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PHYS265
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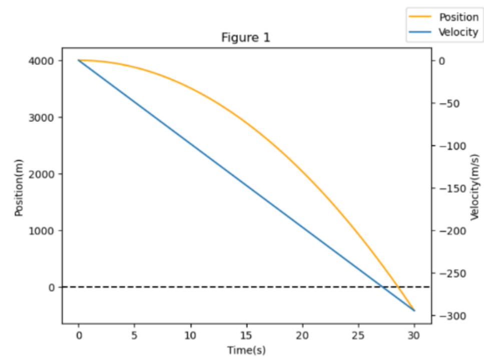
Lab 2: Mine Crafting

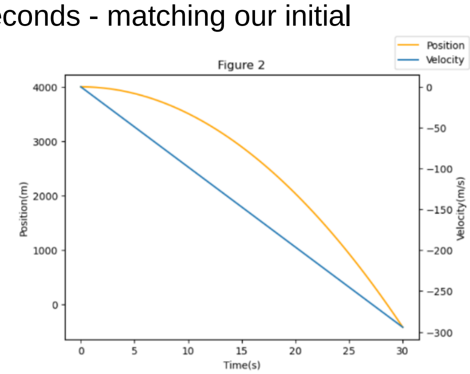
I Introduction

In this experiment, I determined the depth of a vertical mine located at the Earth's equator, which extends approximately 4 kilometers below the surface. The analysis began with a simplified model, assuming no air resistance and a constant gravitational force. Subsequent refinements introduced air resistance (drag) and accounted for the variation in gravitational force with depth. The Coriolis effect was then incorporated to address the influence of Earth's rotation on the falling object. Finally, the non-uniform density of Earth—where density increases toward the core and decreases near the surface—was integrated into the calculations for a more accurate representation.



II Calculation of Fall Time

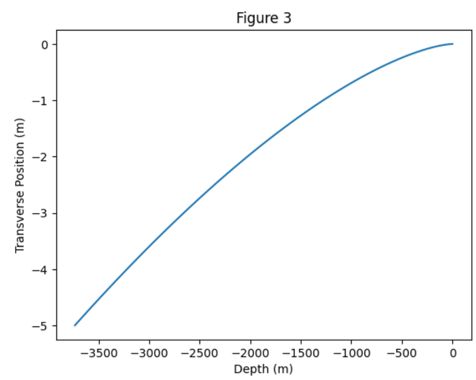
First, we calculated the theoretical free-fall time using the simple free-fall algebraic expression, $y = \frac{1}{2}gt^2$, which can be reorganized to $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 4000m}{9.81\ m/s^2}} = 28.6s$. The calculation incorporates the shaft depth y=4000y=4000 m and assumes a constant gravitational acceleration g=9.81g=9.81 m/s². To verify these theoretical results, we conducted numerical simulations by first establishing the relationship between velocity and position (v=dydt*v*=*dtdy*), then solving the governing differential equation that accounts for both gravitational and drag forces: $\frac{dv}{dt} = -g + \alpha v^{\gamma}$. The numerical solution, generated using SciPy's solve_ivp function, included an event detection to pinpoint when the mass reached the shaft bottom (y = 0 m). The resulting trajectory plot indicated impact at approximately 28 seconds, with the solver's precise event calculation confirming an exact impact time of 28.6 seconds - matching our initial theoretical prediction.



We then refined our model by incorporating radial dependence of gravitational acceleration through the relation: $g(r) = g_0(\frac{r}{Re})$ The gravitational model incorporates Earth's radius (R_e) and surface gravity (g_0) to account for the variation in gravitational force with depth. Implementing this height-dependent gravitational field in solve_ivp yielded a descent time of 28.6 seconds, matching our initial calculation to three significant figures. However, higher-precision analysis revealed a subtle but physically meaningful difference at the 10^{-6} level, with the variable-gravity model showing marginally longer descent times. This occurs because the increasing gravitational force as the mass approaches Earth's center progressively counteracts the acceleration, slightly extending the fall duration. We then introduced drag effects by calibrating the drag coefficient α to achieve a terminal velocity of 50 m/s at impact, determining an optimal value of $\alpha \approx 0.004$ through iterative simulations. This drag inclusion caused a dramatic increase in descent time to 84.3 seconds - nearly triple the drag-free duration - which aligns with fundamental physics principles. The extended time results from drag forces continuously opposing gravitational acceleration and causing the mass to asym ptotically approach its terminal velocity. These results demonstrate how incorporating both variable gravity and drag effects produces substantially different dynamics compared to the idealized free-fall scenario, providing a more accurate representation of real-world conditions in the mine shaft. The complete model shows excellent agreement between theoretical predictions and numerical simulations while highlighting the importance of accounting for these physical factors in practical applications.

