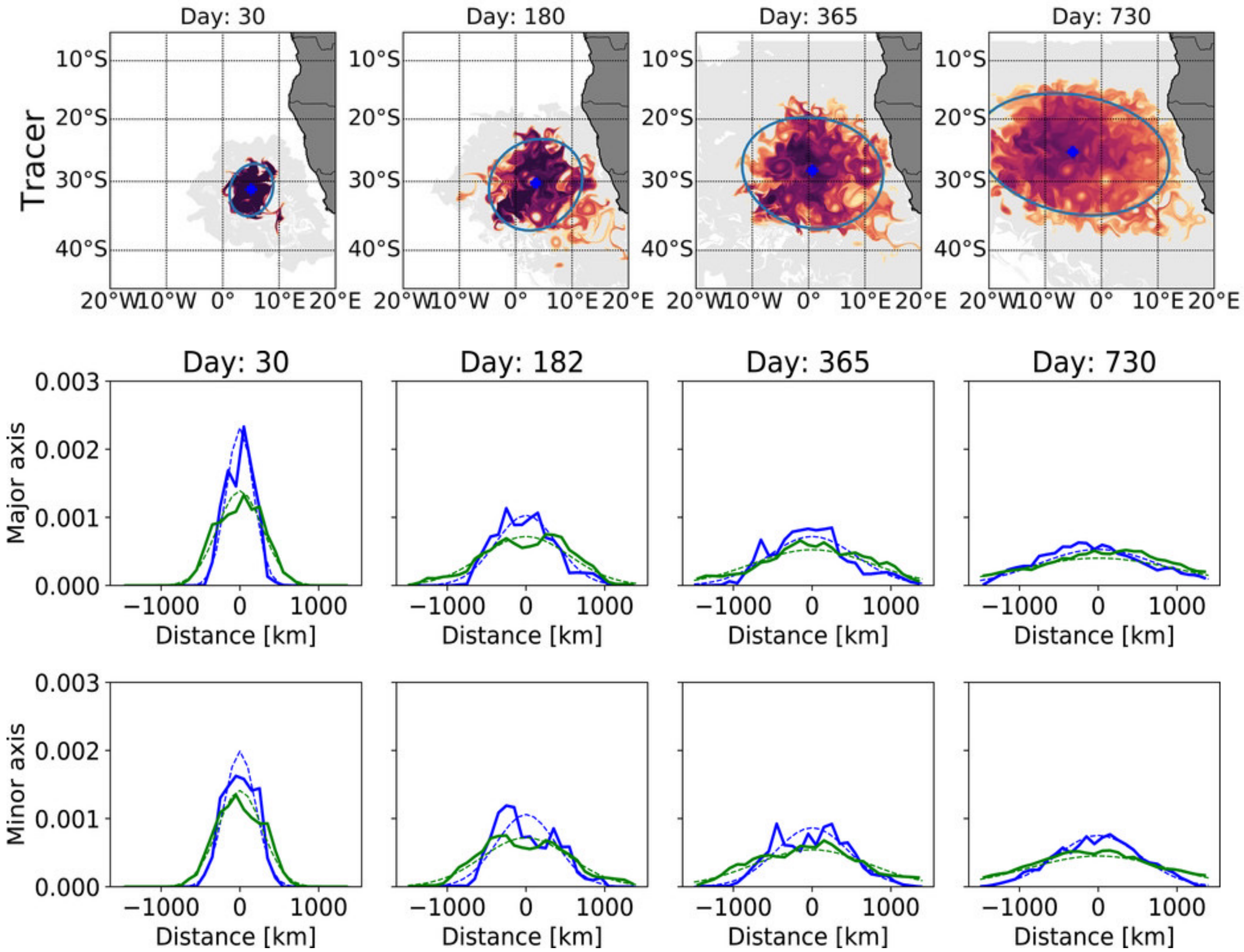
2. Trajectory data

How far do particles travel over time?

How do particle concentrations evolve along a single spatial dimension?

Lyapunov exponents and other tools from dynamical systems theory can be useful for analyzing trajectory spreading.

Remember that in spatial dimensions, distance between two points is measured as the Euclidean or great circle distance!



[Wagner et al, 2019]

Trajectory data: Summary

Depending on the question,

- Can be converted to a **graph** if source and destination are of interest
- Can use **spatial binning** to identify spatial patterns along trajectories
- Can **marginalize** along a single spatial dimension to examine spreading

- Tools from dynamical systems theory can help quantify trajectory uncertainty (e.g., Lyapunov exponents).
- Plenty of applications: flows of traffic, goods, pollutants, biological materials, fluids…

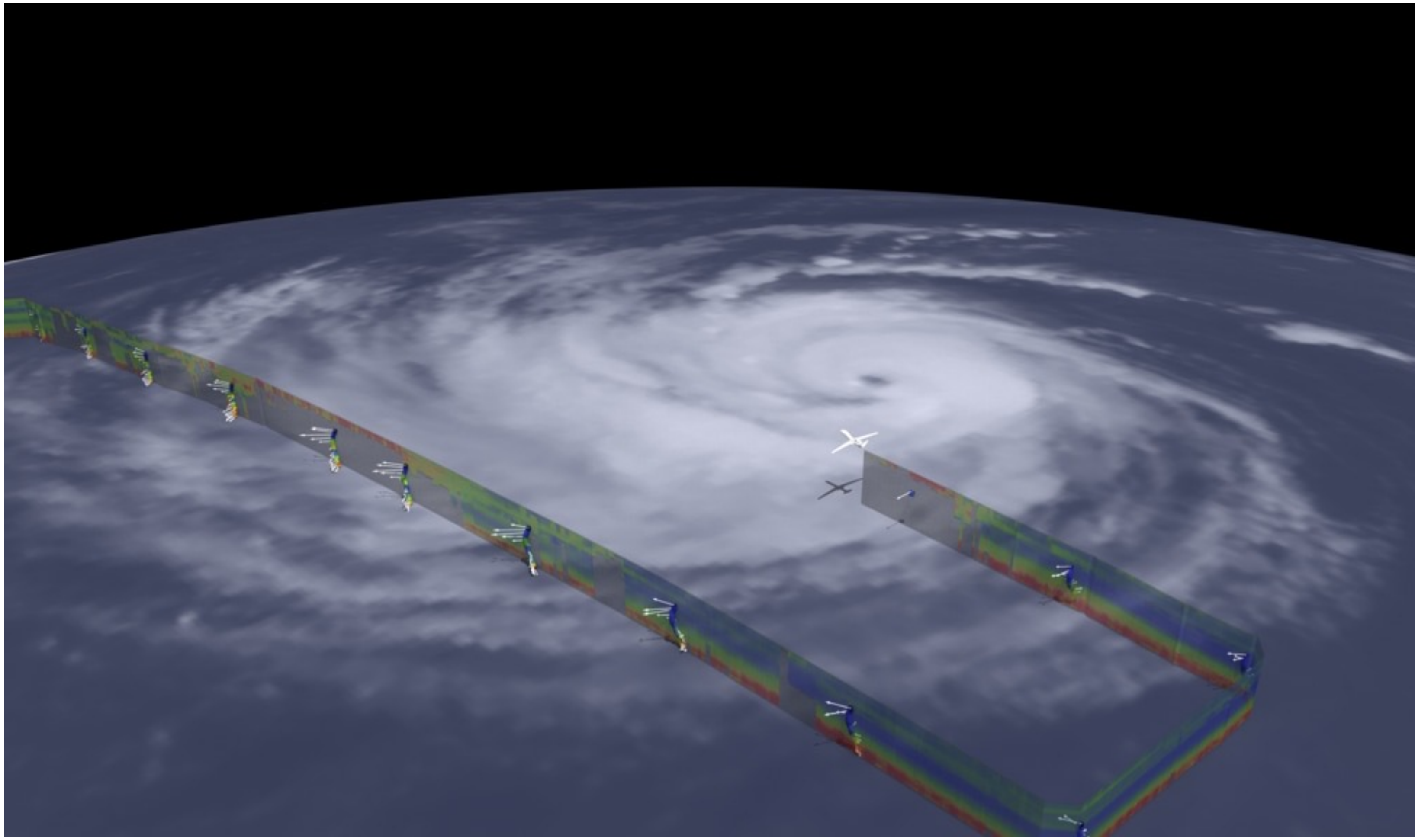
Types of Spatiotemporal Data: Track Data

A sensor traverses the spatial domain, providing measurements over time.

Typically when the sensor (or deploying it) is high-effort/expensive.

E.g.

A NASA mission that flew a plane through a hurricane, deploying sensors that fell to the ocean while making measurements.



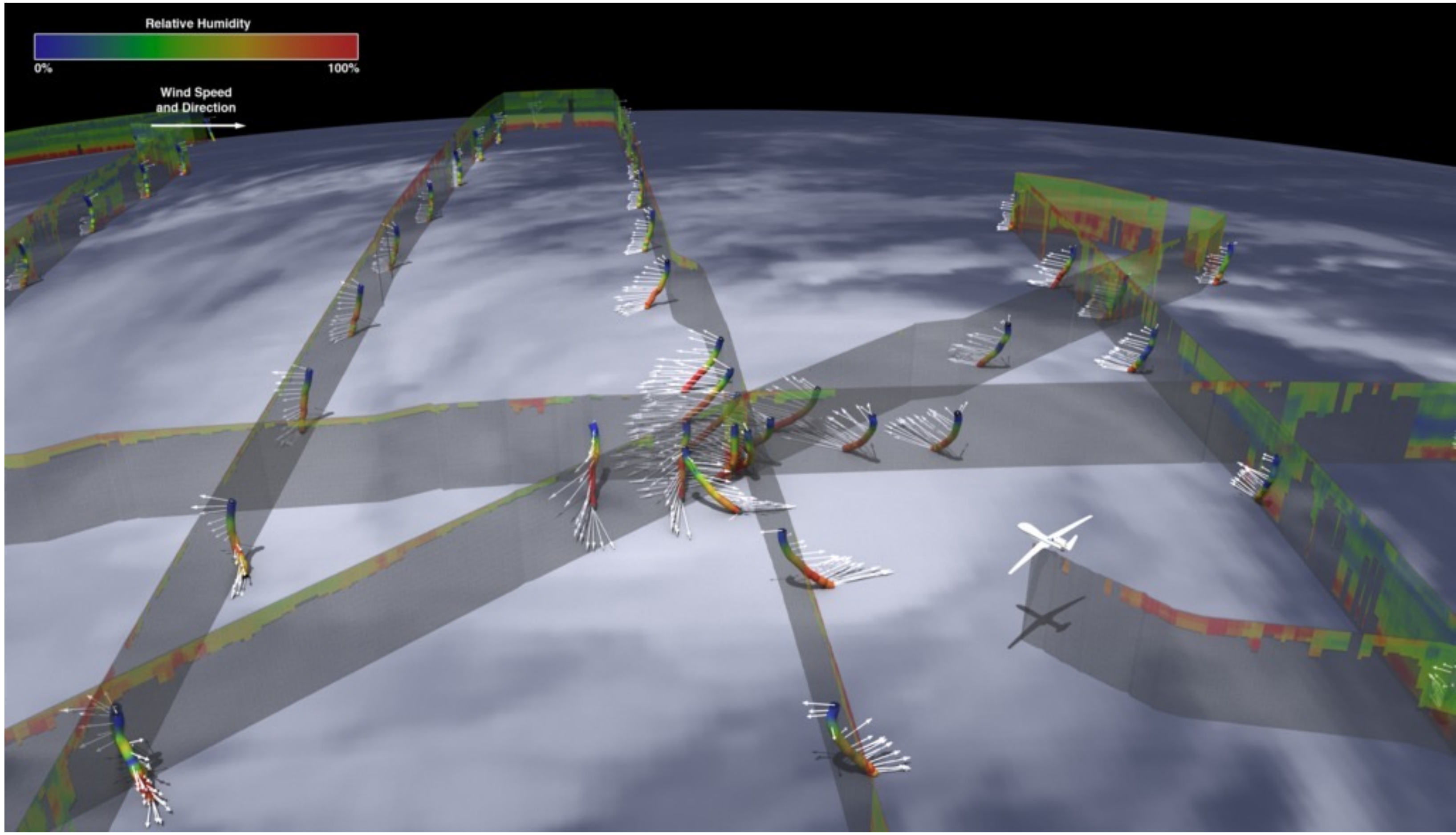
[NASA Visualization Studio: Global Hawk mission, 2014, Hurricane Edouard.]

Types of Spatiotemporal Data: Track Data

A sensor traverses the spatial domain, providing measurements over time.

Complicated, irregular trajectories.

How do we choose the right track to sample the space well (supplemental lecture)?



[NASA Visualization Studio: Global Hawk mission, 2014, Hurricane Edouard.]

Types of Spatiotemporal Data: Summary

We've seen 3 categories of spatiotemporal data:

1. **Field data**
   - **Indexed by spatial location, often on a co-ordinate system**

2. Trajectory data
   - Indexed by moving objects – particles, debris, fluid parcels, vehicles

3. Track/irregular data
   - Indexed by irregularly-spaced co-ordinates, generally constrained by the movement/capability of instruments

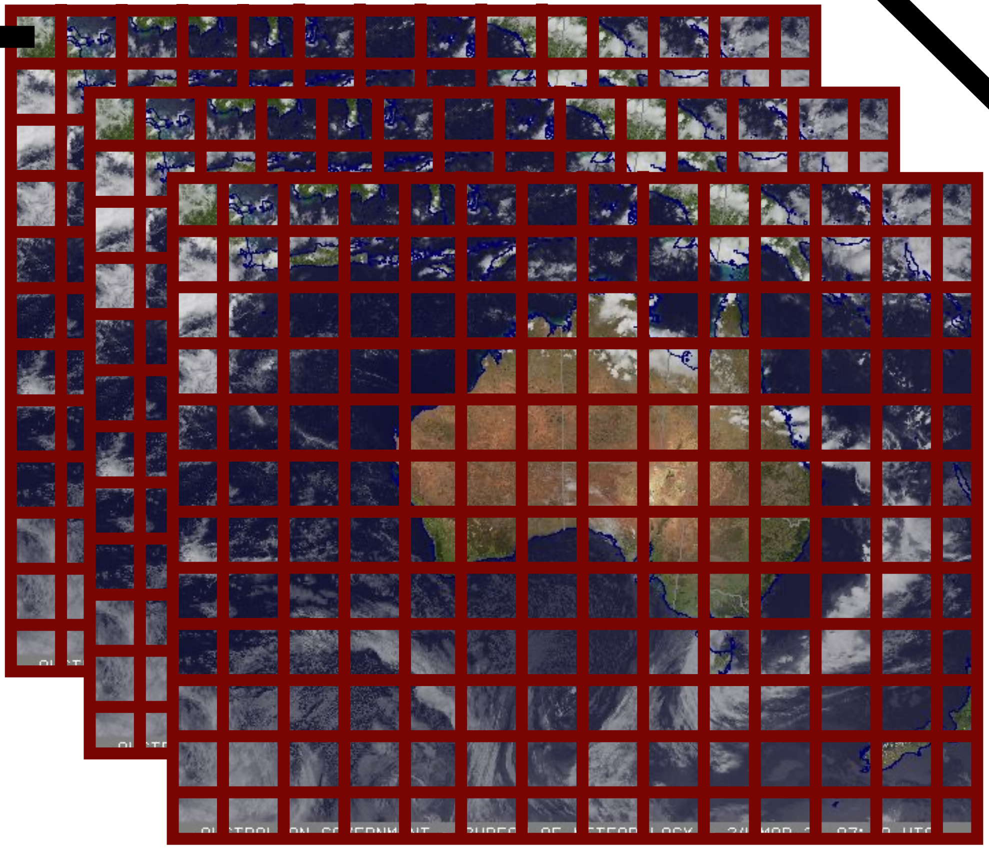
Exploring Spatiotemporal Data

Spatial fields

Co-ordinate Systems

Spatiotemporal field data is generally stored as a matrix, where indices correspond to co-ordinates.

Element
i,j,k



Time

In general, up to 3 space dimensions + 1 time dimension

E.g., ocean temperature

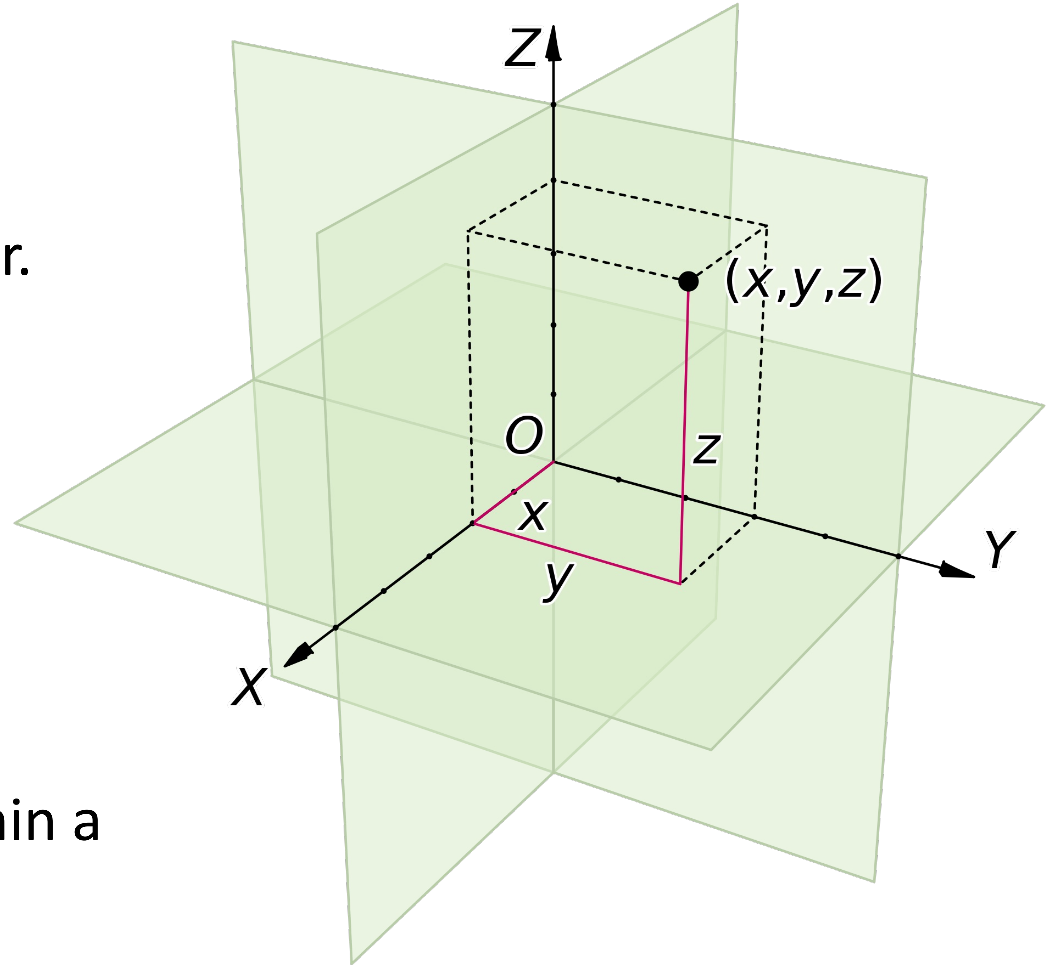
Co-ordinate Systems

- Simple Cartesian Co-ordinates

3 possible axes, at right angles to each other.

Convention:
X horizontal on the page
Y vertical on the page
Z out of the page

Discretized on a grid, so each point lies within a grid cell specified by x,y,z at its centre.



Co-ordinate Systems

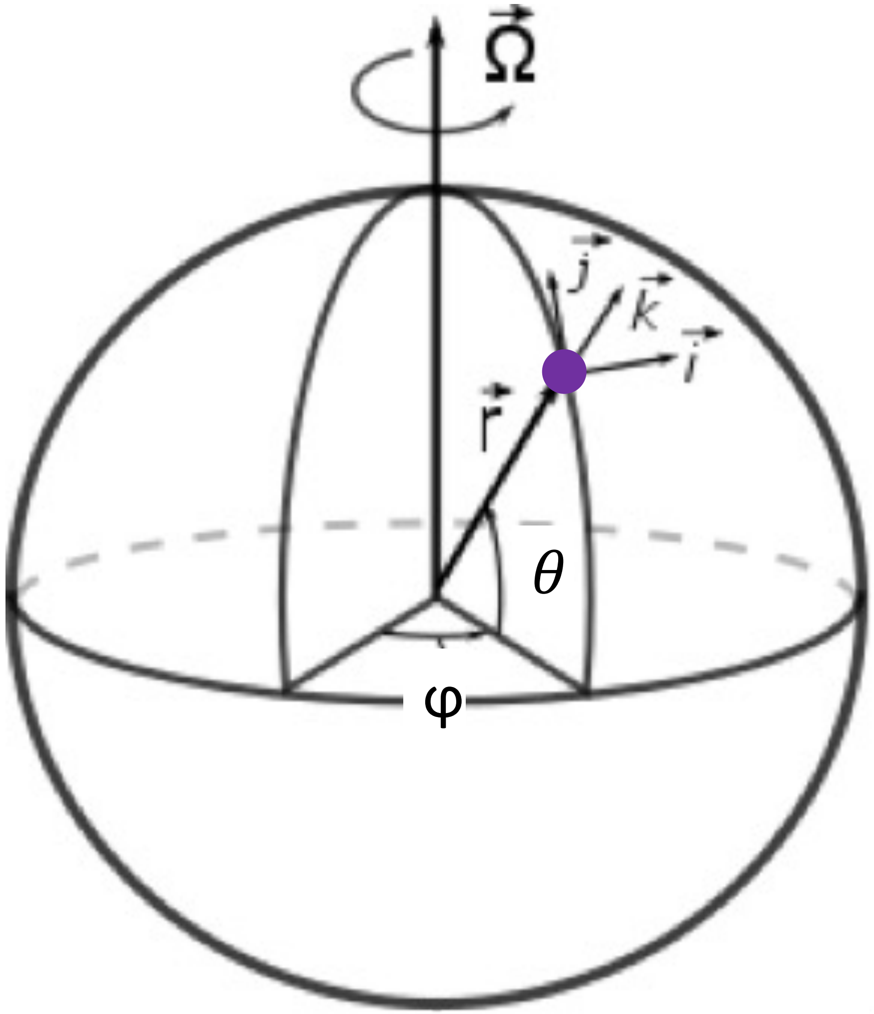
- Spherical Co-ordinates

BUT, the Earth is a sphere! (approximately)

At large enough scales, this matters.

