Lab 2: Vertical Mine Dynamics

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# I. Introduction

This experiment analyzes the descent time of an object dropped into a vertical mine shaft located at the equator, with a depth of approximately 4 kilometers. Initially, the analysis considers a simplified scenario—ignoring drag and assuming a constant gravitational field. The model is then refined by accounting for air resistance and a gravity field that varies with depth. Further refinements include the Coriolis effect due to Earth’s rotation and variations in Earth’s internal density. These progressively complex models provide a more accurate depiction of the fall dynamics.

# II. Fall Time Calculations

To begin, the theoretical fall time was calculated using the classic kinematic equation for free fall. This provides an estimate based on a constant gravitational acceleration and a vertical distance of 4 km. Numerical solutions were then implemented using SciPy’s `solve\_ivp` to simulate position and velocity, confirming the time to impact at roughly 28.6 seconds.

Next, gravitational acceleration was modeled as a function of distance from the Earth’s center. Incorporating this variation slightly increased the fall time compared to the constant-gravity model, due to diminishing gravitational pull with depth.

To simulate a more realistic fall, drag force was added. By tuning the drag coefficient until terminal velocity approximated 50 m/s, it was determined that a coefficient around 0.004 was appropriate. With this, the descent time increased significantly to about 84.3 seconds, as the drag force slowed the object considerably.

# III. Evaluation of Measurement Feasibility

Further modeling examined whether the object would land at the bottom of the shaft or collide with the walls due to lateral deflections. Without drag, the object falls straight down. However, reintroducing drag and the Coriolis effect shows the trajectory veering off-center, causing premature contact with the shaft walls. This emphasizes the importance of launch positioning in precision depth measurements.

# IV. Trans-Earth and Lunar Crossing Time Calculations

A trans-Earth tunnel scenario was explored next. The simulation revealed oscillatory motion, with the object crossing from one side of the planet to the other in 2352 seconds and reaching the Earth's center in about 1200 seconds. Velocity peaked around 7.91 x 10⁻³ m/s at the center. The orbital time was calculated using derived gravitational equations, and found to be approximately twice the trans-Earth crossing time.

The Earth's density distribution was then modeled using different values of the exponent n in a radial density function. As n increased, the object's velocity at the center increased, and the time to reach the center decreased. For n = 2, which closely matches Earth's actual density profile, the descent time was about 1035 seconds.

An analogous simulation was conducted for the Moon, assuming a vertical shaft from pole to pole. The gravity on the Moon was taken as 1.6238 m/s², and the descent time to the center was found to be 1624.9 seconds. The Moon’s density was determined to be about 61% that of Earth’s. The ratio of descent times aligned with the inverse square root of the density ratio, confirming the theoretical relationship between gravitational acceleration and travel time.

# V. Conclusion and Future Exploration

This lab examined the fall dynamics in a 4 km deep mine shaft, beginning with simplified assumptions and progressively introducing real-world factors. The basic fall time was estimated at 28.6 seconds, while the inclusion of drag increased this to 84.3 seconds. The Coriolis effect introduced lateral motion, revealing potential for impact with shaft walls. Further analysis demonstrated that travel time through planetary bodies depends on internal density profiles. The observed correlation between orbital period and density was validated using Earth and Moon models. Future work could extend this study to other planetary bodies, reinforcing or refining the discovered relationships.