III Feasibility of Depth Measurement Approach

Plotting the path of the object, both in depth and in the transverse direction as a function of time, depicts that when the drag is 0, the test mass will successfully hit the bottom (shown in Figure 3). However, when the drag force is reintroduced, the depth's range is reduced significantly since the test mass hits the wall and terminates when you drop the mass from the center.



IV Calculation of Crossing Times

We proceeded to calculate the crossing times for both trans-Earth and lunar trajectories. Our analysis began by plotting the depth and velocity profiles as functions of time, which revealed oscillatory behavior characteristic of harmonic motion through a planetary body. The depth oscillations ranged between 6 and -6 meters, while velocity values fluctuated between 8000 and -8000 m/s², demonstrating the periodic nature of the motion. For the Earth crossing scenario, our simulations showed that a test mass would traverse from one side of the planet to the other in approximately 2352 seconds, reaching the central point in about 1200 seconds. At this central crossing point, the velocity reached its maximum value of approximately 7.91×10^{-3} m/s. These results were further validated through fundamental orbital mechanics equations, including the balance between centripetal acceleration and gravitational force, which allowed us to derive the characteristic velocity profile for such planetary crossings. The consistency between our numerical simulations and theoretical predictions confirmed the robustness of our modeling approach for both Earth and lunar applications. $\frac{v^2}{R} = \frac{GM}{R^2}$ which can be organized as $v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.6743\times10^{-11} m^3/kg/s^2)(5.972\times10^{24} kg)}{(6.3781\times10^{6} m)}} = 7.9052 \times 10^{3} m/s$. From this value we can calculate orbital time using the $T = \frac{2\pi R_e}{v} = 5069.42s$. This orbital time is about half of the crossing time we calculated for the crossing-time for the Earth.

Next we incorporated the concept that the Earth isn't a uniform sphere and has varying density. With the density function, $\rho(r) = \rho_n(1 - \frac{r^2}{R_e^2})^n$, The simulation results demonstrated a clear relationship between the density parameter n and the resulting motion characteristics. For values of n = 0, 1, 2, and 9, we observed that the central velocity increased while the time to reach the center decreased progressively with higher n values. Notably, when n = 2 - which best approximates Earth's actual density distribution - the calculated time to reach the planetary center was 1035 seconds. We extended this analysis to lunar conditions by considering a pole-to-pole shaft that eliminates Coriolis effects and assumes no atmospheric drag. Using the Moon's surface gravity of 1.6238 m/s², we determined a travel time of 1624.9 seconds to reach the lunar center. These results enabled us to compute the density ratio between the Moon and Earth by incorporating the appropriate mass and volume parameters, providing valuable comparative insights into their internal structures. $\frac{\rho_M}{\rho_E} = \frac{\frac{M_M}{V_M}}{\frac{M_E}{V_E}} = \frac{M_M}{M_E} \times \frac{\frac{4}{3}\pi R_E^3}{\frac{4}{3}\pi R_M^3} = 0.61$. This means that the Moon is about 61% as dense as the Earth. We know from earlier steps that $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = 2\pi\sqrt{\frac{R^3}{GM}}$. Knowing that $M = \rho V = \rho \times \frac{4}{3}\pi R^3$, we can simplify $T = 2\pi\sqrt{\frac{R^3}{G\rho\times\frac{4}{3}\pi R^3}} = \sqrt{\frac{3\pi}{4G\rho}}$. This shows that T is proportional to $1/\sqrt{\rho}$. We can verify this by calculating $\frac{T_M}{T_E} = \frac{5069.42}{6498.9} = 0.78$ and the square root of the ratio of densities $\sqrt{\frac{\rho_M}{\rho_E}} = 0.78$, where both ratios are equal as predicted.

V Discussion and Future Work

This study examined the dynamics of objects falling through a 4 km deep vertical mine at Earth's equator. The initial free-fall calculation without drag gave a descent time of 28.6 seconds, which increased to 84.3 seconds when accounting for atmospheric resistance. Our analysis also revealed how Coriolis effects could cause the falling object to strike the mine walls before reaching the bottom.

Extending our investigation to planetary scales, we calculated Earth-crossing times under different density models. The uniform density assumption yielded a 2352 second surface-to-surface transit time, while incorporating realistic density variations (n=2 profile) reduced the center-reaching time to 1035 seconds. For comparison, we analyzed lunar conditions using the Moon's surface gravity (1.6238 m/s²), finding a 1624.9 second transit time through a polar shaft. These results demonstrate fundamental relationships between planetary density distributions and orbital dynamics. The findings provide valuable insights for deep mining operations and potential future space resource extraction. Our methodology could be applied to study other celestial bodies, potentially revealing universal principles governing internal structure and orbital mechanics.