Instead of Cartesian co-ordinates x,y,z, which represent the distance from the origin along a line, we use spherical co-ordinates (φ,θ,r).

The origin is at the centre of the sphere.



Co-ordinate Systems

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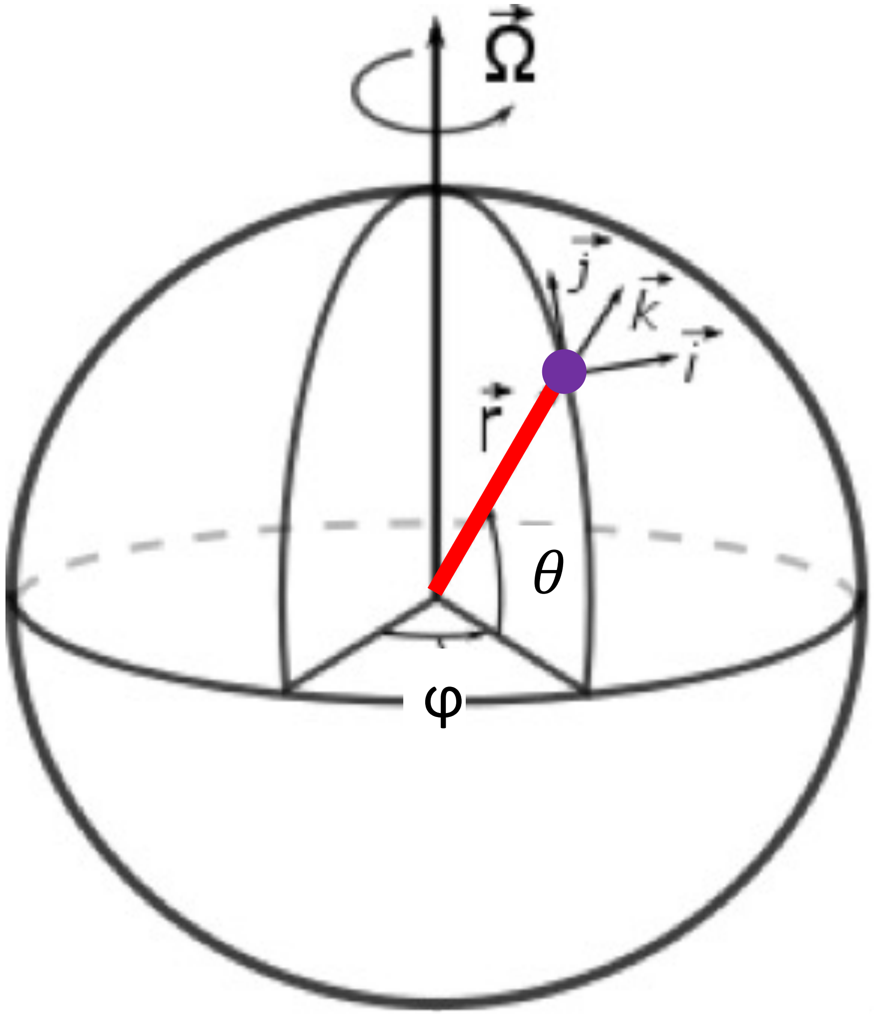
Instead of Cartesian co-ordinates x,y,z, which represent the distance from the origin along a line, we use spherical co-ordinates (φ,θ,**r**).

The origin is at the centre of the sphere.

r is the distance of a point from the origin.

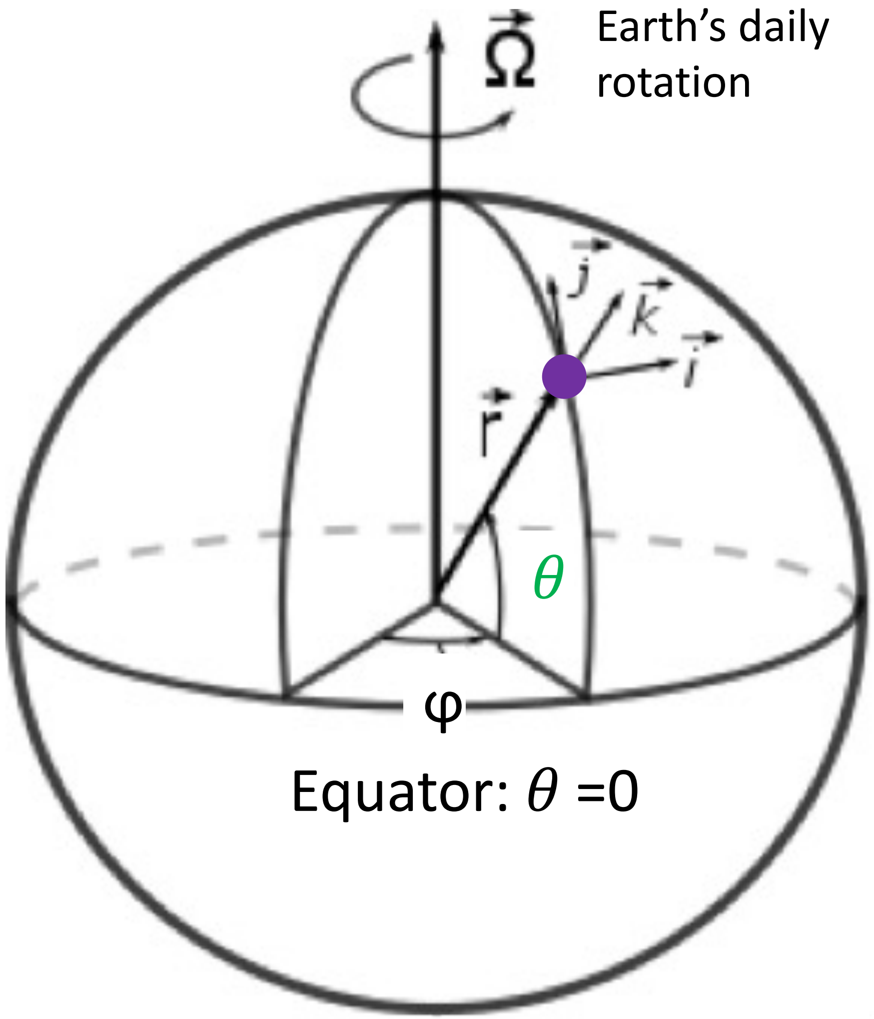
The radial co-ordinate r is the only co-ordinate that can be measured as a distance.



Co-ordinate Systems

- Spherical Co-ordinates

The other 2 co-ordinates are measured as angles with respect to planes.

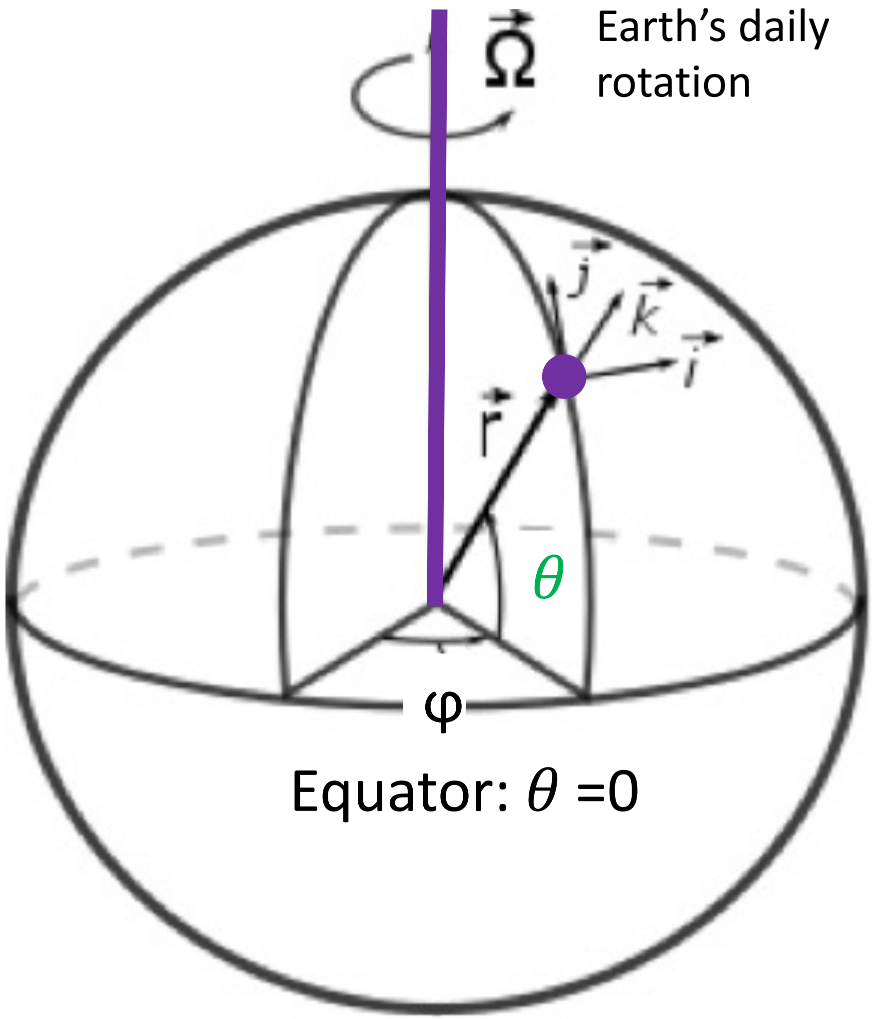


Co-ordinate Systems

- Spherical Co-ordinates

The other 2 co-ordinates are measured as angles with respect to planes.

We choose a line that passes through the origin as a reference. On Earth, it makes sense to use the axis of rotation.



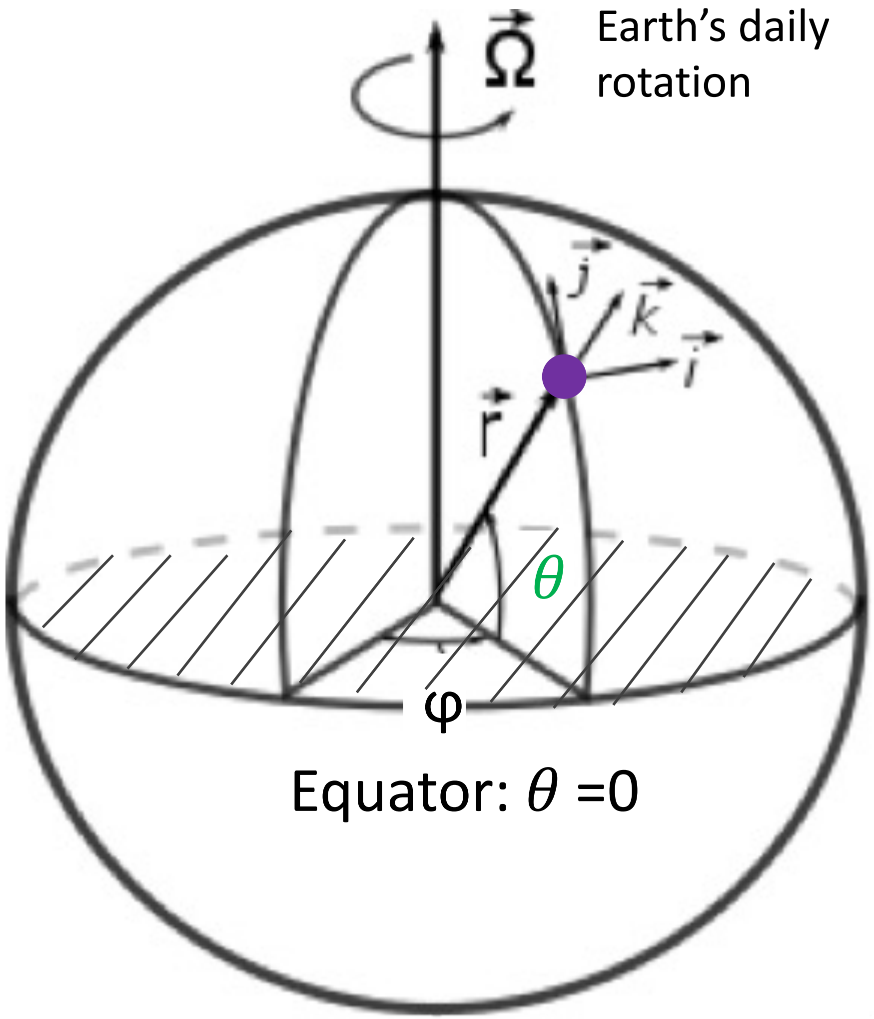
Co-ordinate Systems

- Spherical Co-ordinates

The other 2 co-ordinates are measured as angles with respect to planes.

We choose a line that passes through the origin as a reference. On Earth, it makes sense to use the axis of rotation.

The plane perpendicular to this line, and passing through the origin, is the Equator, where θ = 0.



Co-ordinate Systems

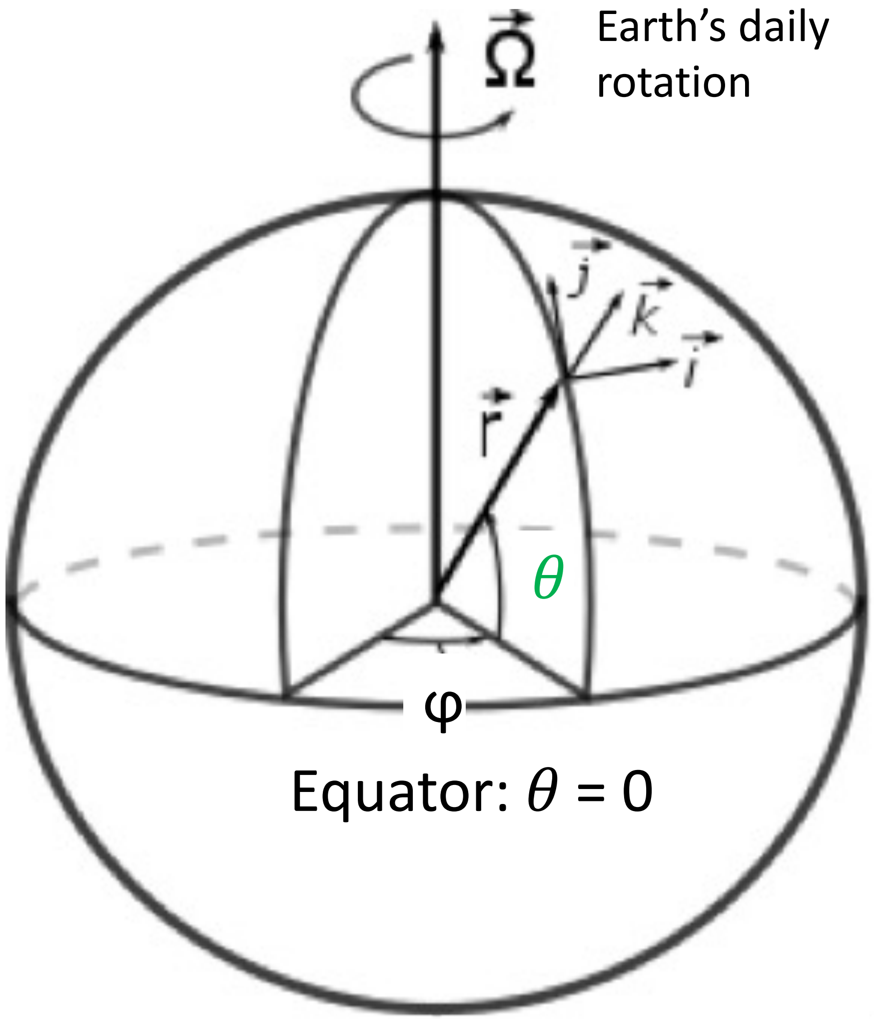
- Spherical Co-ordinates

θ is then measured as the angle between the line connecting a point to the origin, and the equatorial plane.

What's θ at the North Pole?

θ is sometimes called the elevation angle.

On Earth, this is **latitude**.



Co-ordinate Systems

- Spherical Co-ordinates

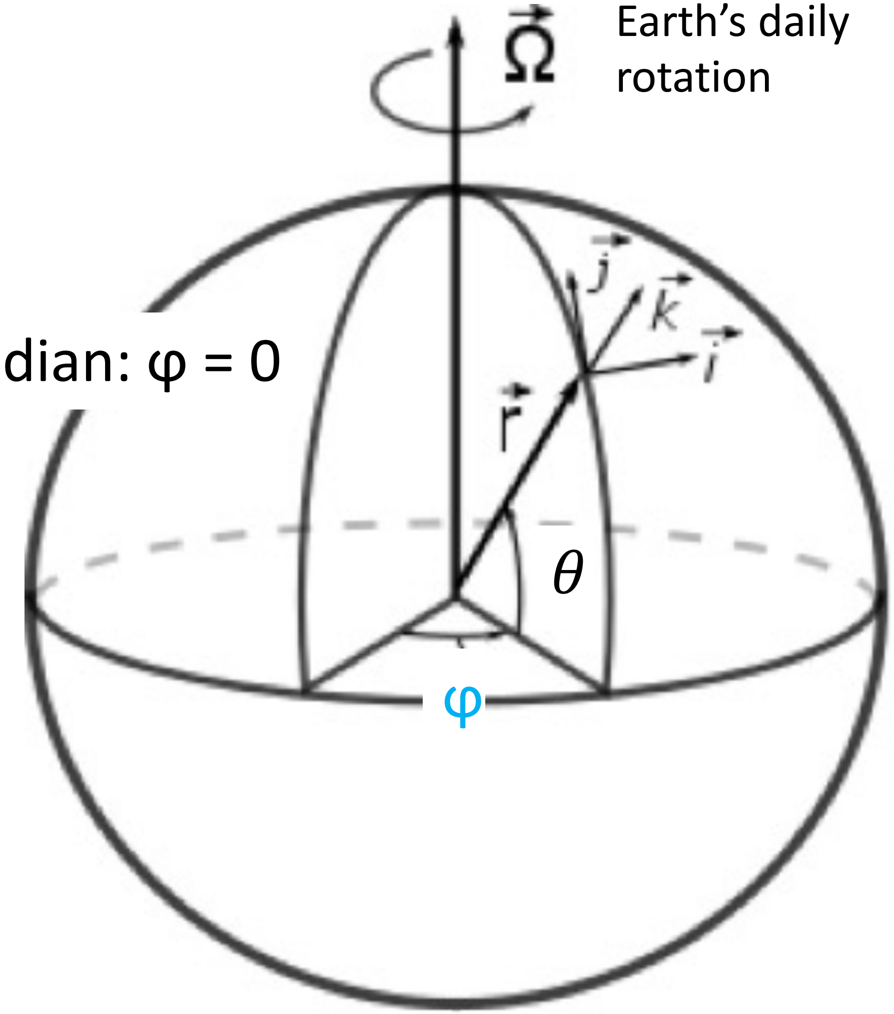
φ is defined based on a plane that passes through the origin, and must be perpendicular to the equatorial plane.

Prime Meridian: φ = 0

The choice of plane is arbitrary. On Earth, we use the Prime Meridian.

φ or the azimuth is then the **angle** between the projection of the r vector onto the equatorial plane and this plane.

On Earth, we call φ **longitude**.



