GRAPH THEORY

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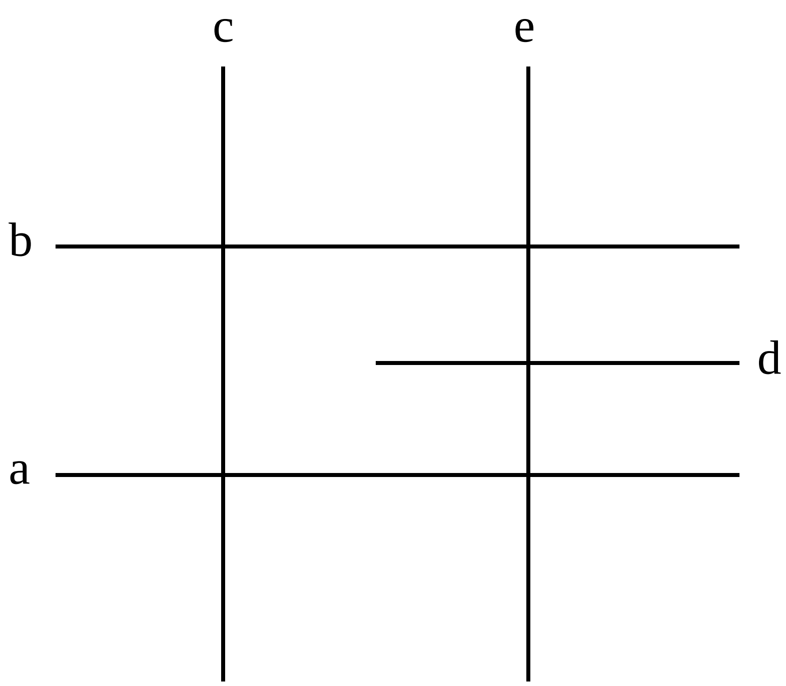
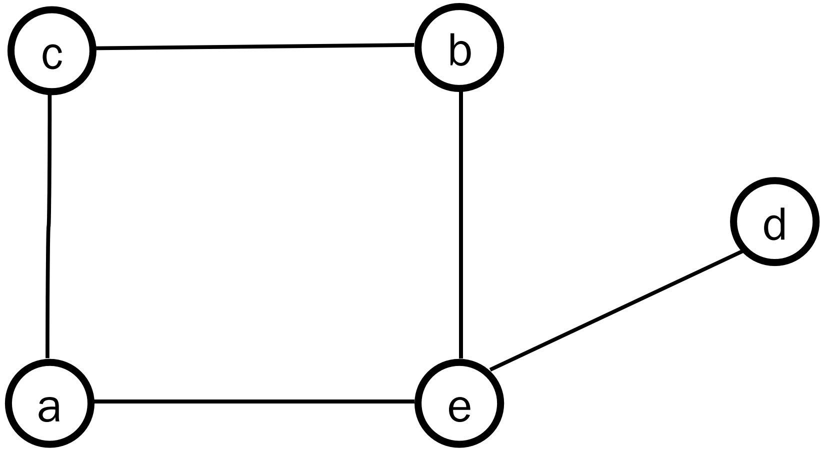
Objective

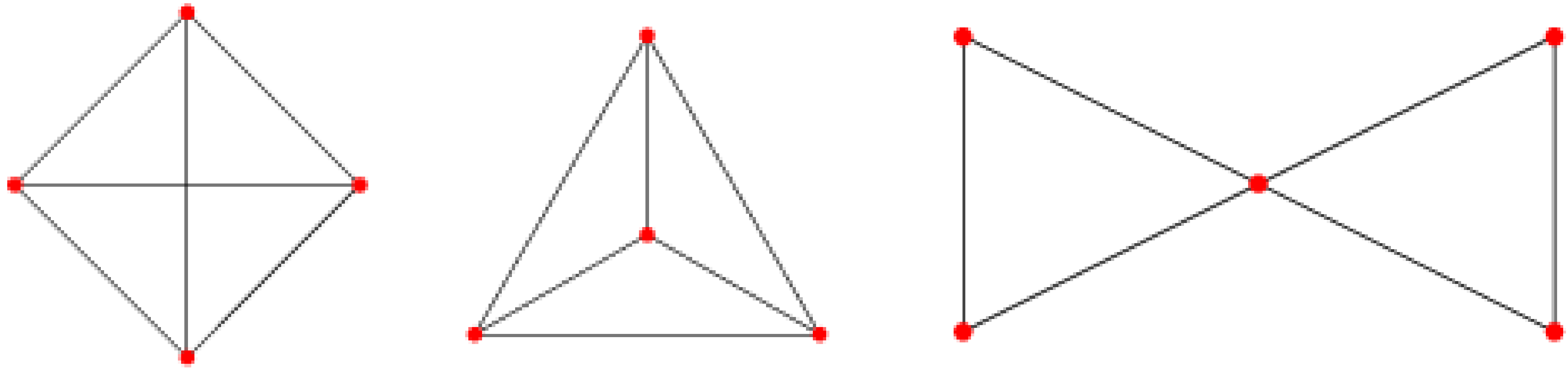
To characterize the **outerplanar graphs** which are also **B_0-VPG graphs**

