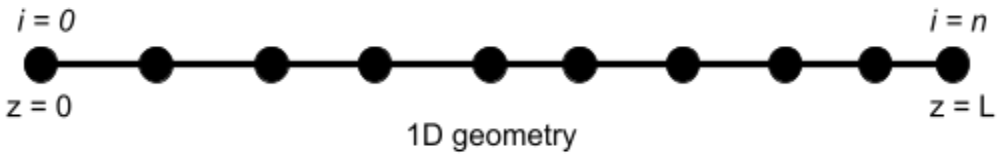
Problem Statement- NDP Modeling



Problem:

This is a problem encountered in modeling the neck-down profile of the fiber during the drawing process. Consider a 1D geometry starting from z=0 and ending at z=L. This geometry is discretized into n grid points, the first point at z=0 and the last at z=L, with the intermediate points between z=0 and z=L. The z-axis is discretized in n grid points which are indexed using index *i.* The functions T(z) and R(z) are defined on z, and thus, at all points *i.*

Suppose that R(z) is known at index *i* of the domain as:

R = R1 * (1 / (1 + exp(-a + *i* / (n / b)))), where a,b,n, R1 are given in the constants list.

T(z) needs to be solved using the following equation:

$$T'' + f(T)T' + g(T) = 0$$

$$f(T) = \frac{3pT^2 + \frac{2R'(K_c + pT^3)}{R} - \frac{\rho C_p R_1^2 V_1}{R^2}}{K_c + pT^3}$$

$$g(T) = \frac{\frac{2}{R}\left[Q_{2-1} - \epsilon\sigma\left(T^4 - T_0^4\right) - h\left(T - T_0\right)\right]}{K_c + pT^3}$$

In the above equations, only R and R'(dR/dz) are functions of z, while the rest of the terms are constants given below.

Boundary Conditions:

T = Ts , at z = 0;
T = Ts, at z = L.

Constants:

p, Kc, ρ, Cp, V1, R1, Q21, ∈, σ, T0, Ts, h, L are known constants given below:

```
a = 5,
b = 10
R1 = 0.6  # cm
R2 = 50*1e-4  # cm
V2 = 100 # cm/s
V1 = (R2**2 * V2)/(R1**2)  # cm/s
gamma = 2.0  # cm-1 - absorption coefficient
Kc = 0.05  # W/cmC
rho = 2.2  # g/cm3
Cp = 0.25  # cal/gC
Ts = 1600  # C
T0 = 1100  # C
g = 980  # cm/s2
alpha = 300  #dyn/cm

# Emmisivity- currently assumed to be constant, but will be later replaced with a function
epsilon = 0.3 # unitless

# heat transfer coefficient, currently assumed to be a constant, but will be later replaced with a function
h = 2*1e-2 #W/cm2C

sigma = 5.67*1e-4  # in kg cm2 s-2 K-1
n0 = 1.47
Q21 = 1000 # some arbitrary positive number, choose as per requirement
p = (16*(n0**2)*sigma)/(3*gamma)

L = 20 # length of the domain in cm
dz = 0.1  # Distance between two grid points in centimeters, choose as per requirement
n = int(L/dz)  # number of grid points
```

i is used for indexing the points. The z coordinate of a point at index *i* would be:
z(i) = i*dz

Similarly, R(z) and T(z) are defined in terms of index *i* as:
R(z) = R(i*dz) = R(i)
T(z) = T(i*dz) = T(i)

Provide a solution for T at all points *i* lying in the geometry.