Co-ordinate Systems

- Spherical Co-ordinates

One more thing about r:

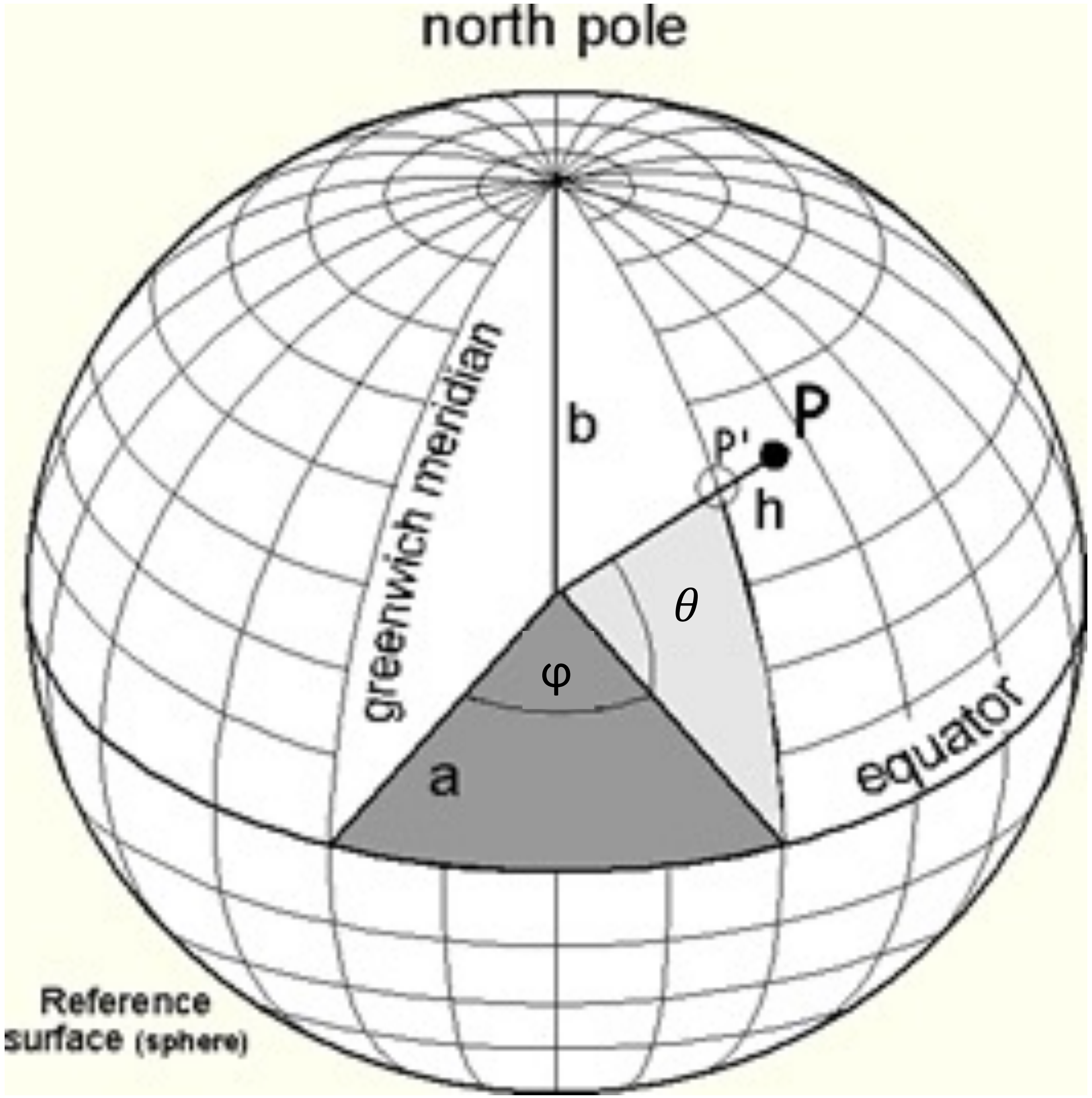
For objects near the Earth's surface, we subtract the radius of the Earth.

Most of what we study will be close to the surface:

The oceans are at most 11 km deep.

Most weather happens below 15 km.

The radius of the Earth is **6371 km**.



Co-ordinate Systems

- Spherical Co-ordinates summary:

3 Co-ordinates:

φ = longitude

Lines run north-south

Ranges from -180 to 180

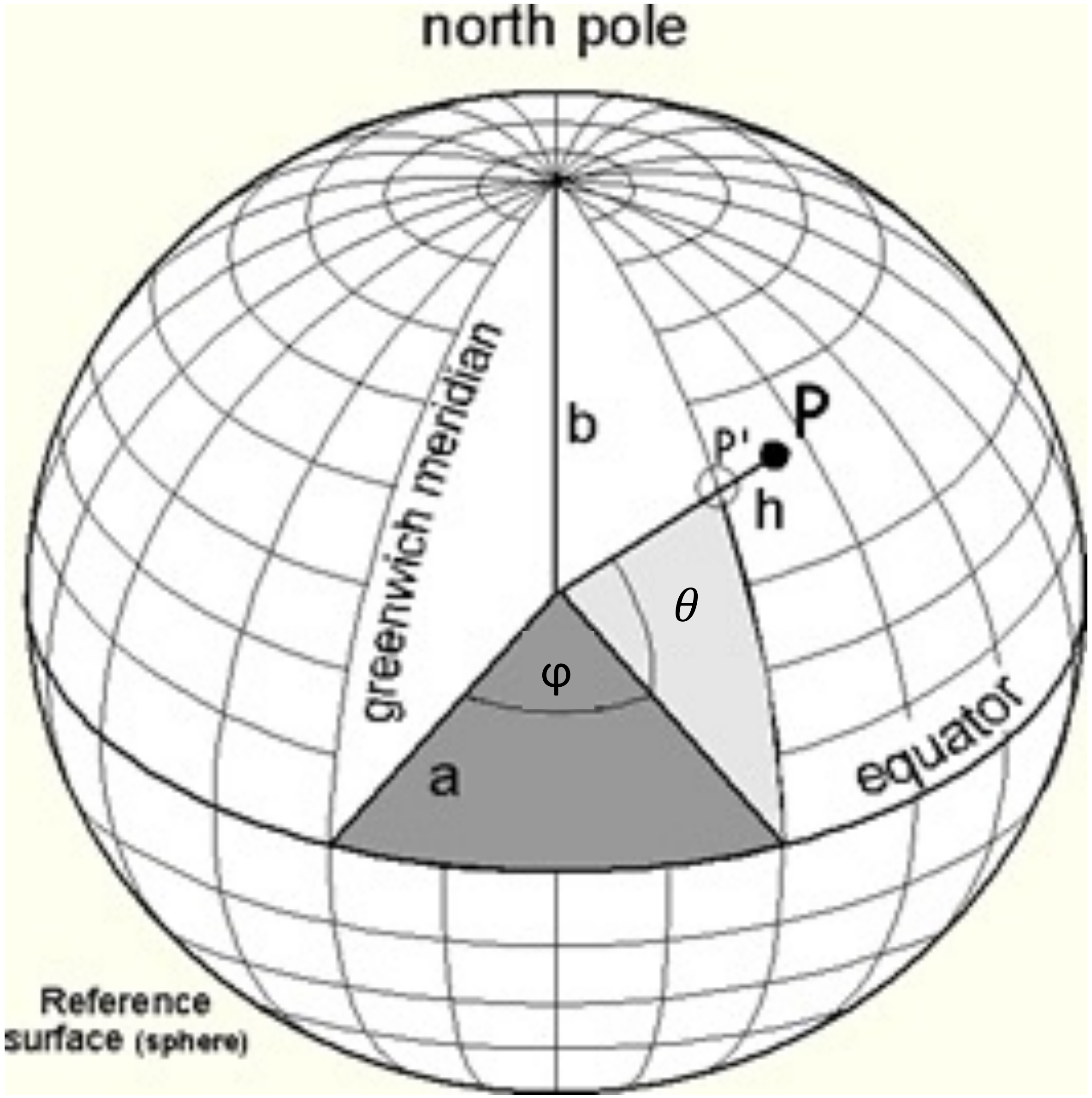
θ = latitude

Lines run east-west

Ranges from -90 to 90

R or h = altitude

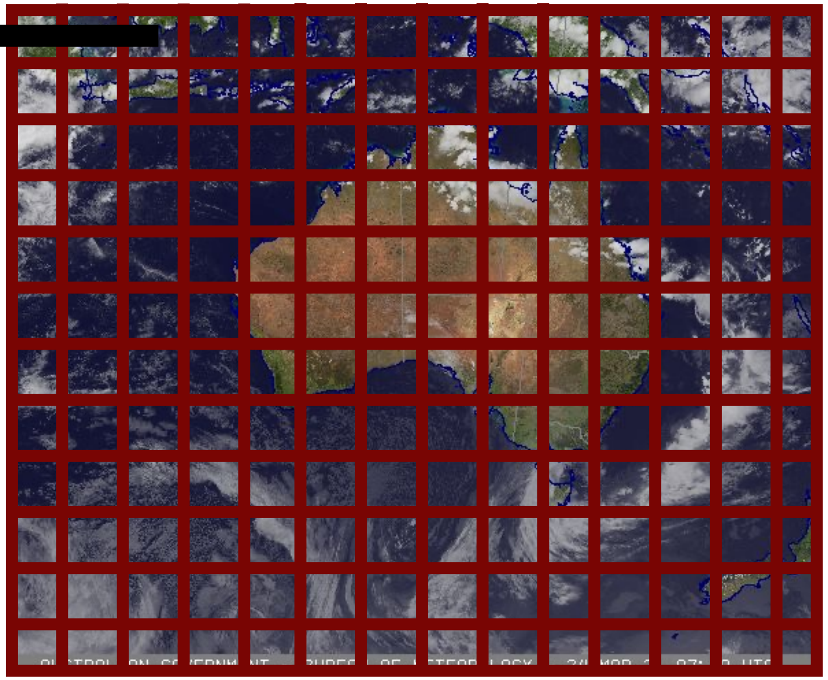
Measured from the surface of the Earth



Spherical Co-ordinate Systems and Indexing

Generally stored as a matrix, where indices correspond to grid cells in spherical coordinates.

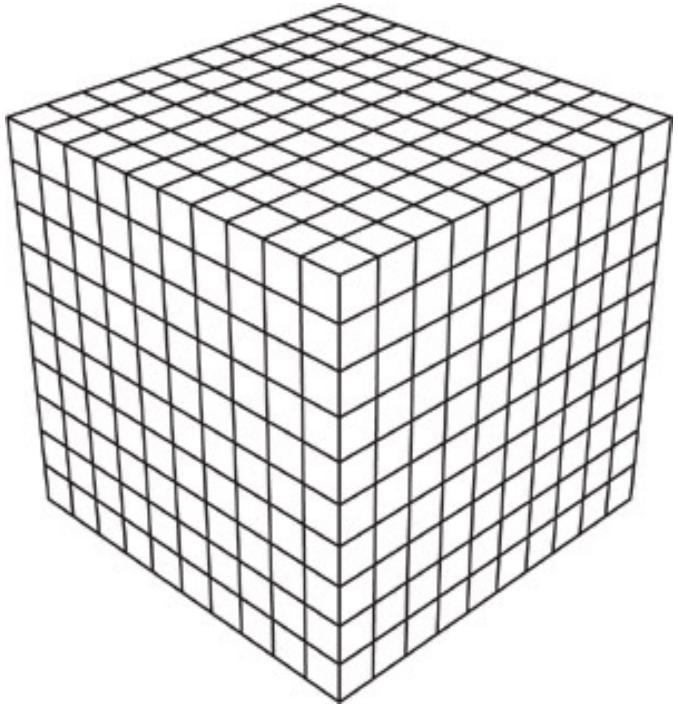
Element *i,j*
is at
longitude φ
latitude θ

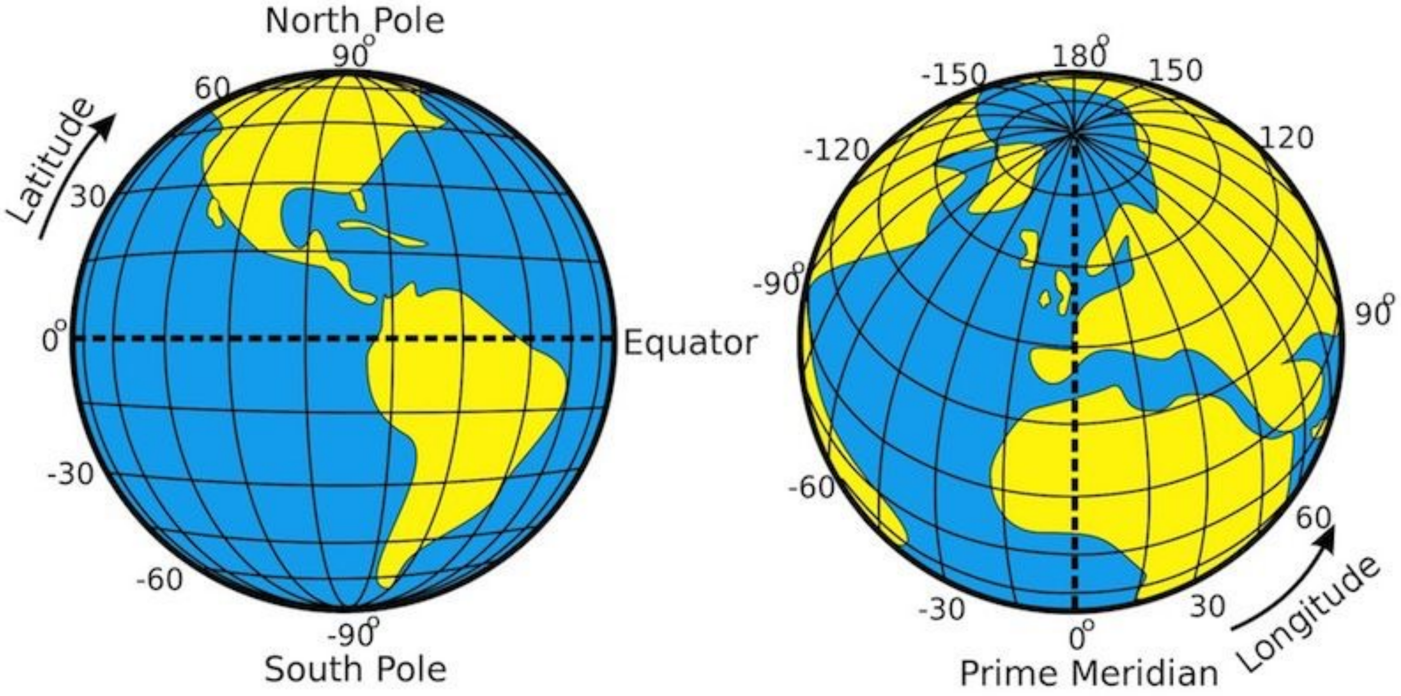


Element i,j may be either the average value over its grid cell or a measurement taken at the grid's center.

Spherical Co-ordinate Systems and Indexing

Unlike in Cartesian co-ordinates, grid cells in spherical co-ordinates **do not have equal volume.**





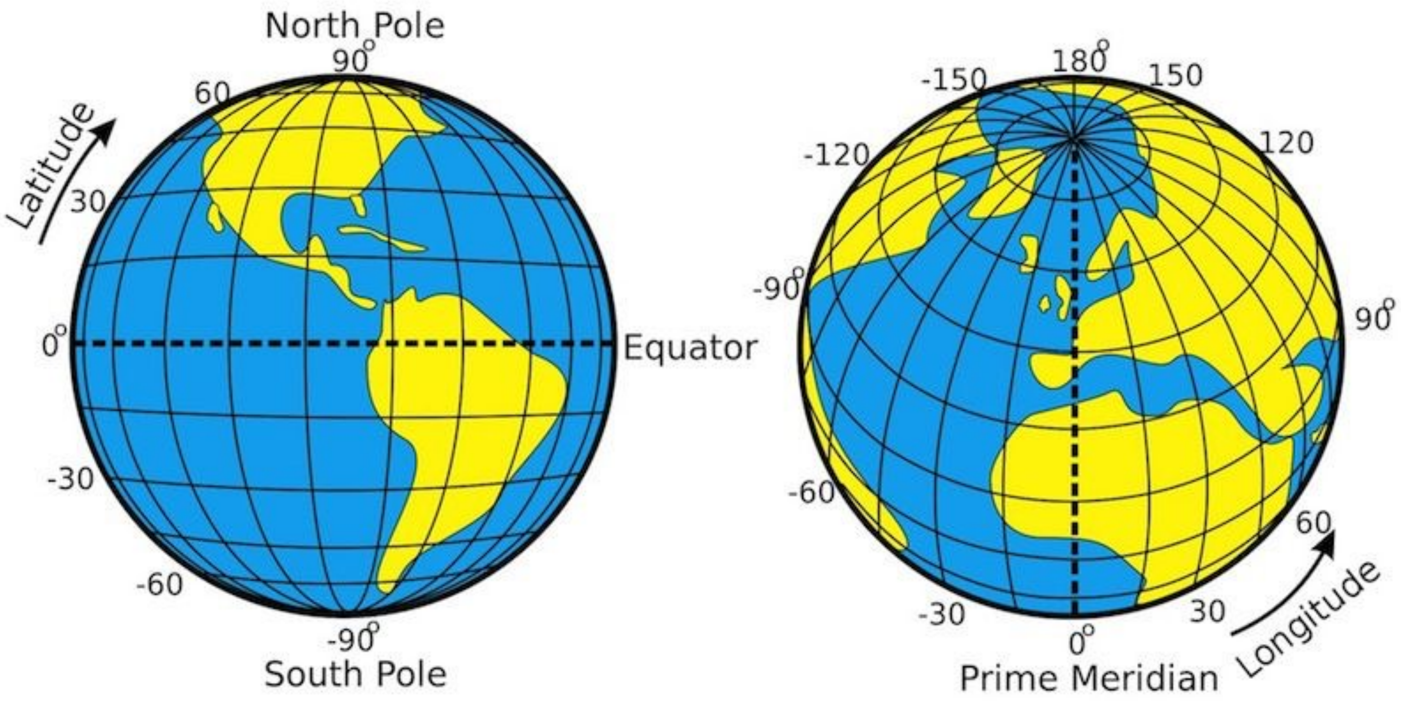
The distance between longitude lines goes to zero at the poles.
At the Equator, they are 111 km apart.

Spherical Co-ordinate Systems and Indexing

Unlike in Cartesian co-ordinates, grid cells in spherical co-ordinates **do not have equal volume.**

This means that elements of the data matrix can represent quantities averaged over a different volume.

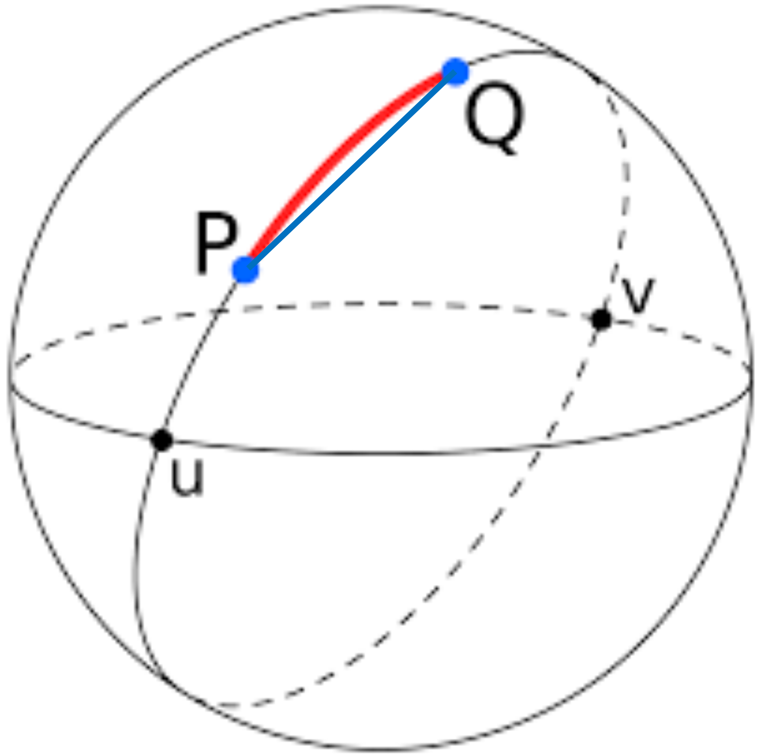
To measure distance or a spatial area, you need to correct for this. **It matters every time you calculate an average or sum!**



Spherical Co-ordinate Systems and Distances

Great circle distance

The shortest distance between P & Q on the Earth's surface is an arc along the great circle connecting the two points. *No burrowing through the Earth!*



A great circle on a sphere is a circle whose center is collocated with that of the sphere.

Lines of longitude are great circles. Lines of latitude, except for the equator, are not.

Spherical Co-ordinate Systems and Distances

Great circle distance

This is why long-haul flight paths are not straight lines on a map.

Spherical Co-ordinate Systems and Distances

Great circle distance

$$d = 2r\,arcsin\left(\sqrt{sin^2\left(\frac{\theta_2 - \theta_1}{2}\right) + cos(\theta_1)cos(\theta_2)sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right)}\right)$$

You can use pre-built functions to calculate this:

rdist.earth() from **fields** in R

Geodesic() from **geopy.distance** in Python

These functions also account for the ellipsoidal shape of the Earth: it's a little flatter at the poles.