B_0-VPG Graph

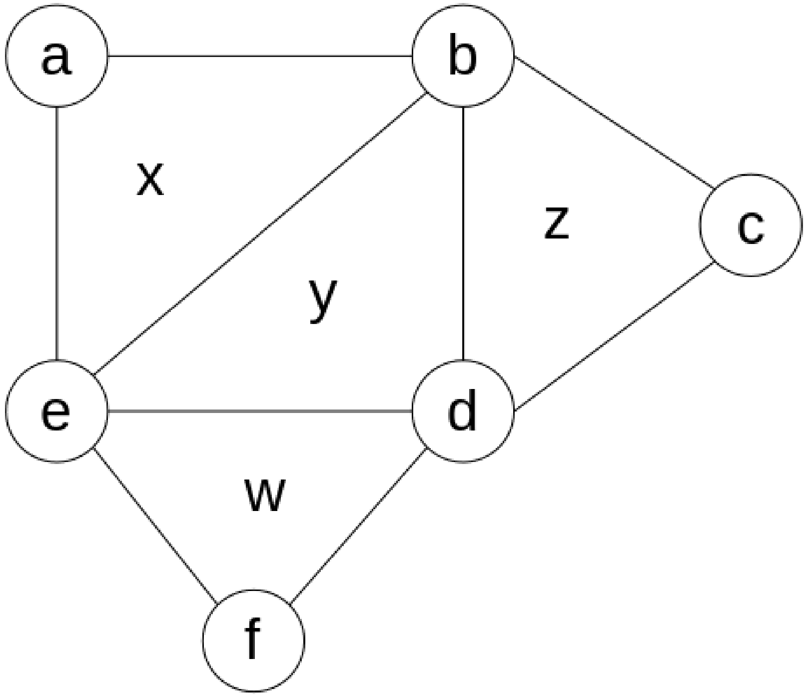


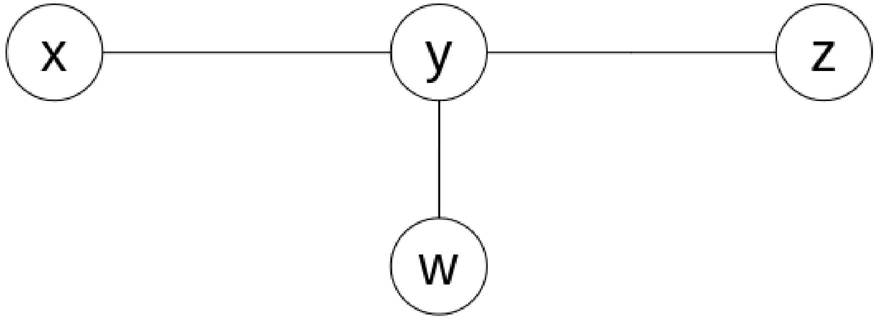


Planar and Outerplanar Graphs

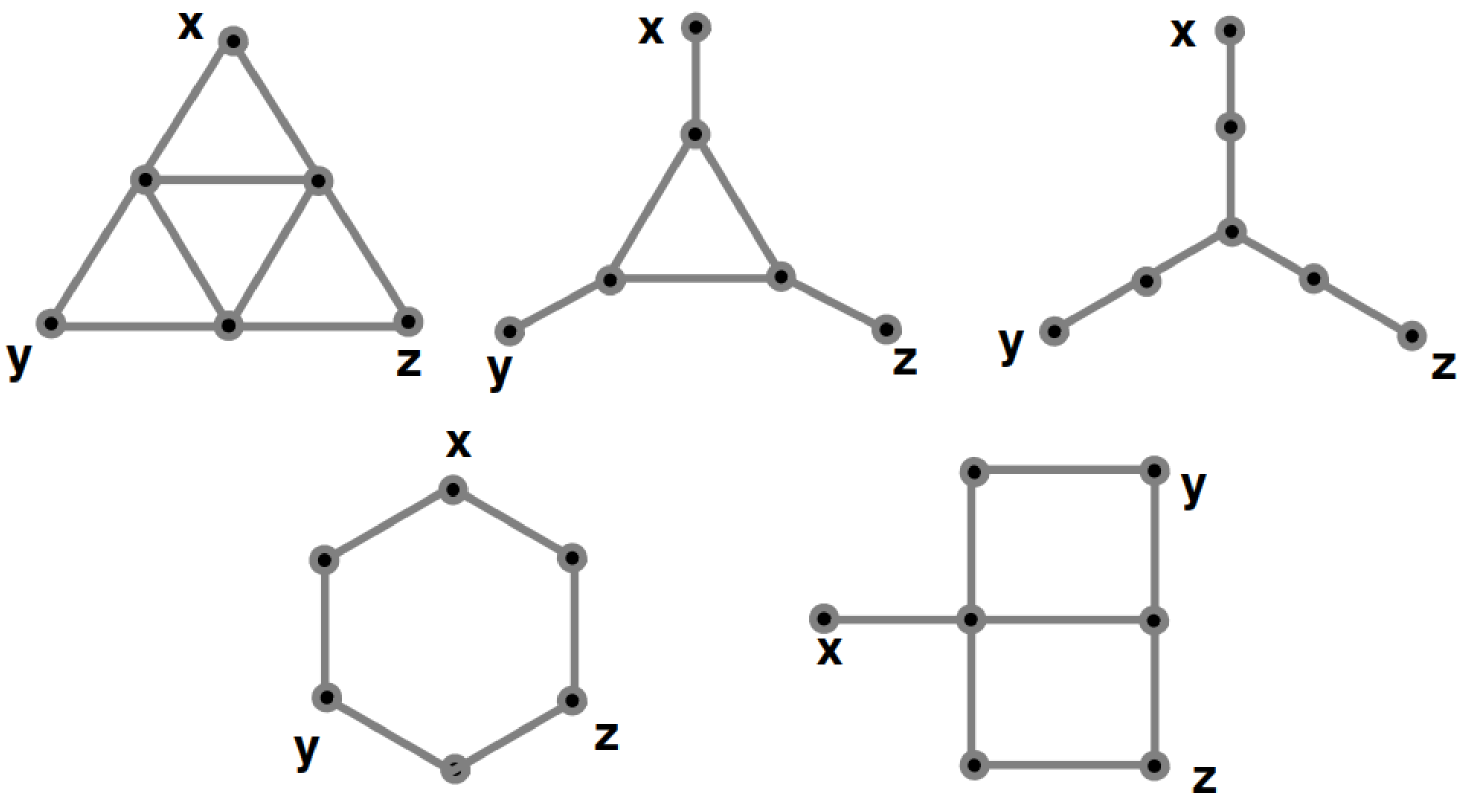


Weak Dual Graph

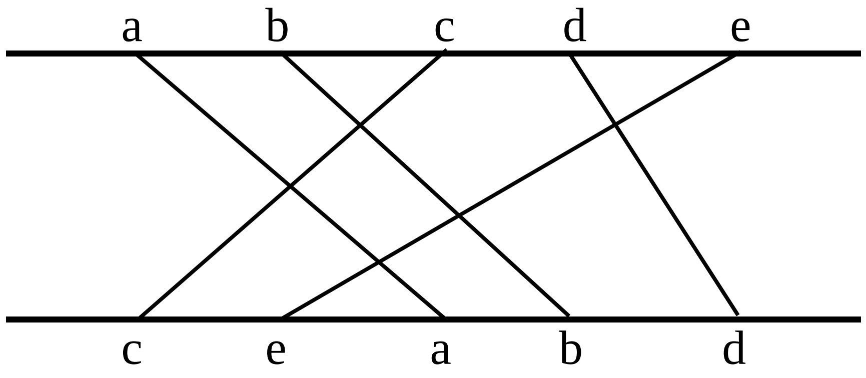


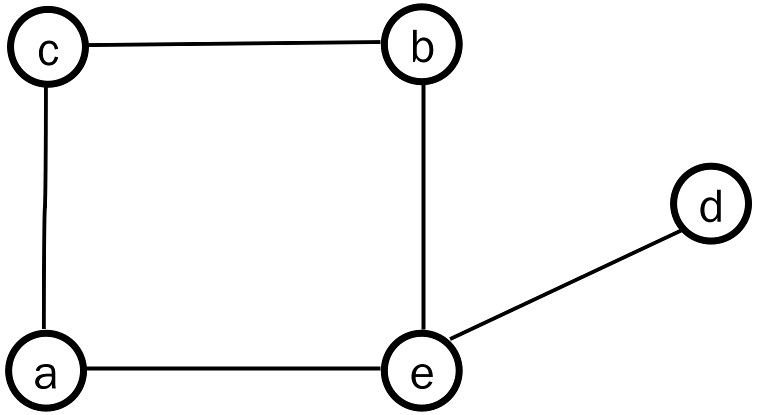