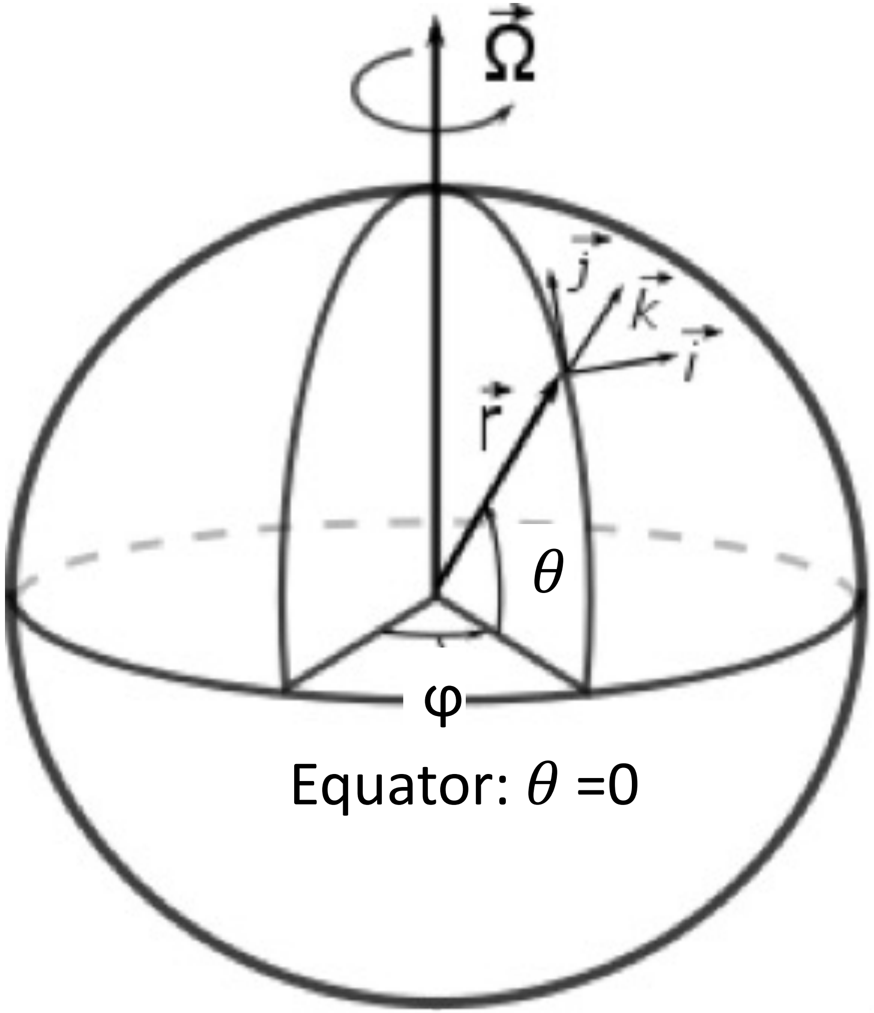
Spherical Co-ordinate Systems and Distances

A trick to calculate area-weighted quantities:

Multiply the field by $cos(\theta)$ at each θ (latitude), then calculate the quantity.

Intuition: This gives a weight of 1 at the equator and 0 at the poles.

Always remember to check units of longitude and latitude: degrees or radians?



Spherical Co-ordinate Systems and Distances

Another way around this issue: use different co-ordinates!

E.g. **UTM (Universal Transverse Mercator)**, which expresses distances in metres.

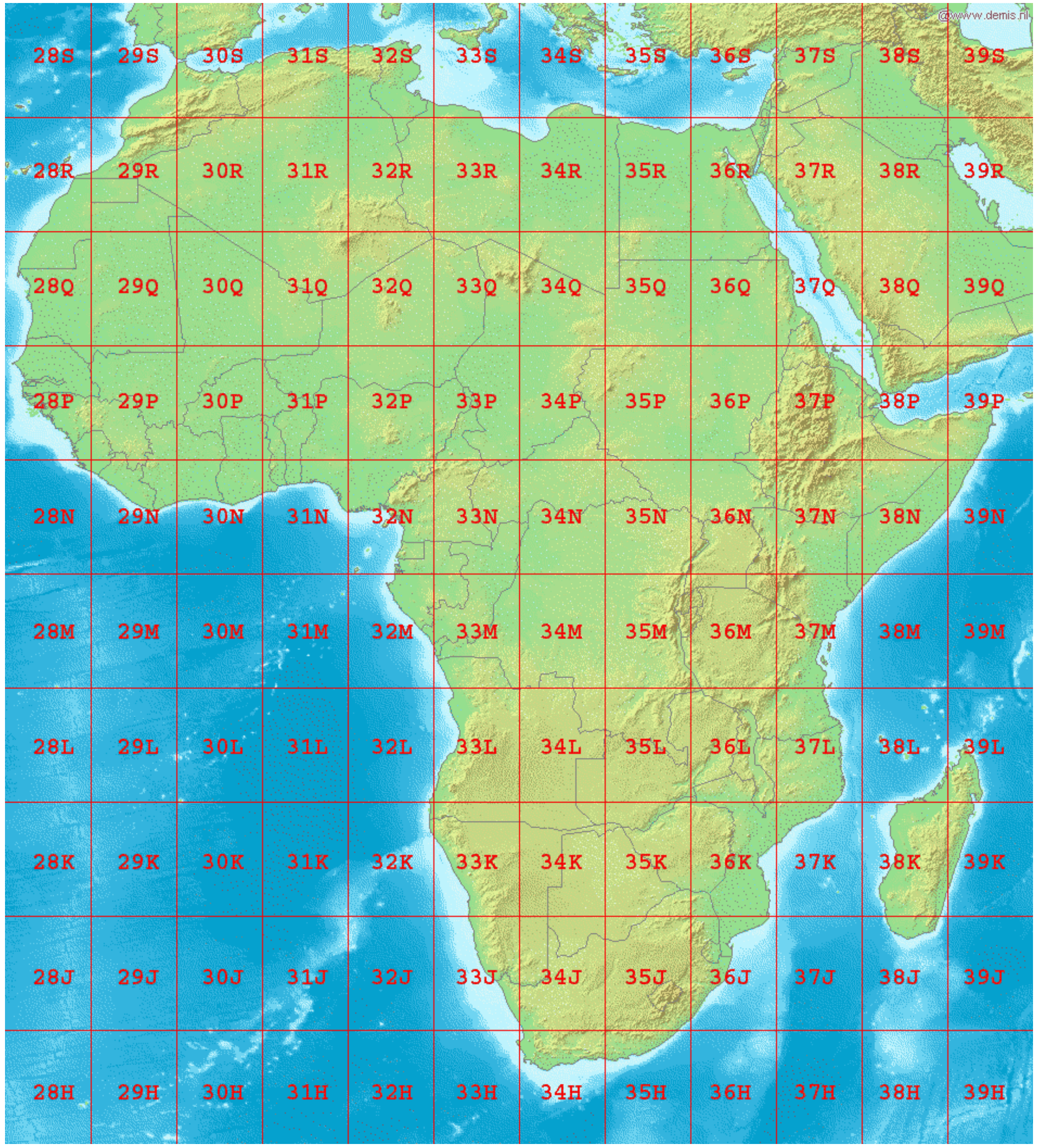
Numbers: E-W

Letters: S-N

Locations are expressed as distances east and north in metres from the southwest corner.

Disadvantage: not valid across box boundaries.

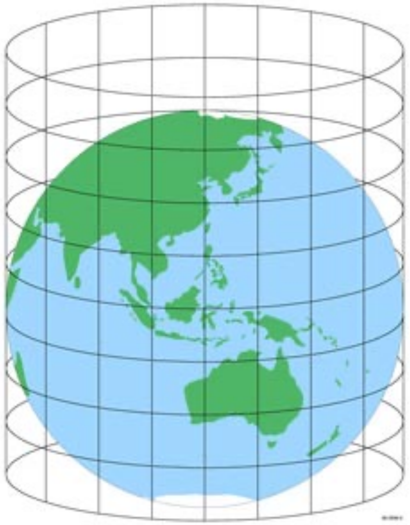
A good resource for working with co-ordinate reference systems: [*Geocomputation with R* by Lovelace et al.]



Visualizing a Spherical Surface

The Earth is a sphere, our screens are flat.

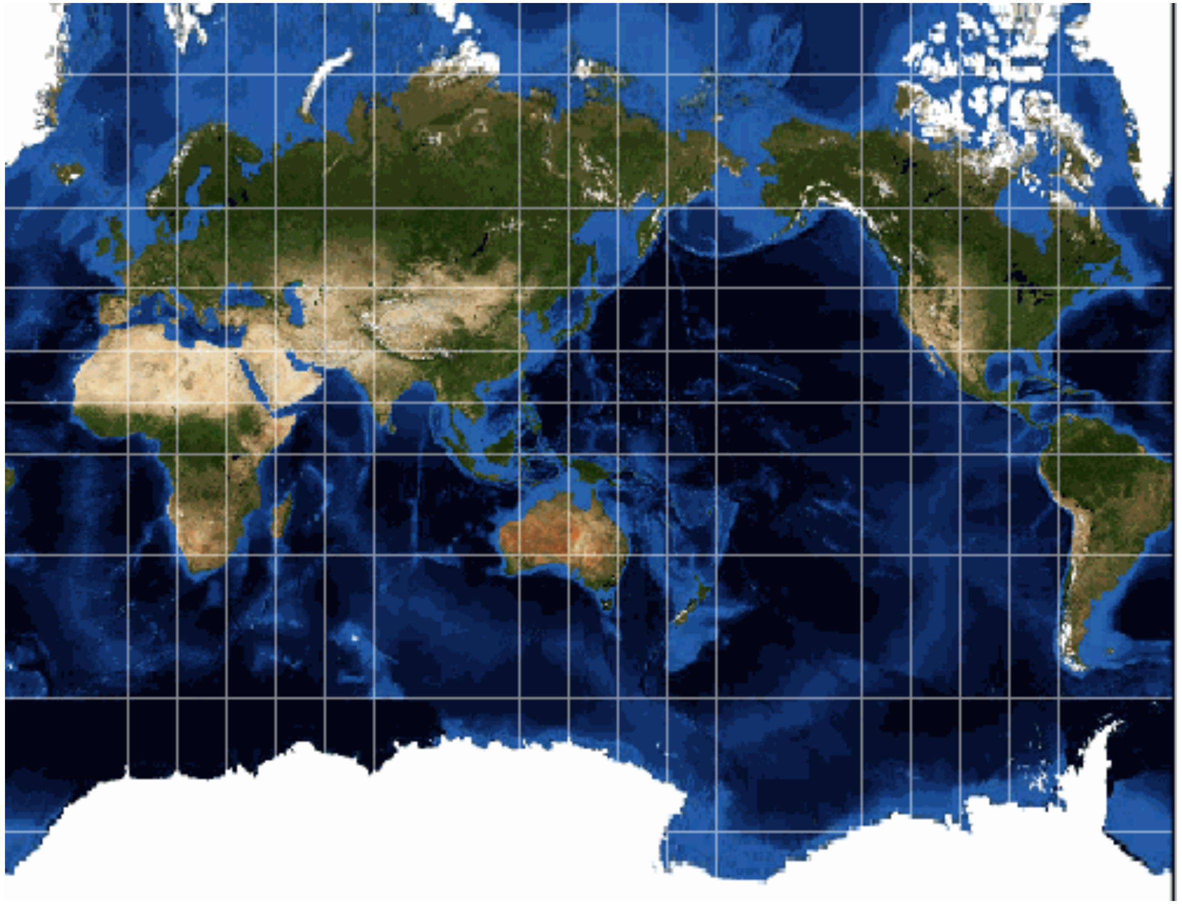
Finding the best map projection depends on what you're most interested in.



1. Mercator

good for

- areas near the equator
- navigation



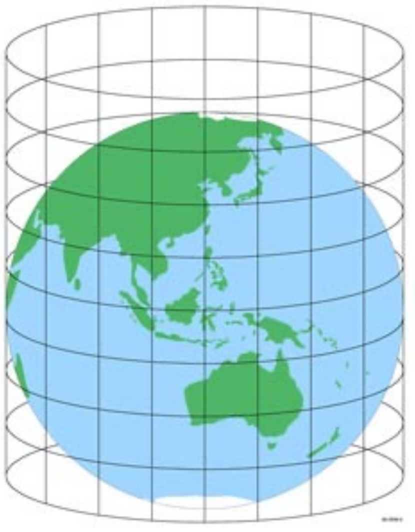
[Intergovernmental Committee on Surveying and Mapping]

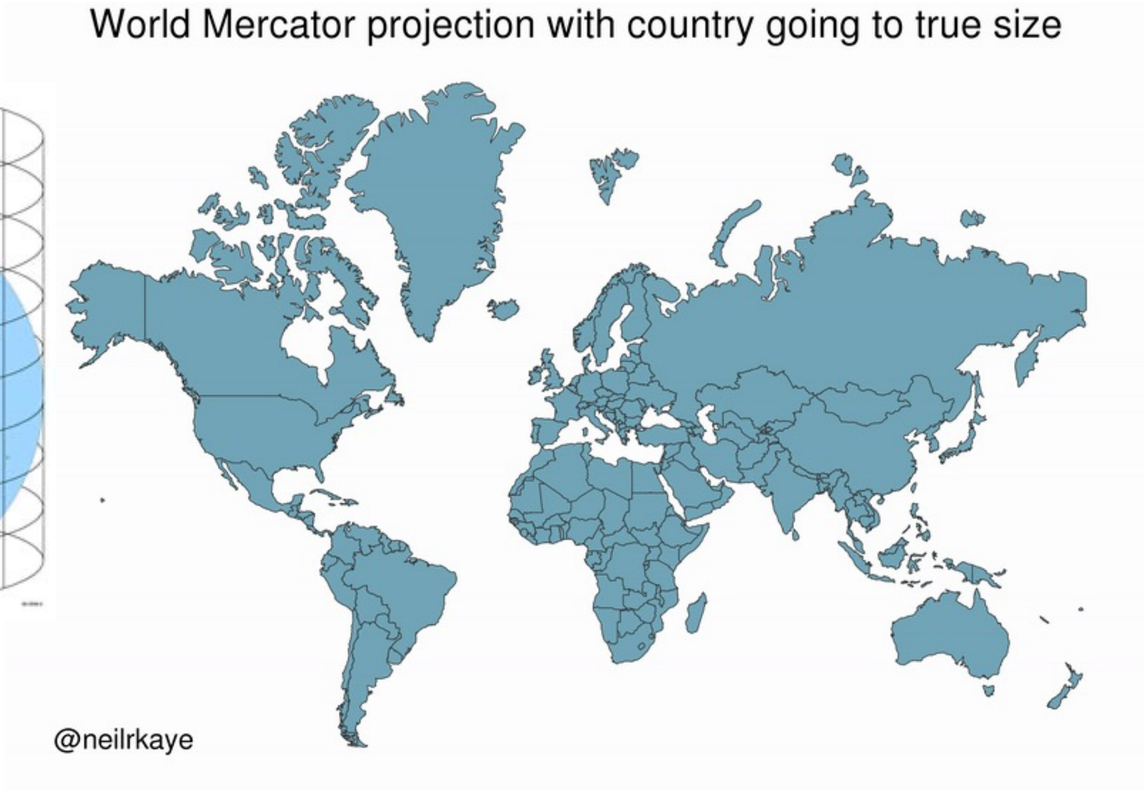
Visualizing a Spherical Surface

The Earth is a sphere, our screens are flat.

Finding the best map projection depends on what you're most interested in.

1. Mercator





<u>Disadvantage:</u>
Extreme distortion of area towards the poles.

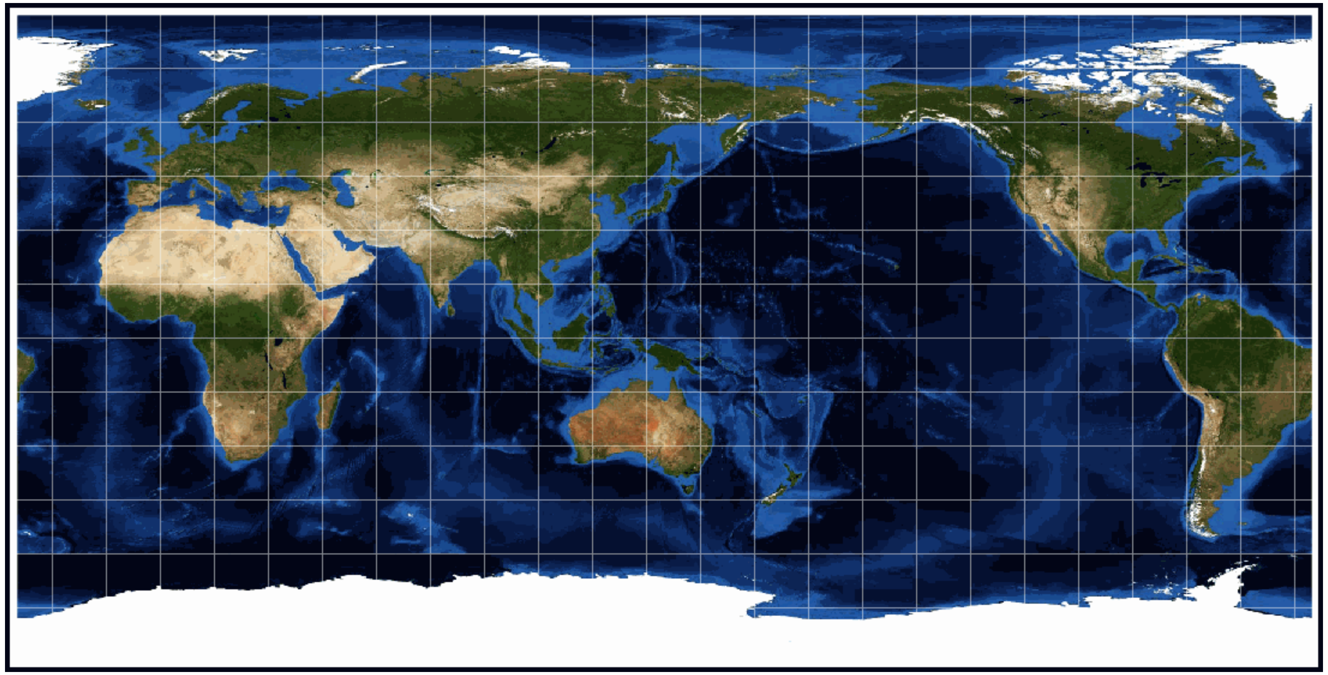
Visualizing a Spherical Surface

2. Plate-Carrée

Assumes all lat-lon boxes are squares.

Good for:

- Reducing area distortion
- Quick plots of gridded data



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Visualizing a Spherical Surface

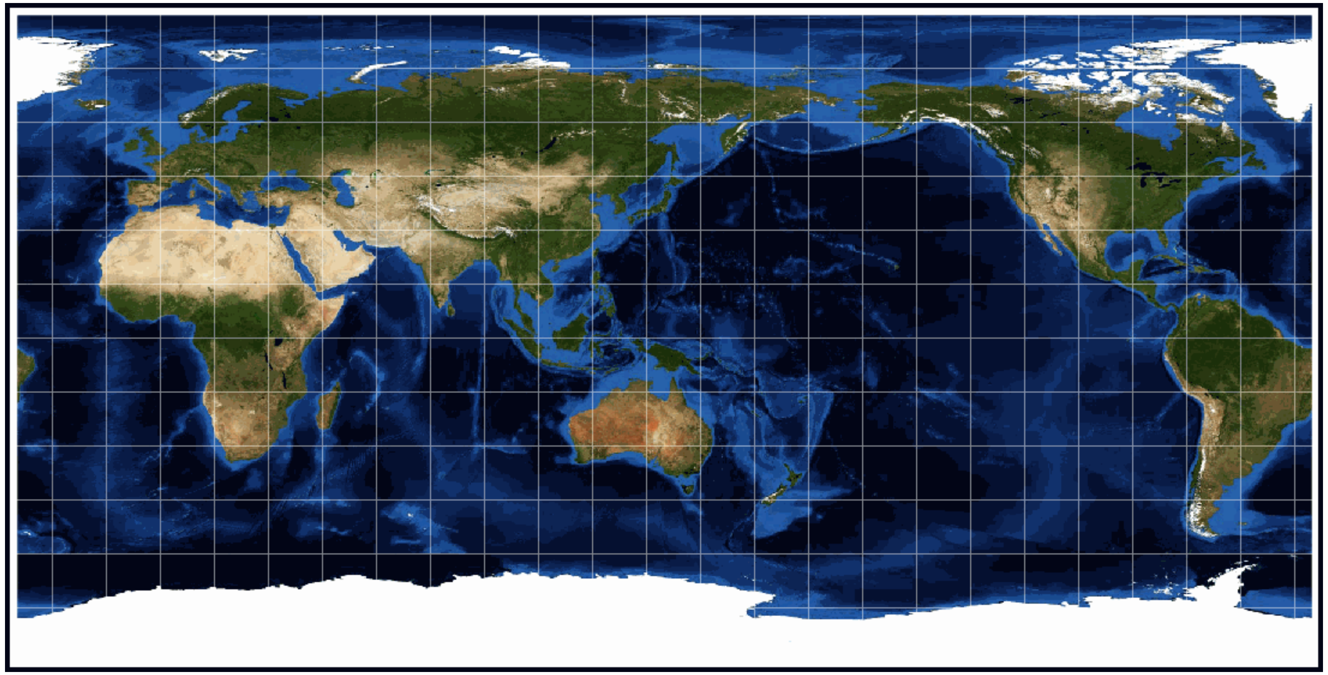
2. Plate-Carrée

Assumes all lat-lon boxes are squares.