Asteroidal Triple

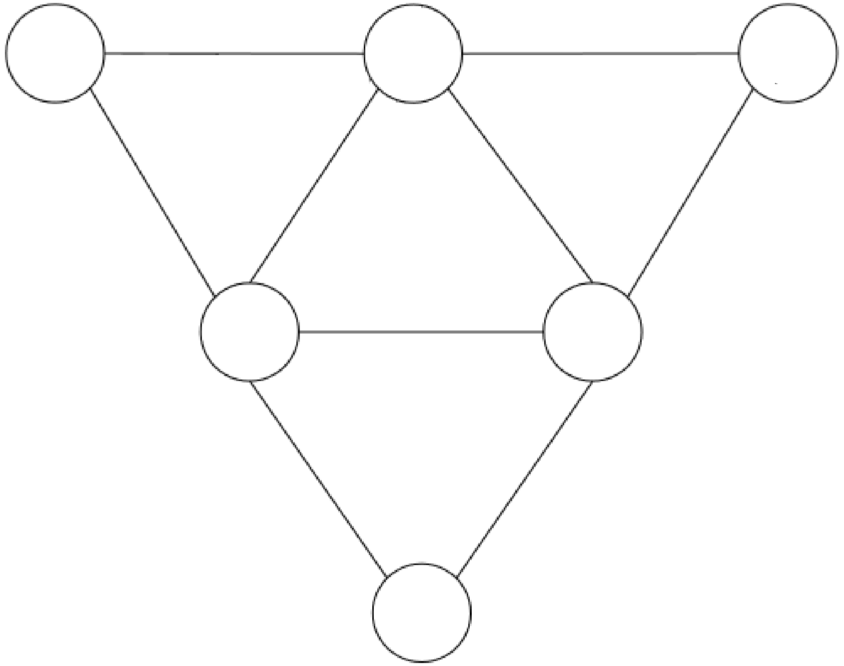


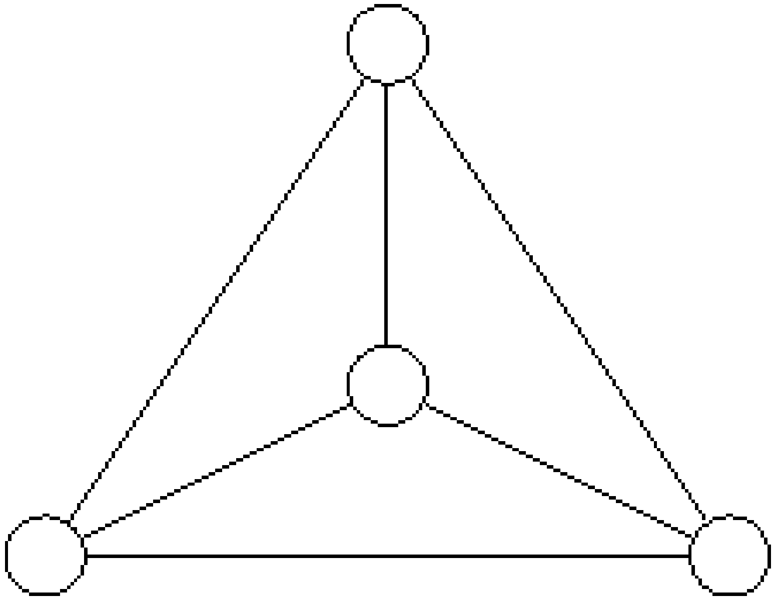
Permutation Graph





Counter Examples!





Prior Works and Important Theorems

- Bipartite permutation graphs, i.e. permutation graphs which are also bipartite graphs, are B_0 VPG graphs [1]
- A graph G is a permutation graph if and only if both G and its compliment G^c are comparability graphs [3]

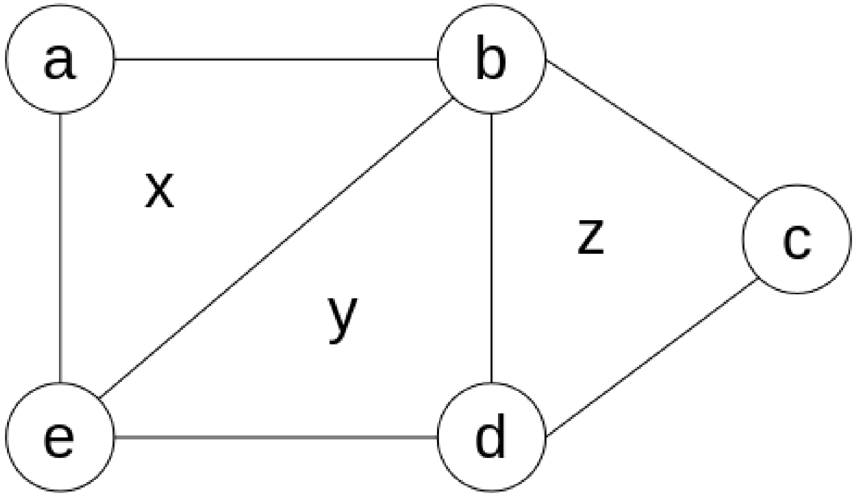
Prior Works and Important Theorems

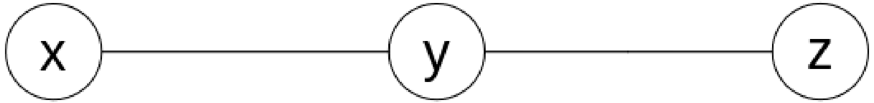
- Permutation graphs have no induced cycles of lengths 5 or greater [2]
- The graph C_4 has a unique B0 VPG representation; it consists of two horizontal parallel paths intersecting with two vertical parallel paths [4]

Prior Works and Important Theorems

- A planar graph is outerplanar if and only if its weak dual is a forest [5]. A simple corollary is that the weak dual of a 2-connected outerplanar graph is a tree.
- If a graph G is the compliment of a comparability graph, then G contains no asteroidal triple (i.e. G is AT-free) [6]. Thus, permutation graphs are also AT-free.