Disadvantage:

- Does not preserve shapes

(esp. near poles)



Intergovernmental Committee on Surveying and Mapping

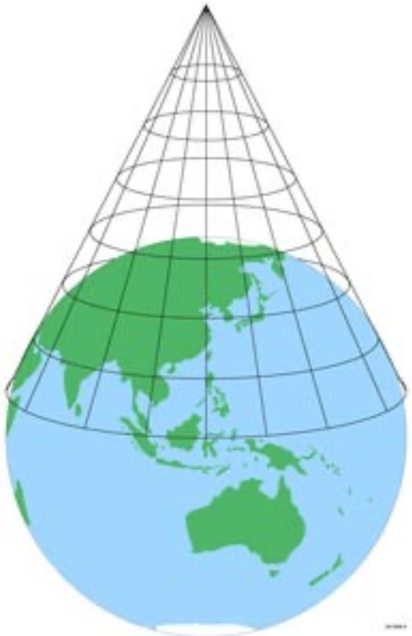
Visualizing a Spherical Surface

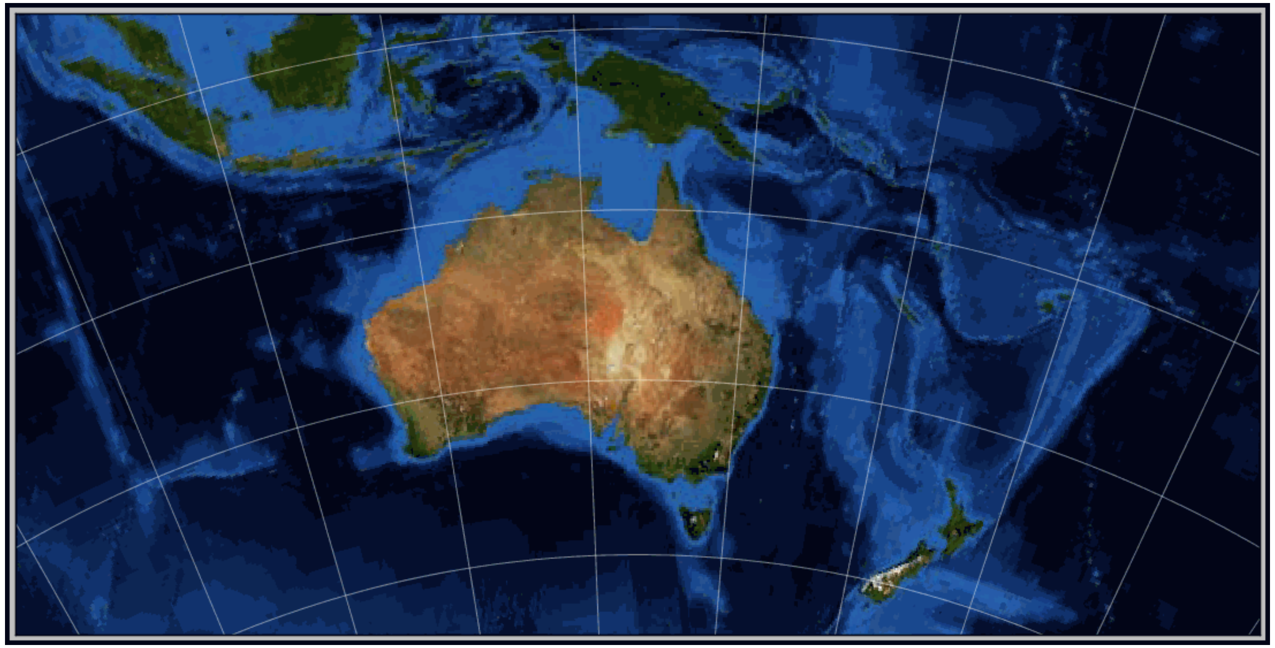
3. Lambert (conical)

Good for:

Preserving shapes and areas in mid-latitudes.

Minimizing distortion in a selected region of interest.





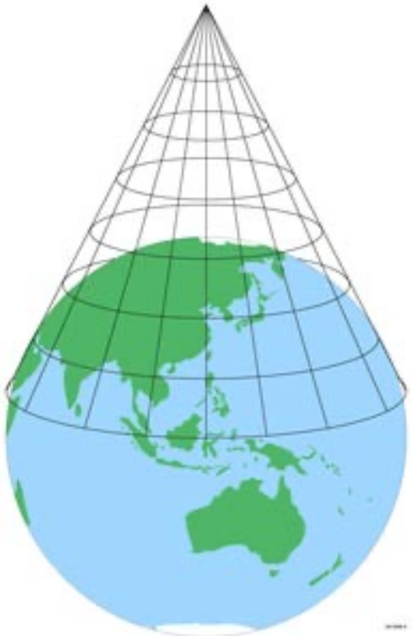
[Intergovernmental Committee on Surveying and Mapping]

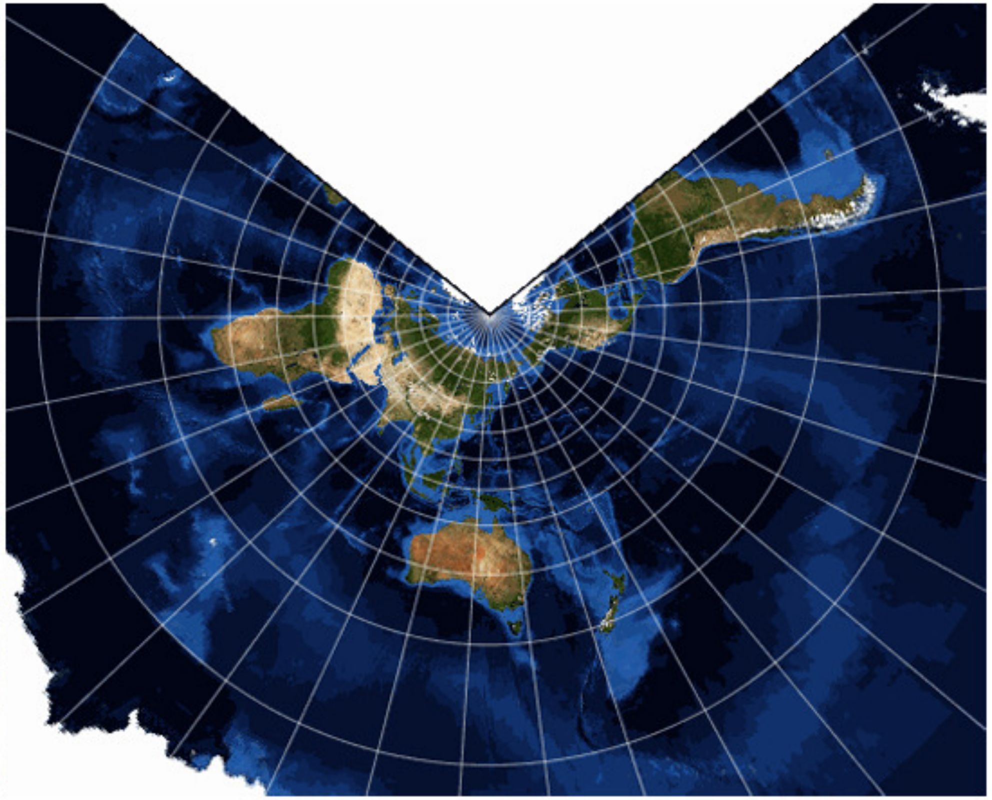
Visualizing a Spherical Surface

3. Lambert (conical)

Disadvantage:

Not practical for maps of the whole globe.





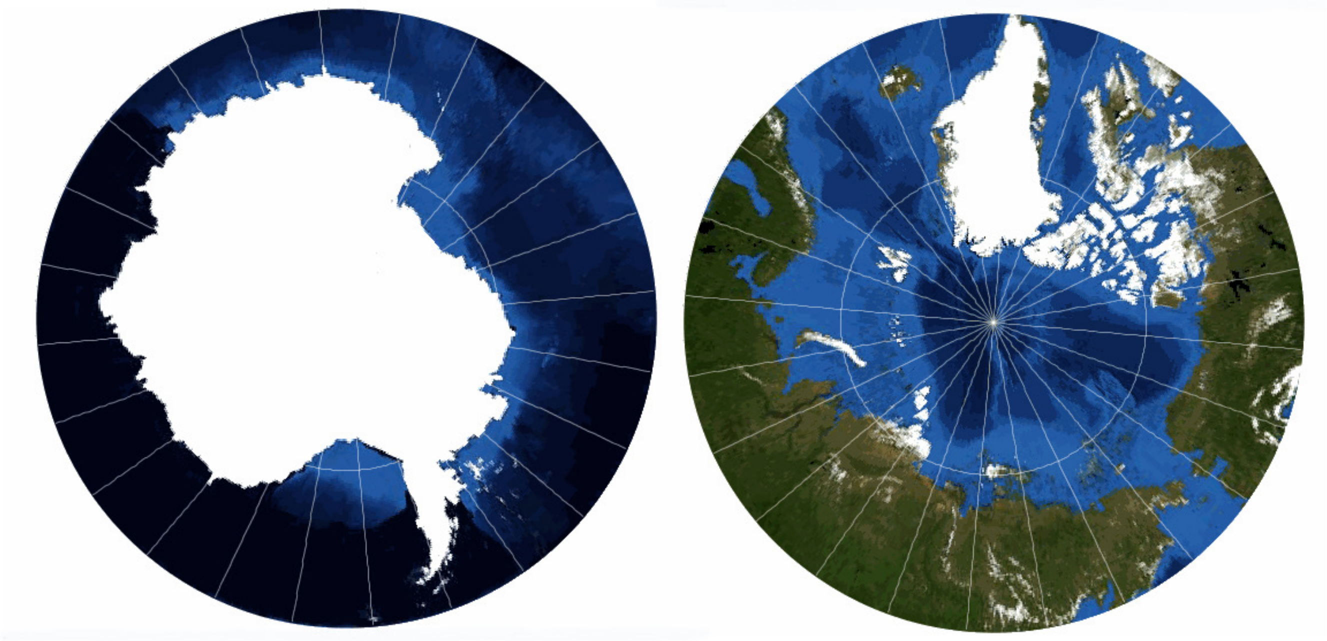
[Intergovernmental Committee on Surveying and Mapping]

Visualizing a Spherical Surface

4. Stereographic (Polar)

Good for:

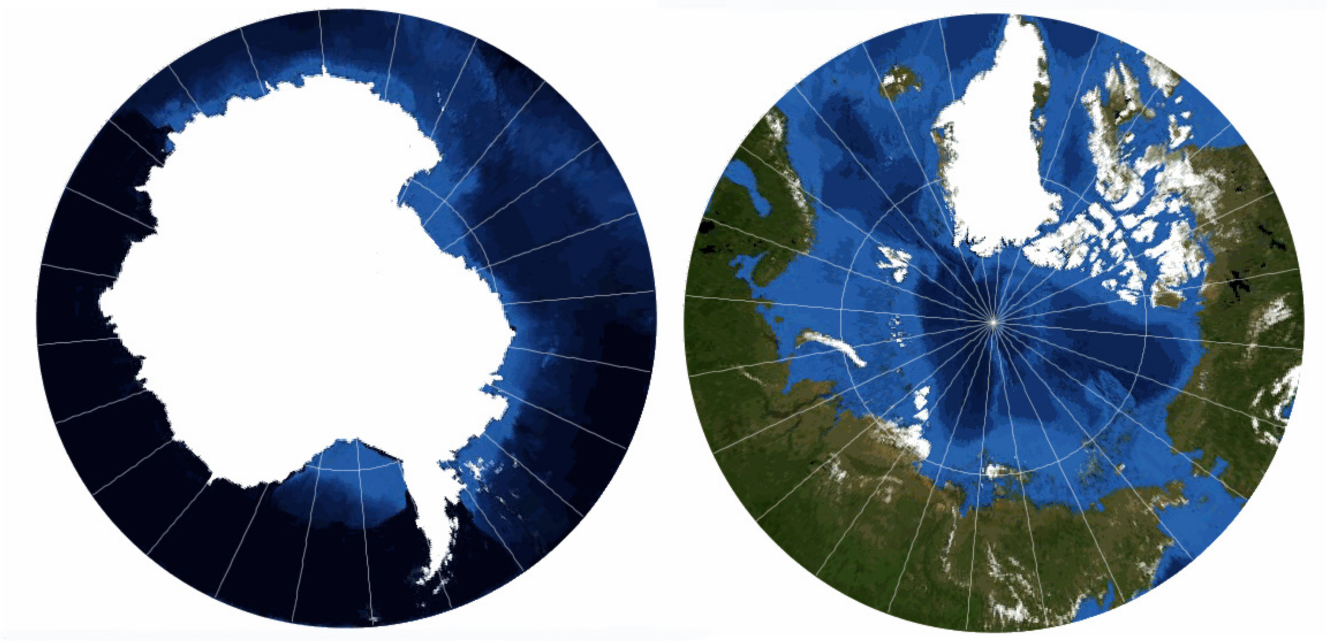
- Polar regions



[Intergovernmental Committee on Surveying and Mapping]

Visualizing a Spherical Surface

4. Stereographic (Polar)



Disadvantage:

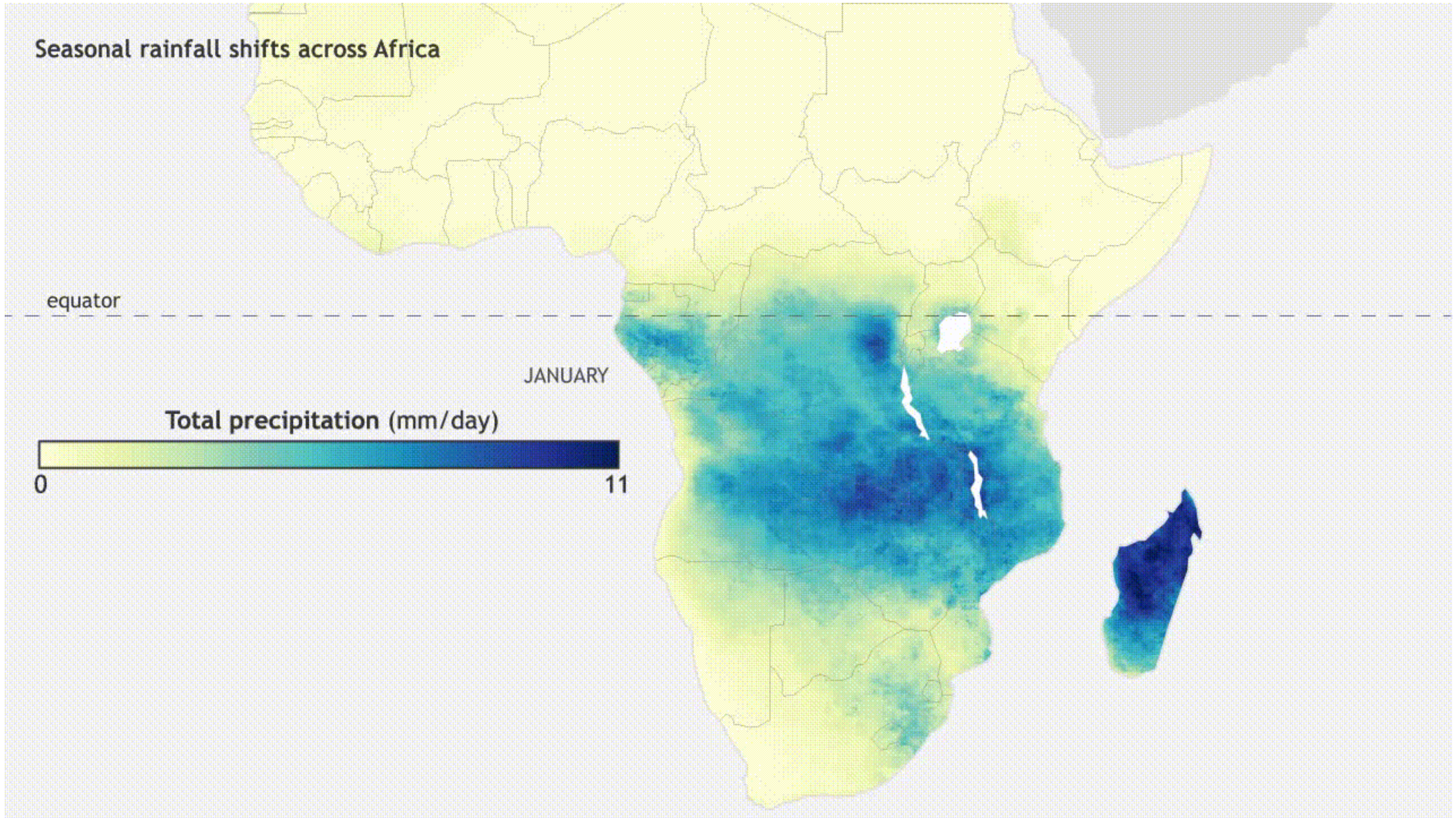
- Cannot be used globally (extreme distortion near the equator).

[Intergovernmental Committee on Surveying and Mapping]

Visualizing Spatiotemporal Data

We only have 2 dimensions for visualization.

- Maps – marginalize over time
- Time series/trace – marginalize over space



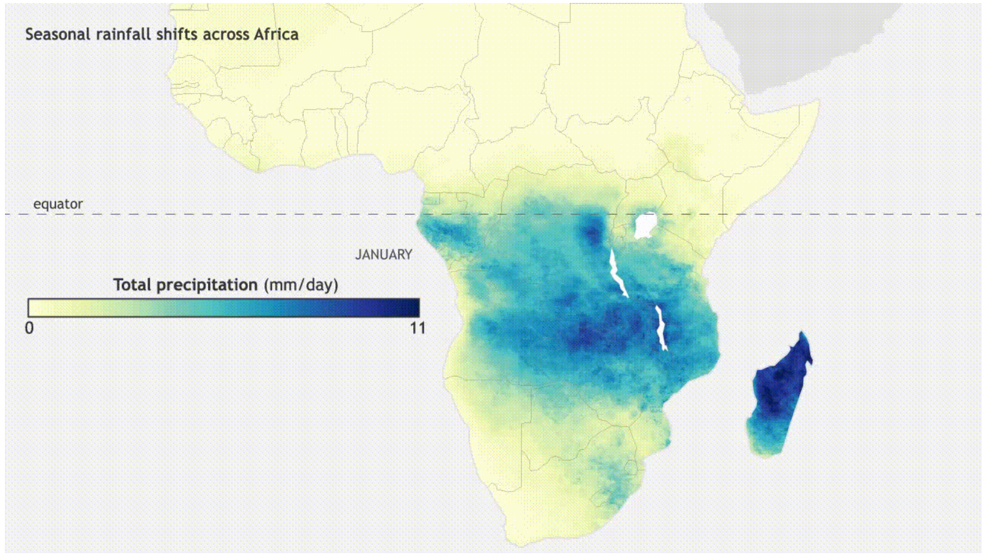
Climate.gov

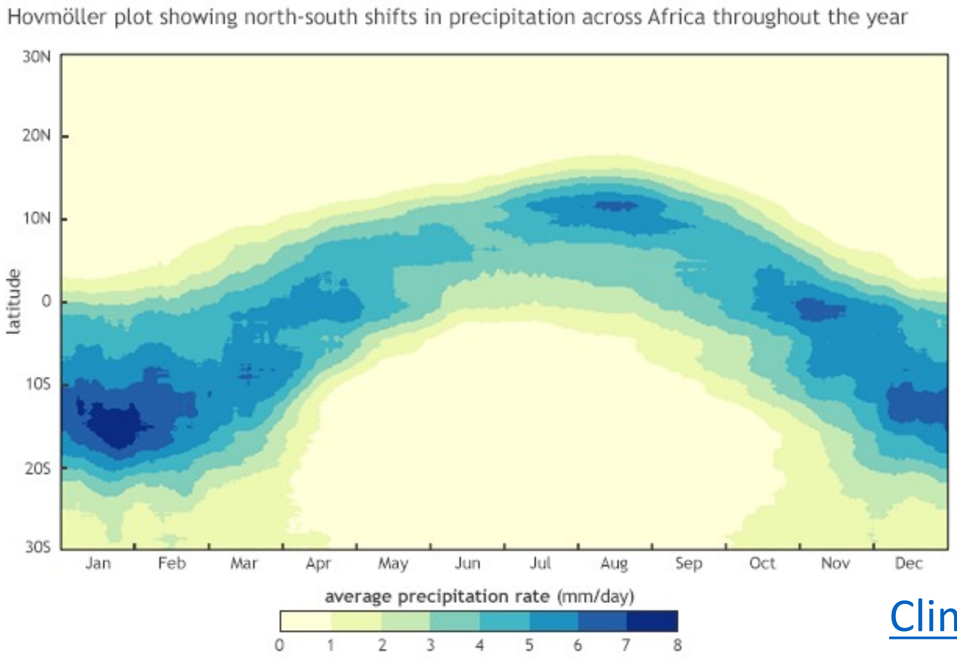
Visualizing Spatiotemporal Data

We only have 2 dimensions for visualization.

- Maps – marginalize over time
- Time series/trace – marginalize over space
- **Hovmoeller diagrams** – average over a spatial co-ordinate

Useful for propagating phenomena: storms, waves, fronts





Climate.gov

Dimension Reduction for Spatiotemporal Data

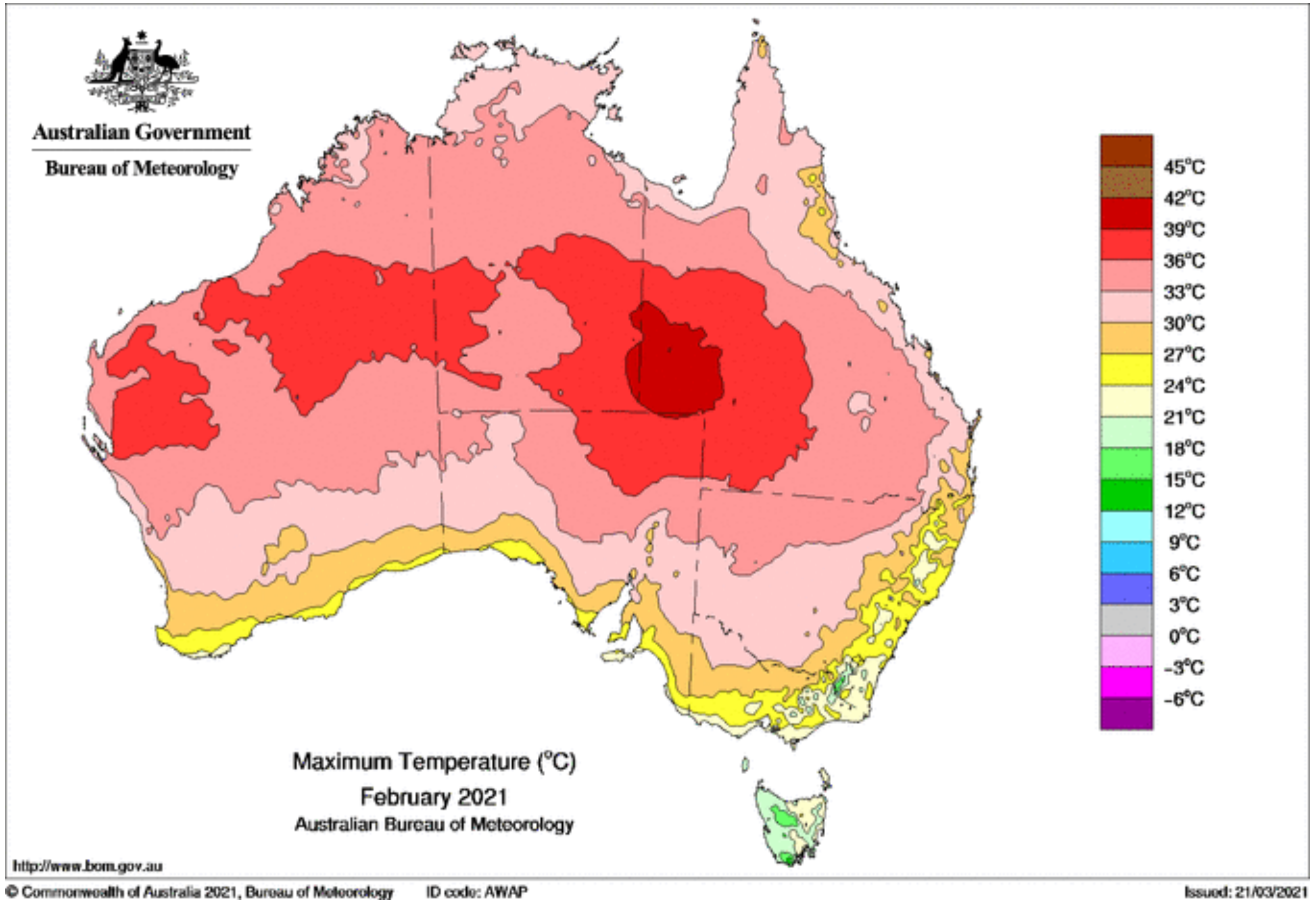
Empirical Orthogonal Functions

Principal Component Analysis tailored to spatiotemporal data.

(let's switch to using "dimension" in the usual mathematical sense from here on)

Our data is high-dimensional: each sample (i.e., snapshot in time) consists of a grid filled with values.

BUT, these values are far from independent, meaning we have a lot of redundant information.

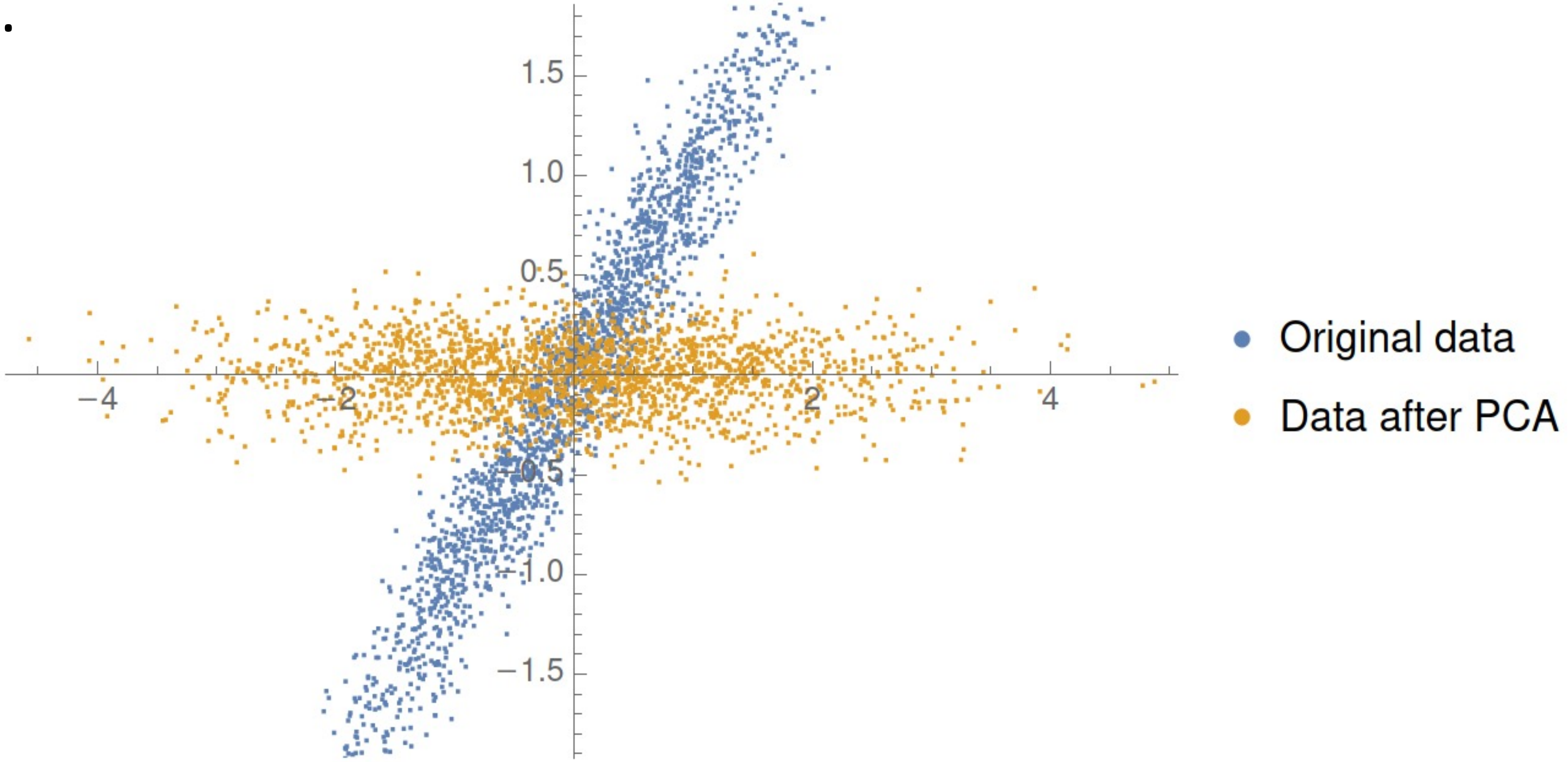


E.g., surface temperature is a smoothly-varying quantity.

Review of PCA

Objective: to **change the basis** of the data such that individual dimensions of the resulting data are uncorrelated.

The first few axes of this new basis should explain **most of the variance** of the data.



Review of PCA

We're looking for the right set of weights to use to perform this rotation.

We start with the empirical covariance matrix.

$$\hat{C} = \frac{1}{n-1}\sum_{i=1}^{n}(\vec{x}_i - \bar{x})(\vec{x}_i - \bar{x})^T \approx C = \mathbb{E}[(X - \mathbb{E}[X])(X - \mathbb{E}[X])^T]$$

Where the *p*-dimensional vectors $\vec{x}_i = \left(x_{1i}, \dots, x_{pi}\right)$ are individual realizations of X and $\bar{x}$ is the empirical mean.

Review of PCA

The optimal weights can be found by solving an eigenvalue problem.

$\hat{C}$ can be diagonalized such that:

$$\hat{C} = W \Lambda W^T$$

Where

Λ is a diagonal matrix containing the eigenvalues in descending order, i.e., $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$.

W contains the orthogonal eigenvectors $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_p$.

Review of PCA

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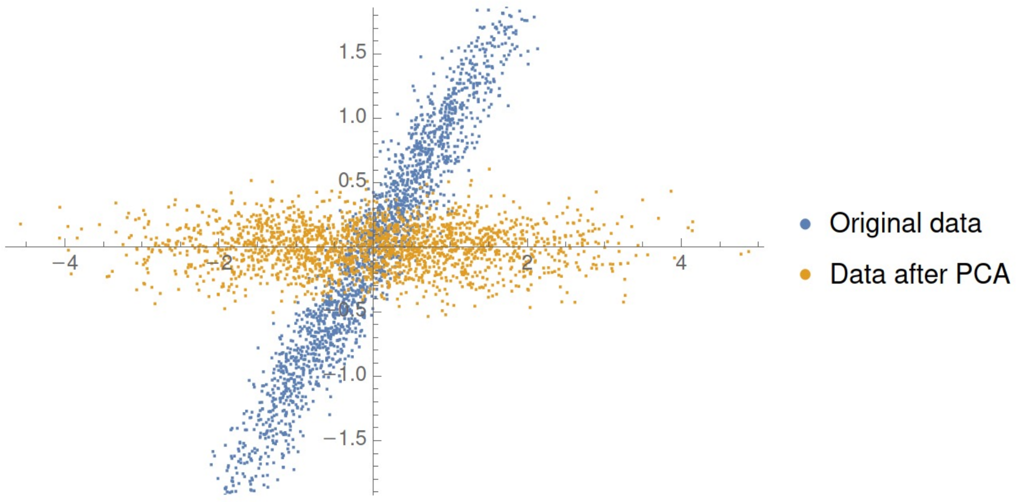
W contains the orthogonal eigenvectors $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_p$. → weights to rotate the data

The rotated data is then

$$A_i = a_{1i}, a_{2i}, \ldots, a_{pi} = \mathrm{W}^T \vec{x}_i$$



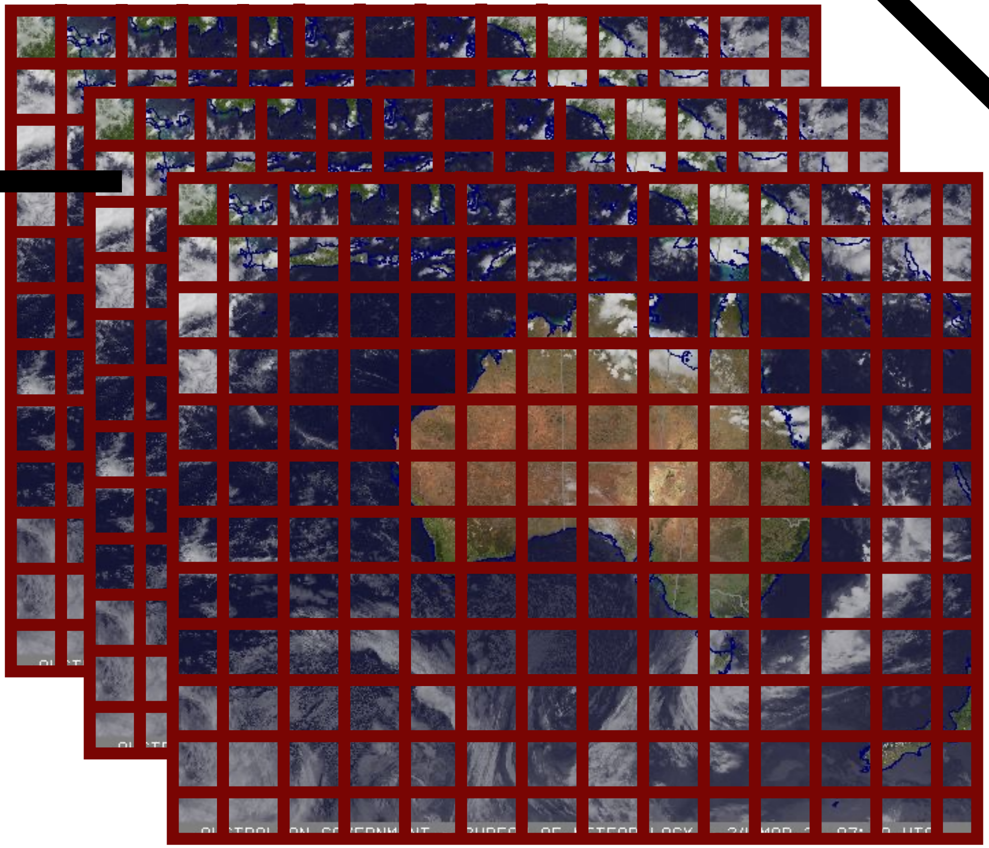
Using spatial co-ordinates in PCA

In the spatiotemporal case

Grid cell → dimension,   and   “snapshot” in time → a sample

Grid cell j,k

Each cell j,k is a dimension of a sample i



Time i

Each picture is a sample

Using spatial co-ordinates in PCA

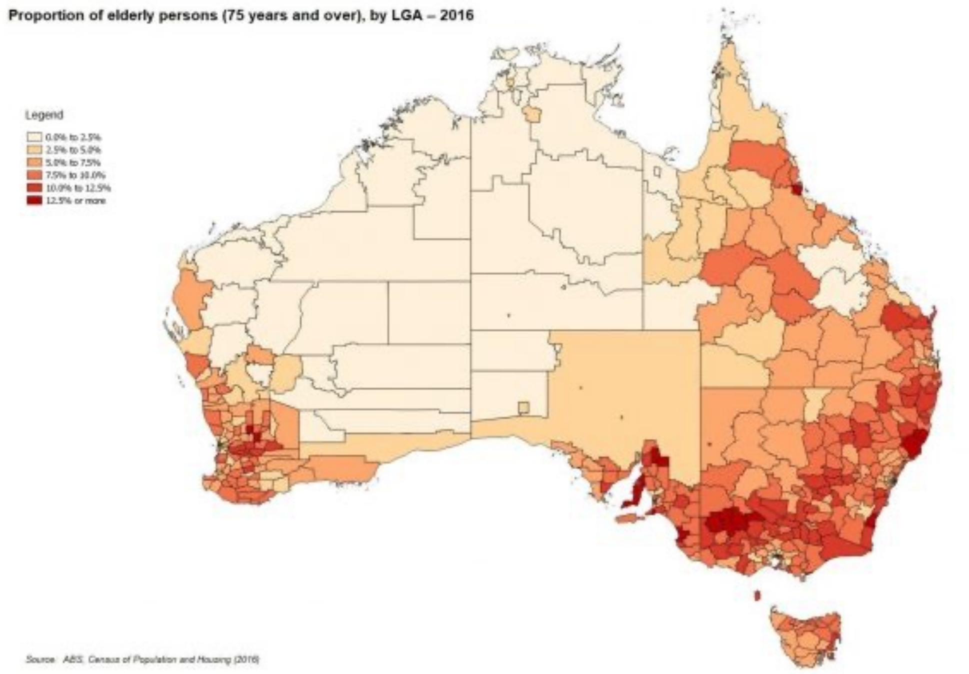
In the spatiotemporal case, our data

$$\vec{x}_i = (x_{1i}, \ldots, x_{pi})$$

Is indexed by

spatial location 1,2,…,p and

Time i



In the case of irregularly spaced data (e.g., by state, or weather station), 1 to p is the spatial index.

Using spatial co-ordinates in PCA

In the spatiotemporal case, our data

$$\vec{x}_i = (x_{1i}, \dots, x_{pi})$$

Is indexed by

spatial location 1,2,…,p and

Time i

For gridded 2-D spatial data, we “vectorize” by unwrapping the matrix such that p = J x K

Using spatial co-ordinates in PCA

Then, solving for the weights as before

$$\hat{C} = \Psi\Lambda\Psi^T$$

Now gives us the *spatially-indexed* weight matrix Ψ.

Each column of Ψ is an eigenvector of length *p* containing a weight for each spatial index (and could be plotted on a map).

These columns $\psi_1, \psi_2, \ldots$ are known as **Empirical Orthogonal Functions** (EOFs).

Using spatial co-ordinates in PCA

Each column of Ψ is an eigenvector of length p containing a weight for each spatial index.

The rotated data is

$$A_i = a_{1i}, a_{2i}, \dots, a_{pi} = \Psi^T \vec{x}_i$$

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,T} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,T} \\ \vdots & & \ddots & \vdots \\ a_{(p-1),1} & a_{(p-1),2} & \cdots & a_{(p-1),T} \\ a_{p,1} & a_{p,2} & \cdots & a_{p,T} \end{bmatrix}$$

Using spatial co-ordinates in PCA

Each column of Ψ is an eigenvector of length p containing a weight for each spatial index.

The rotated data is

$$A_i = a_{1i}, a_{2i}, \dots, a_{pi} = \Psi^T \vec{x}_i$$

Since i is a time index, A is a collection of p time series

$$A = [A_1, \dots, A_T] \in \mathbb{R}^{p \times T}$$

$$A = \begin{bmatrix} \boxed{\begin{matrix} a_{1,1} \\ a_{2,1} \\ \vdots \\ a_{(p-1),1} \\ a_{p,1} \end{matrix}} & \begin{matrix} a_{1,2} \\ a_{2,2} \\ \\ a_{(p-1),2} \\ a_{p,2} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \\ \cdots \end{matrix} & \begin{matrix} a_{1,T} \\ a_{2,T} \\ \vdots \\ a_{(p-1),T} \\ a_{p,T} \end{matrix} \end{bmatrix} \qquad A = \begin{bmatrix} \boxed{\begin{matrix} a_{1,1} & a_{1,2} & \cdots & a_{1,T} \end{matrix}} \\ \begin{matrix} a_{2,1} & a_{2,2} & \cdots & a_{2,T} \\ \vdots & & \ddots & \vdots \\ a_{(p-1),1} & a_{(p-1),2} & \cdots & a_{(p-1),T} \\ a_{p,1} & a_{p,2} & \cdots & a_{p,T} \end{matrix} \end{bmatrix}$$

Using spatial co-ordinates in PCA

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Since *i* is a time index, *A* is a collection of *p* time series

$$A = [A_1, \ldots, A_T] \in \mathbb{R}^{p \times T}$$

These are known as the Principal Component (PC) time series.

$$A = \begin{bmatrix} \boxed{a_{1,1} & a_{1,2} & \cdots & a_{1,T}} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,T} \\ \vdots & & \ddots & \vdots \\ a_{(p-1),1} & a_{(p-1),2} & \cdots & a_{(p-1),T} \\ a_{p,1} & a_{p,2} & \cdots & a_{p,T} \end{bmatrix}$$

Interpreting EOFs

- The first **EOF ψ_1** represents the **spatial pattern** that maximizes covariance between points in space.
- The **variance explained** by the 1st EOF is the 1st **eigenvalue λ_1** and is maximized by design.
- The 1st **PC time series** represents the **variations** of the first EOF **over time.**

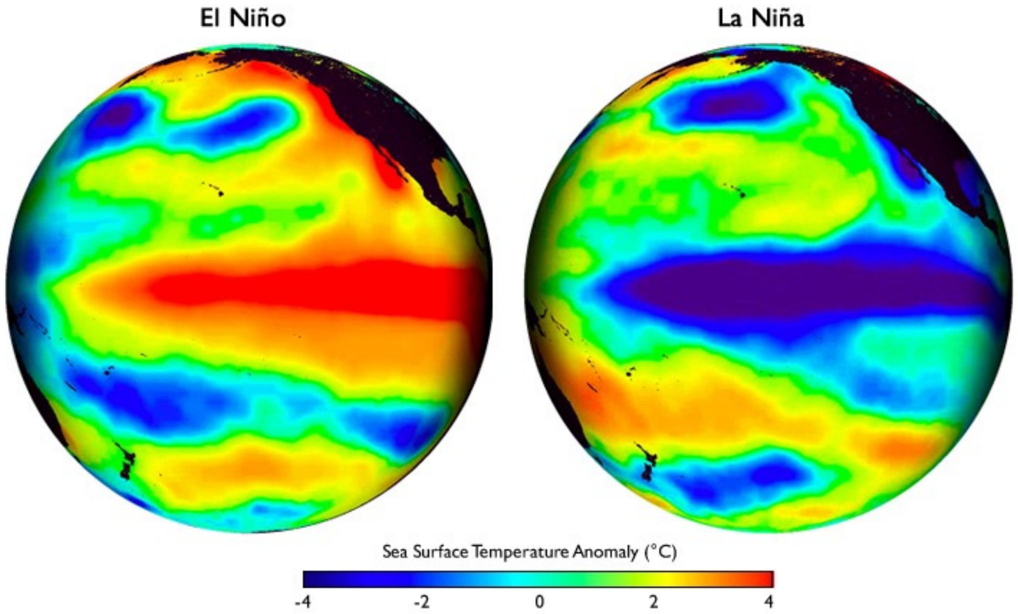
- The 2nd EOF, variance explained, and PC time series are constrained to be orthogonal to the 1st, so may not hold physical meaning.

How to use EOFs

- Sometimes used for compression
- Generating synthetic data
- Dimension reduction
- **Detecting spatial structure of covariance**

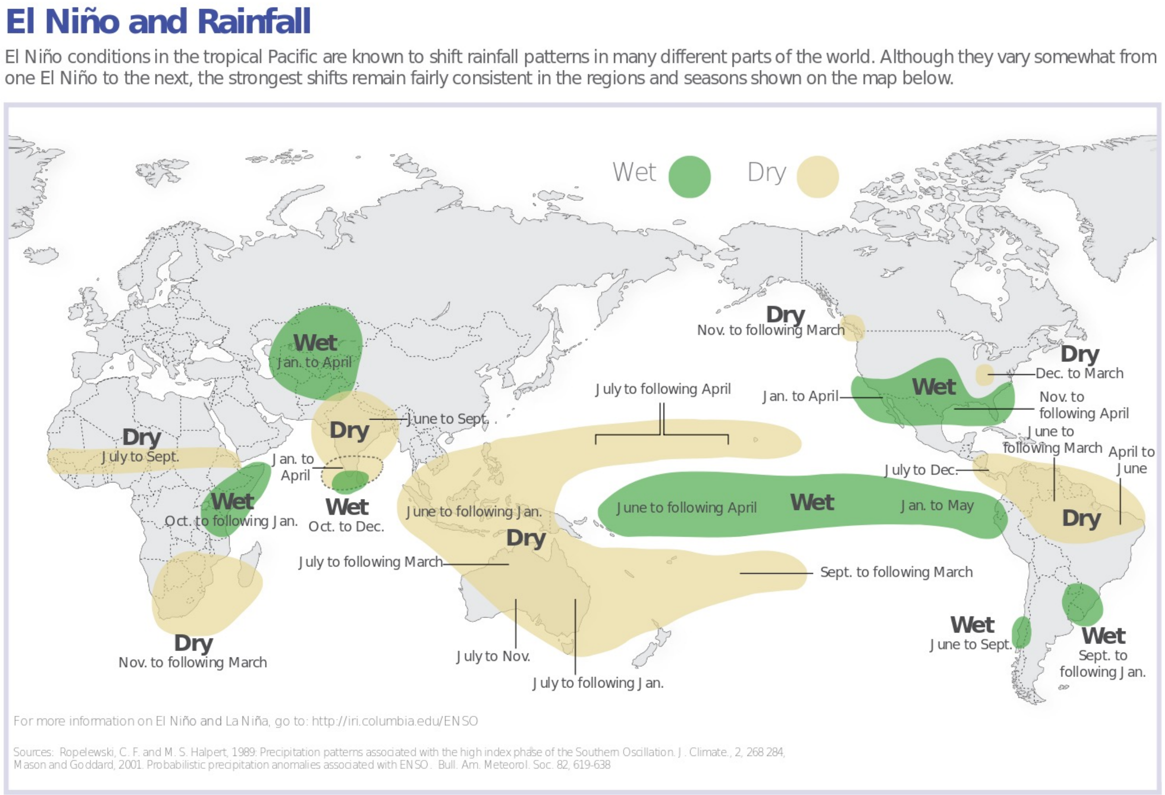
EOF Example: El Niño-Southern Oscillation

An oscillation between **cool (La Niña)** and **warm (El Niño)** states.



ENSO affects rainfall and temperatures around the world – its signature can be seen in agriculture, natural disasters, and more…

[Examples of conditions during El Niño and La Niña events.]()

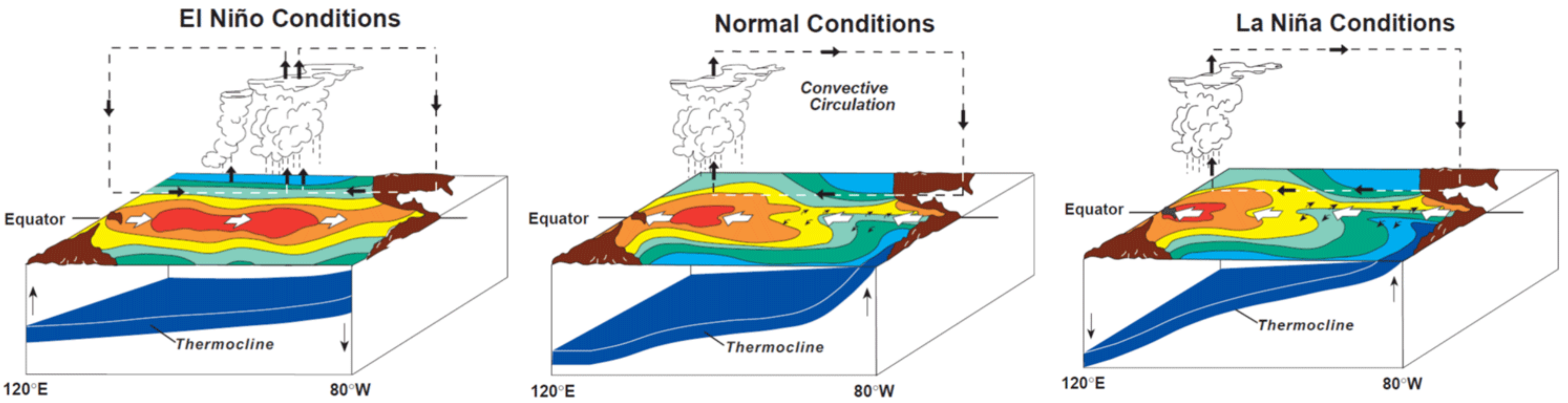


EOF Example: El Niño-Southern Oscillation

An oscillation between **cool (La Niña)** and **warm (El Niño)** states.

NO fixed characteristic frequency (quasi-periodic).

NO fixed spatial pattern (each event looks a little different).

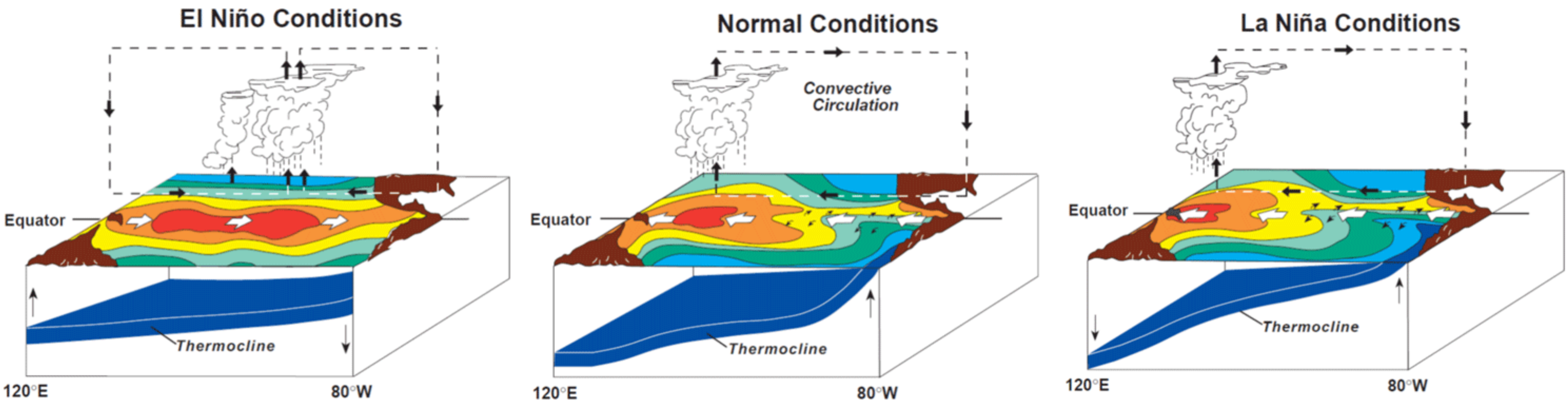


EOF Example: El Niño-Southern Oscillation

An oscillation between **cool (La Niña)** and **warm (El Niño)** states.

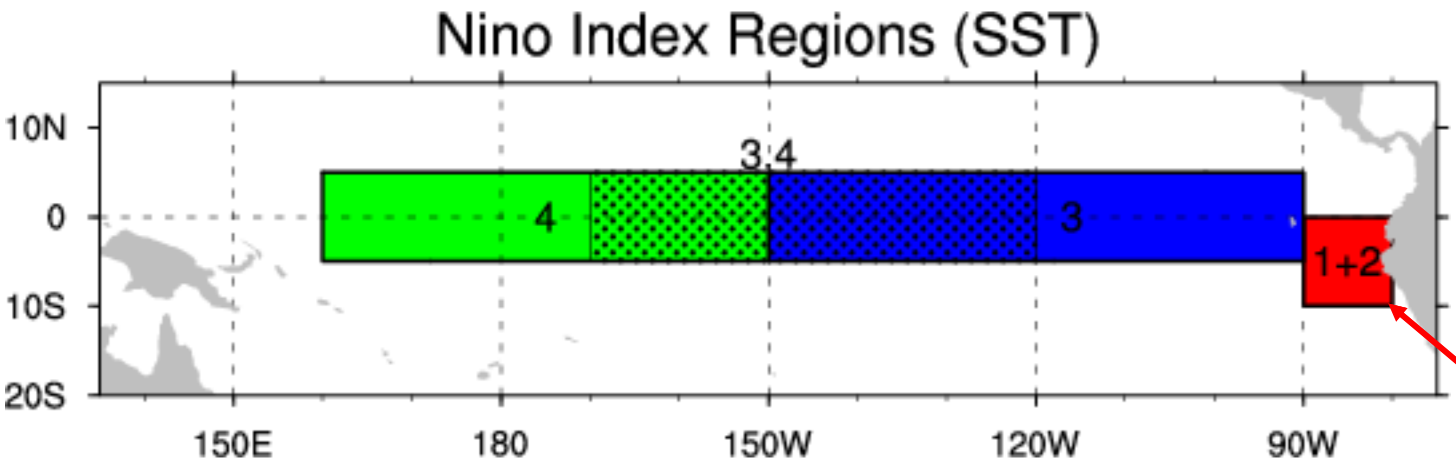
NO fixed characteristic frequency (quasi-periodic). **But we know it happens every 2-7 years.**

NO fixed spatial pattern (each event looks a little different). **But we know it always involves surface temperature anomalies in the Tropical Pacific.**



EOF Example: El Niño-Southern Oscillation

How do we characterize this phenomenon?



[From UCAR Climate Data Guide]

When fishermen off the coast of Peru first recognized this phenomenon in the 1700s, they thought temperatures in this area mattered the most.

EOF Example: El Niño-Southern Oscillation

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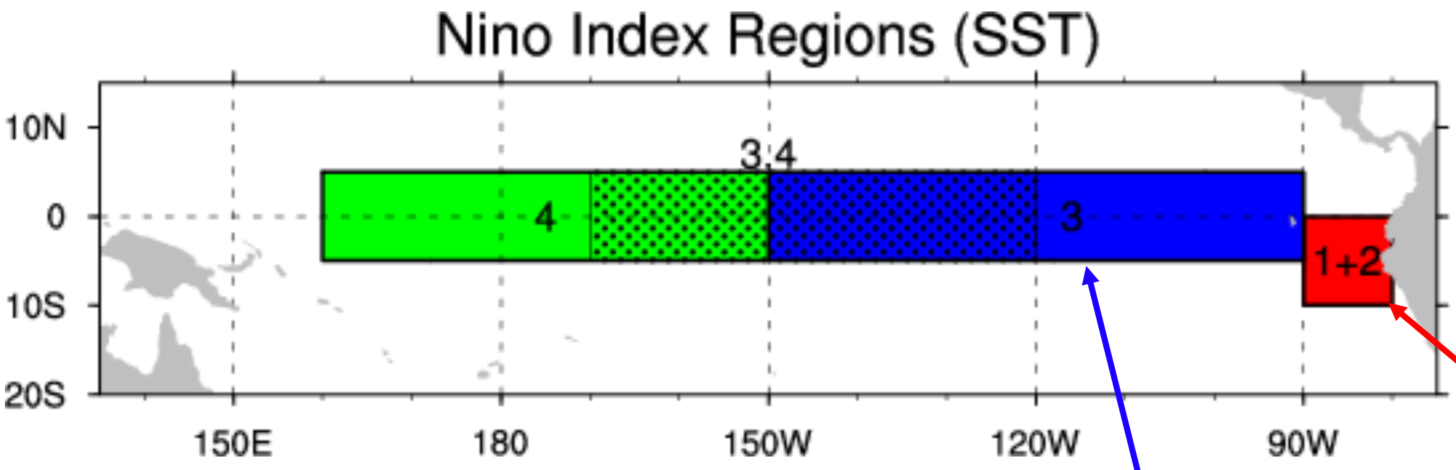

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As scientists started to study the phenomenon, they thought this was the main region of temperature anomalies.

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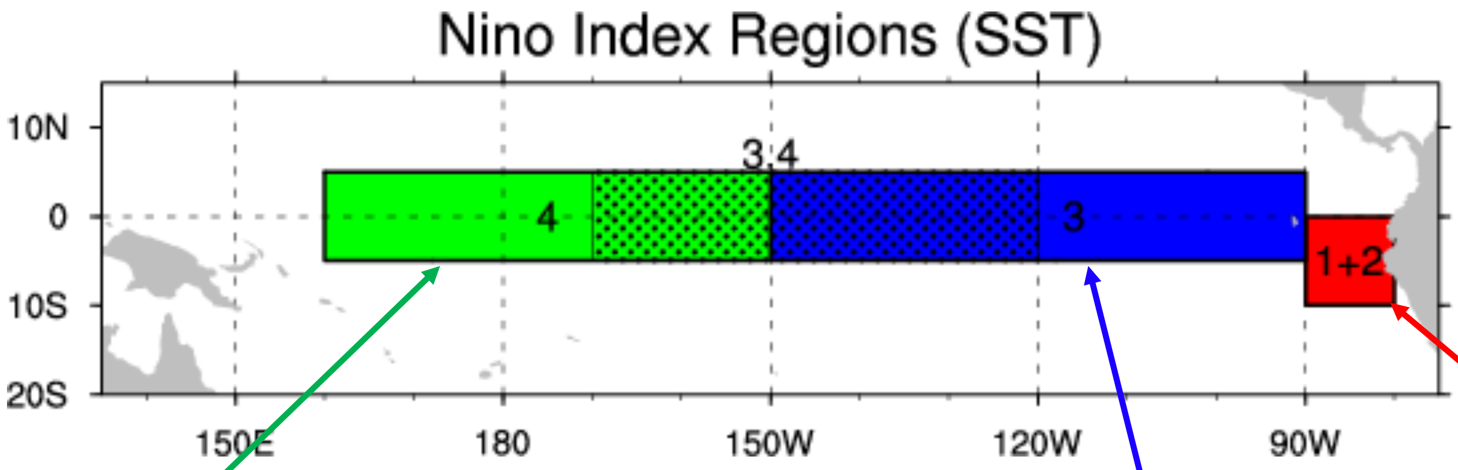

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As satellite measurements became available, they realized this area was involved, too.

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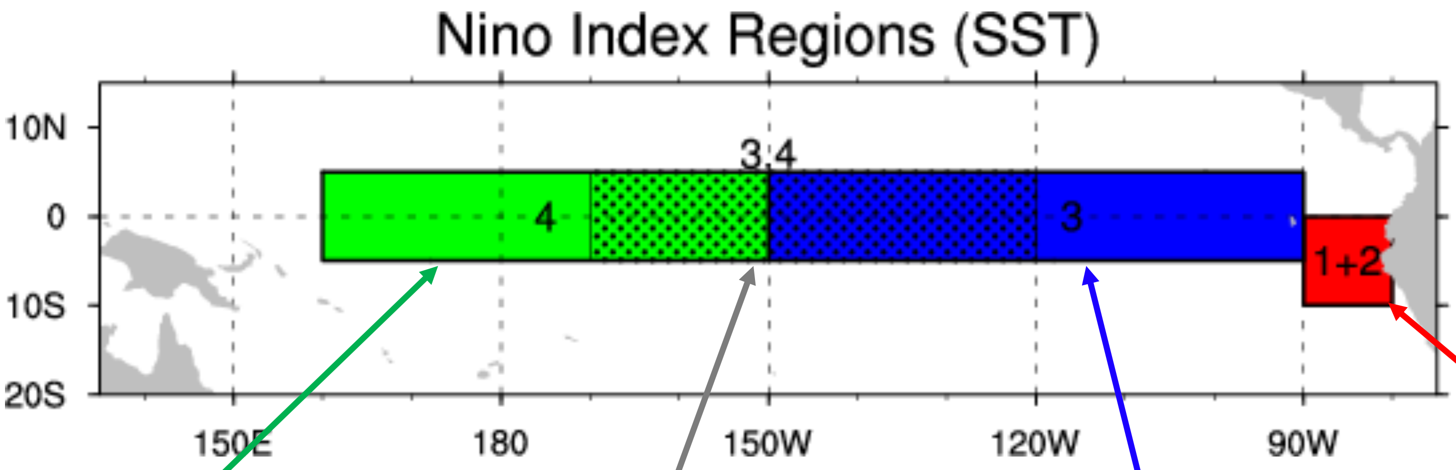


From UCAR Climate Data Guide

As satellite measurements became available, they realized this area was involved, too.

We now commonly use the spatial average over this “compromise” area

As scientists started to study the phenomenon, they thought this was the main region of temperature anomalies.

When fishermen off the coast of Peru first recognized this phenomenon in the 1700s, they thought temperatures in this area mattered the most.

EOF Example: El Niño-Southern Oscillation

A better way to characterize this:

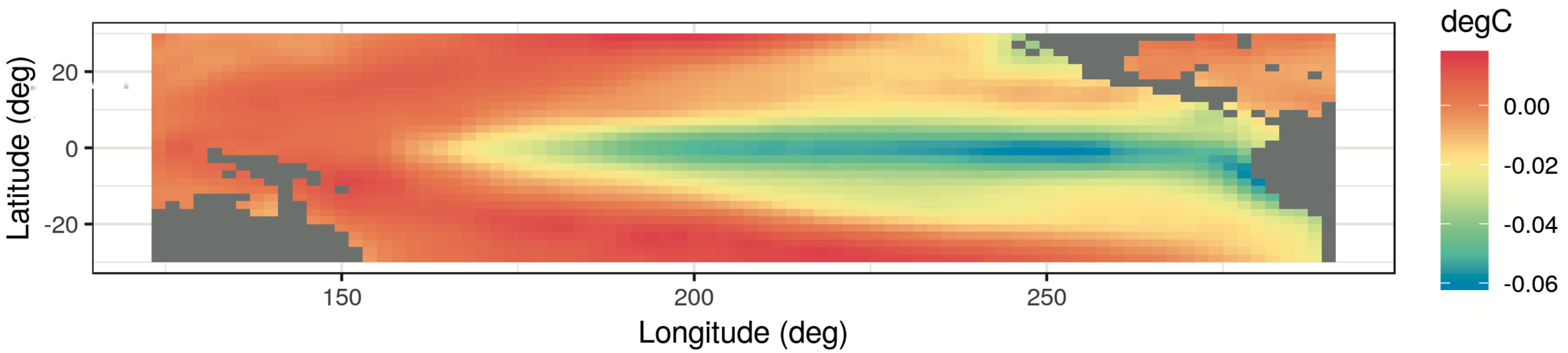
ENSO is a high-variance phenomenon that engages much of the Pacific basin.

EOFs

- identify the spatial pattern that maximizes covariance between spatial points (the weight matrix)
- Give us a time series of how this pattern varies over time (the PC time series)
- Tell us how much variance we have explained (the eigenvalues)

EOF Example: El Niño-Southern Oscillation

The 1st EOF of monthly sea surface temperatures over the tropical Pacific:

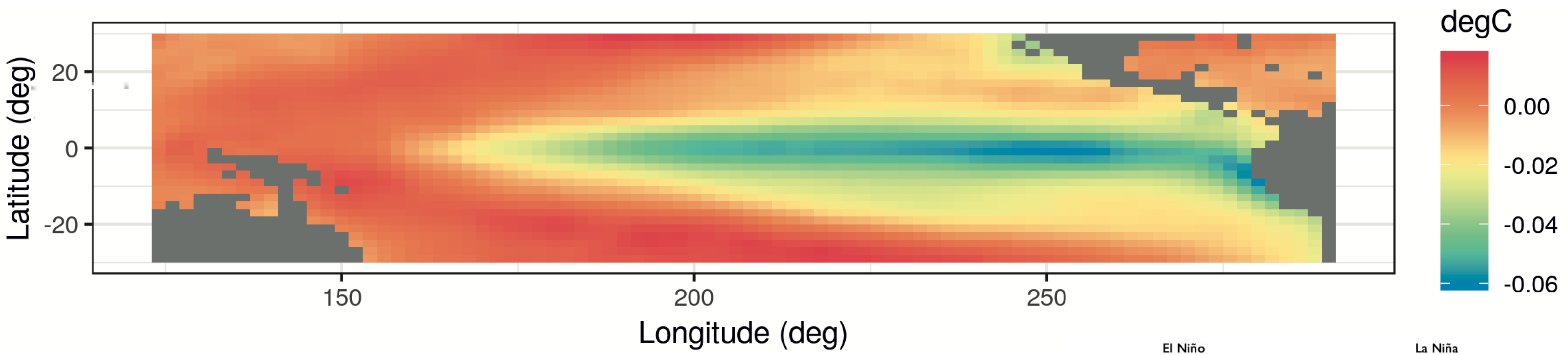


[Example from Wikle, Zammit-Mangion, Cressie]()

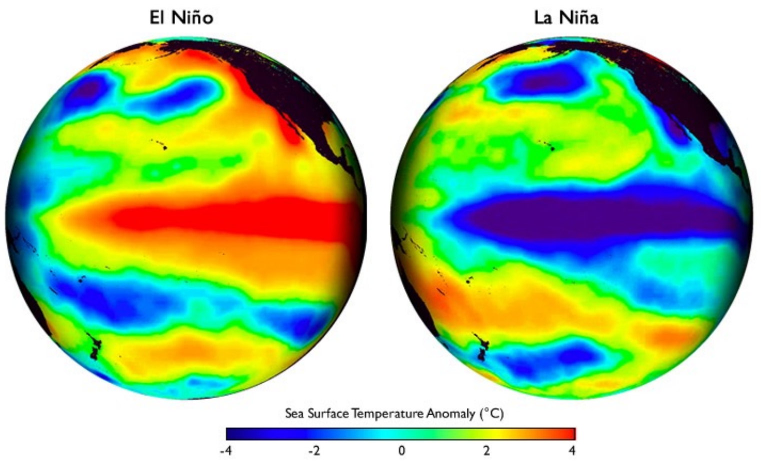
This is the 1st column of Ψ, re-ordered onto the original j,k indices to give a map.

EOF Example: El Niño-Southern Oscillation

The 1st EOF of monthly sea surface temperatures over the tropical Pacific:

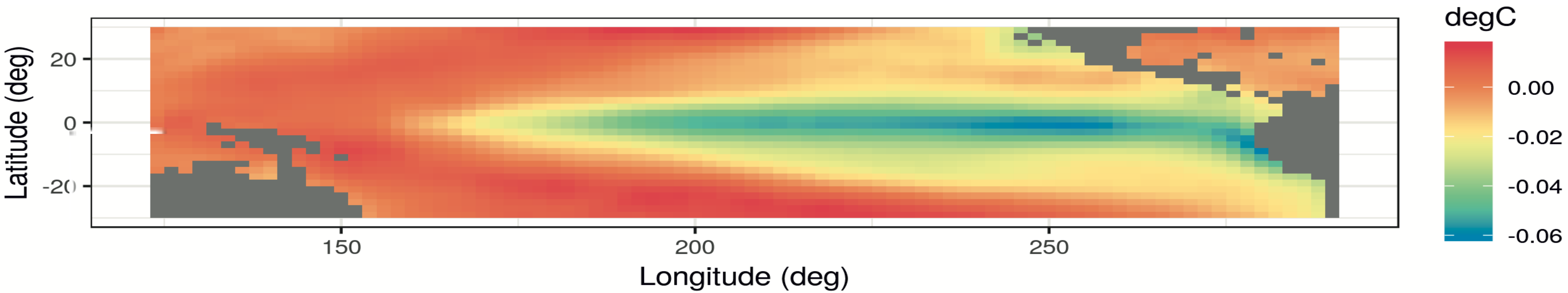


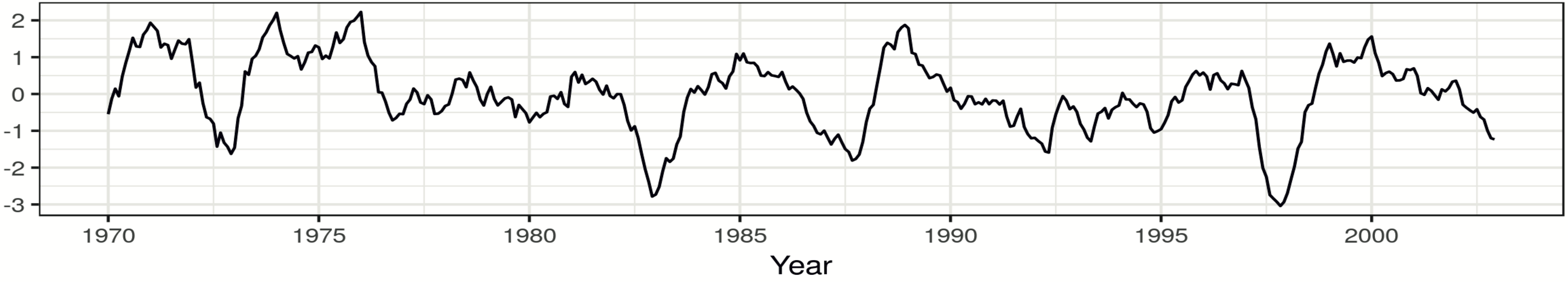
- Remarkably similar to the ENSO pattern we saw on the maps.
- Captures the irregular shape of the temperature anomaly (i.e., not a box).
- Explains approx. 40% of the variance.



EOF Example: El Niño-Southern Oscillation

The 1st EOF & PC time series of monthly sea surface temperatures over the tropical Pacific:





This gives the projection of the data onto this spatial pattern at each point in time.

EOF Example: El Niño-Southern Oscillation

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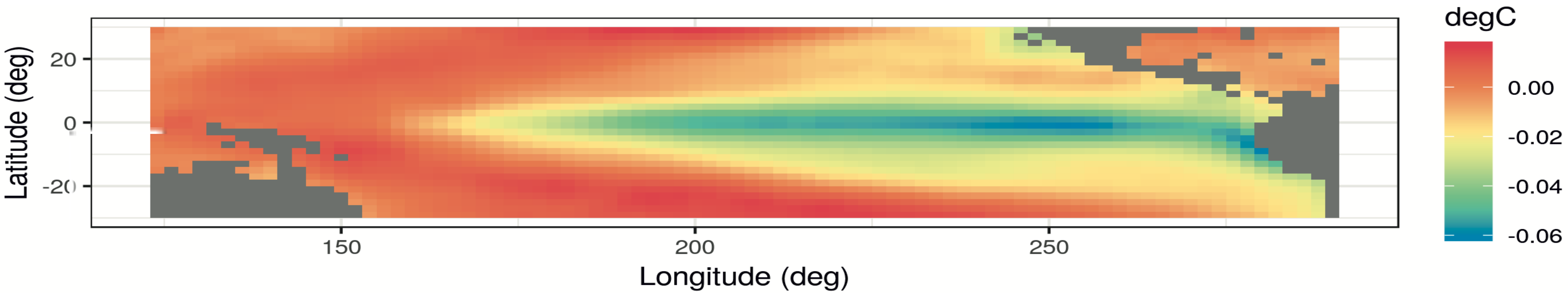
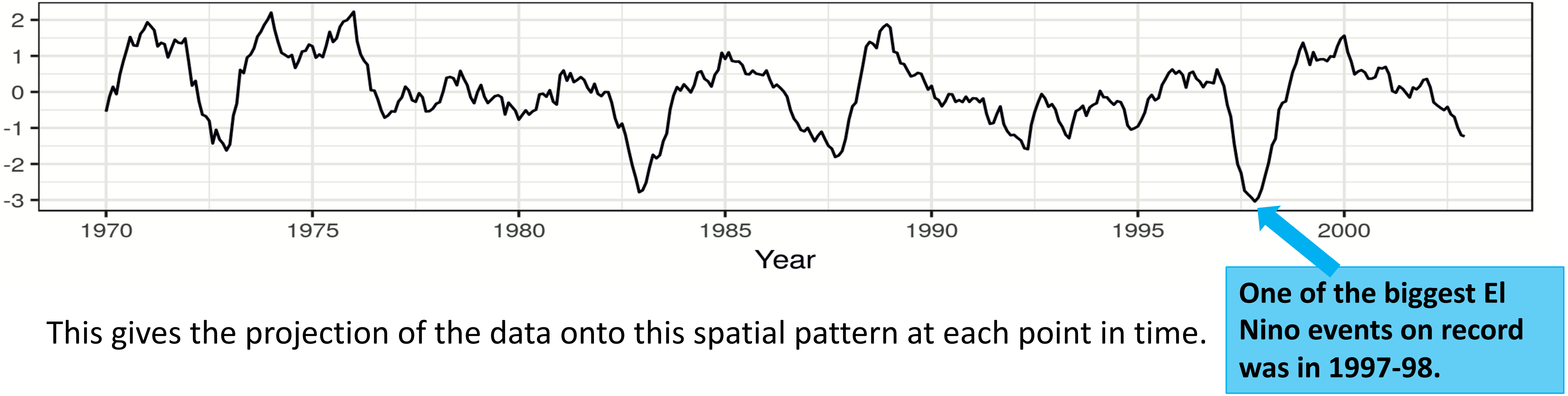
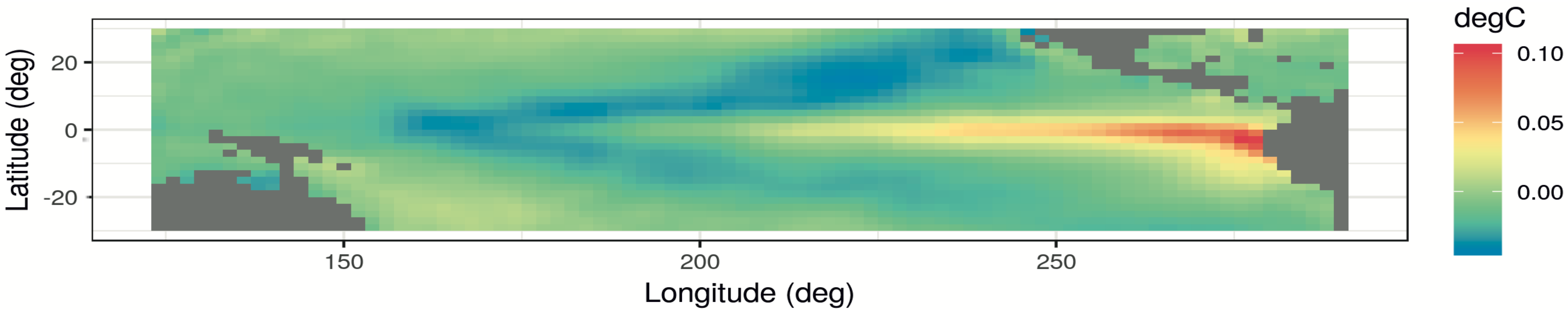


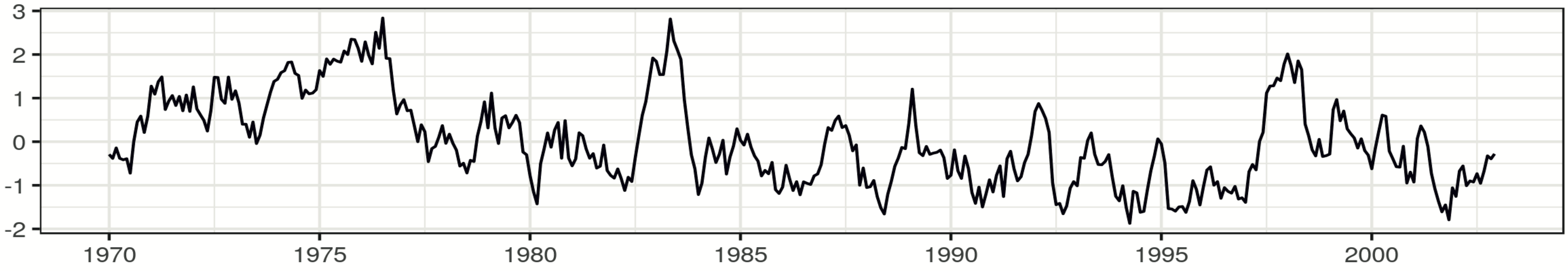


This gives the projection of the data onto this spatial pattern at each point in time.

EOF Example: El Niño-Southern Oscillation

The 2nd EOF & PC time series of monthly sea surface temperatures over the tropical Pacific:





This is constrained to be orthogonal to the first EOF: does **not** have physical meaning.

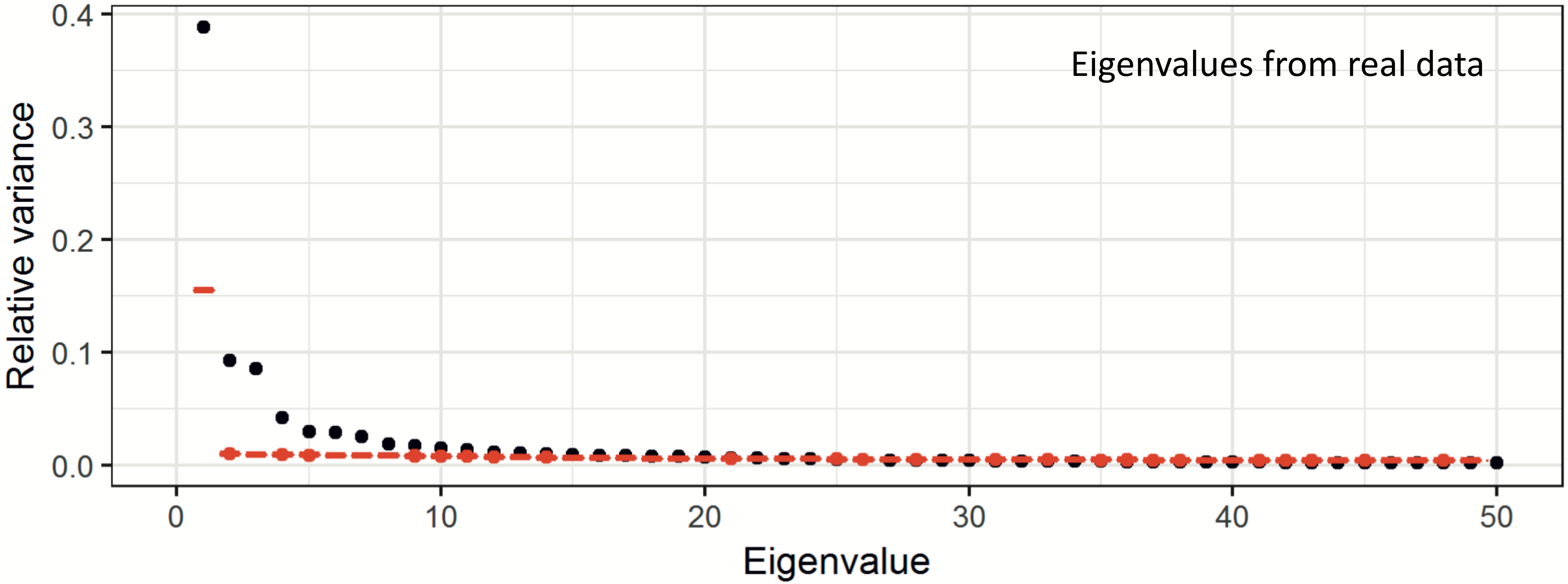
Reconstructing the signal

In this case, p = 2261 and T = 399. That's 902,139 data points!

Most of the variance is explained by the 1st 12 EOFs.

So instead, we could represent the data as 12 time series and 12 spatial matrices, i.e., 12 x (399 + 2261) = 31,920 data points. (~ 30 times smaller!)

"Scree plot" of eigenvalues λ



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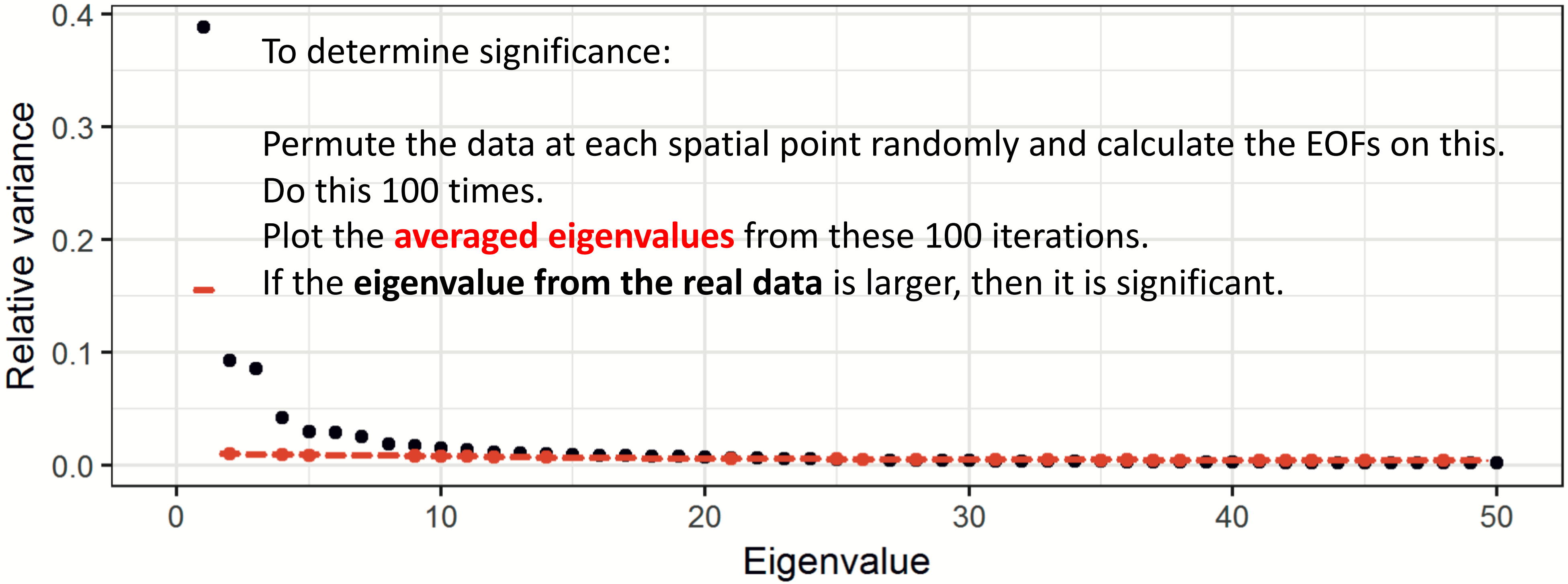
"Scree plot" of eigenvalues λ



To determine significance:

Permute the data at each spatial point randomly and calculate the EOFs on this.
Do this 100 times.
Plot the **averaged eigenvalues** from these 100 iterations.
If the **eigenvalue from the real data** is larger, then it is significant.

EOF Example: El Niño-Southern Oscillation

Summary: How should we interpret/use this result?

- NOT mechanistic
- NOT to infer causal/physical structure from the second or higher PC
- As an indicator of ENSO activity
- To characterize a spatial pattern
- To compress the data

Empirical Orthogonal Functions: wrap-up

- A specialized application of PCA
- Can be used in a variety of ways to characterize/analyze spatiotemporal data containing spatially-covarying phenomena
- Cannot be used directly for inference
- Forms the basis for methods used to extract interpretable information (esp as a first-pass) from complex, multivariate spatiotemporal fields, e.g. MCA, image filtering.

Spatiotemporal Data Analysis: wrap-up

- 3 kinds of spatiotemporal data: fields, trajectories, and irregular/track
- At large enough scales, we need to use spherical co-ordinates: longitude, latitude, altitude.
  - Distance, area, and visualization need to be handled carefully.
- PCA can be used to do more with spatiotemporal data.
  - In general, statistical methods can be adapted by accounting for spatiotemporal ordering of the data.