Define: Chain

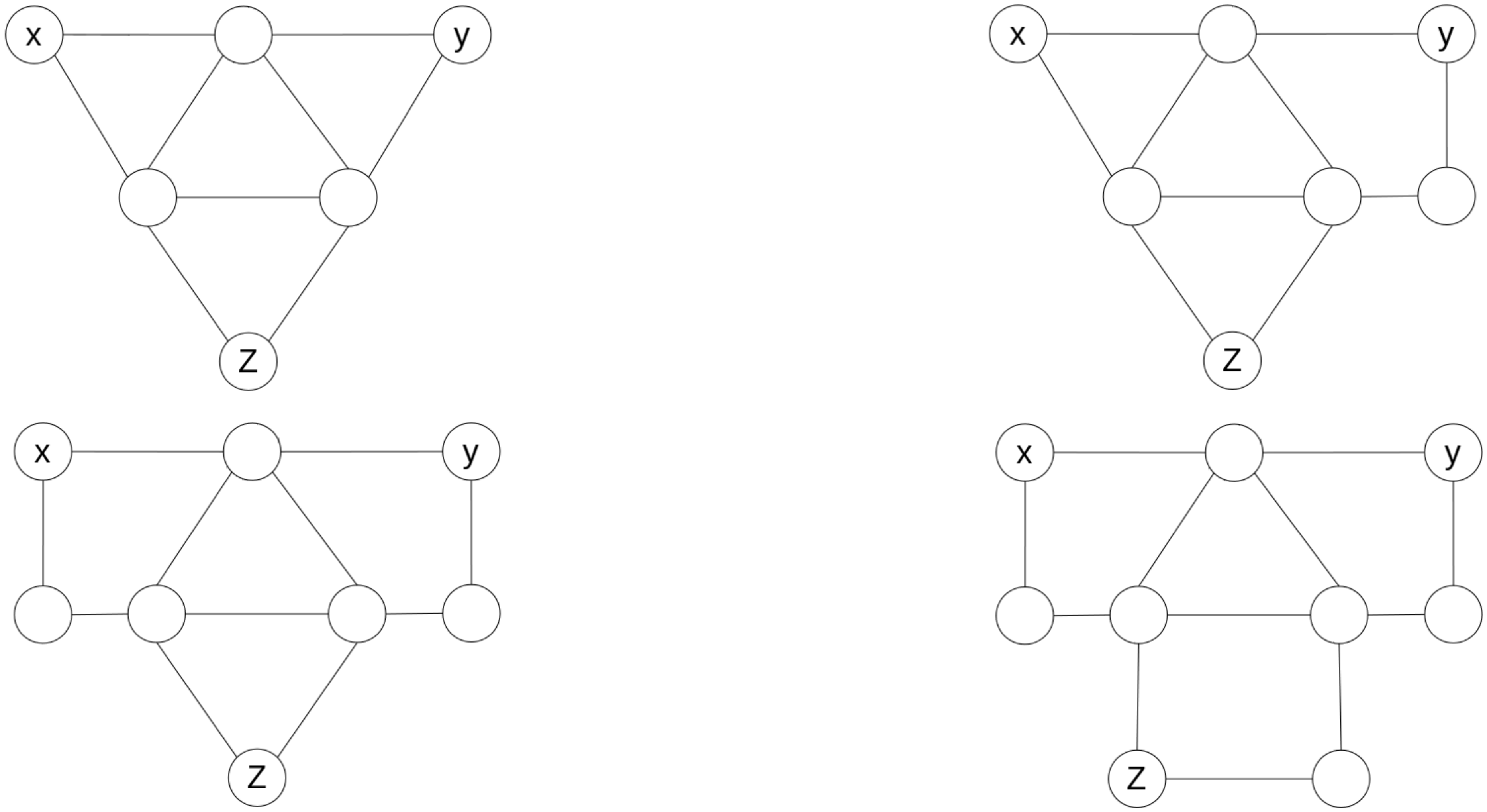




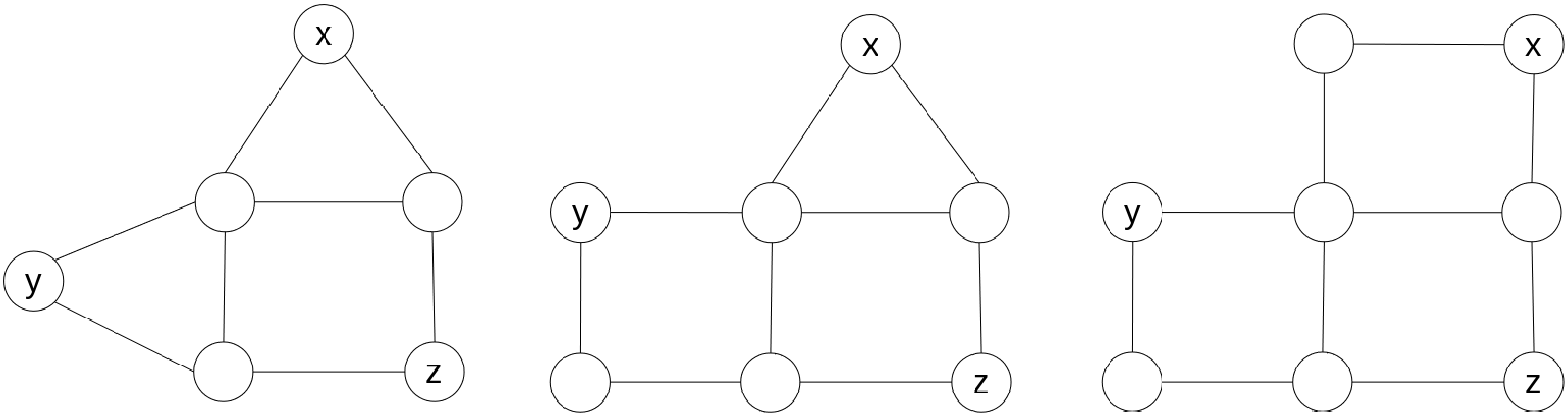
2-Connected Outerplanar Permutation Graphs are Chains

- **Claim:** Weak dual of a 2-Connected Outerplanar Permutation Graphs G is a path.
- **Proof:** Weak dual of a 2-Connected Outerplanar Graph is known to be a tree [5]. Thus it is sufficient to show that the maximum degree of weak dual of G is 2. Note that a bounded face of the graph G can either be a C_3 or a C_4 since Permutation Graphs cannot have induced cycles of length 5 or more [2].

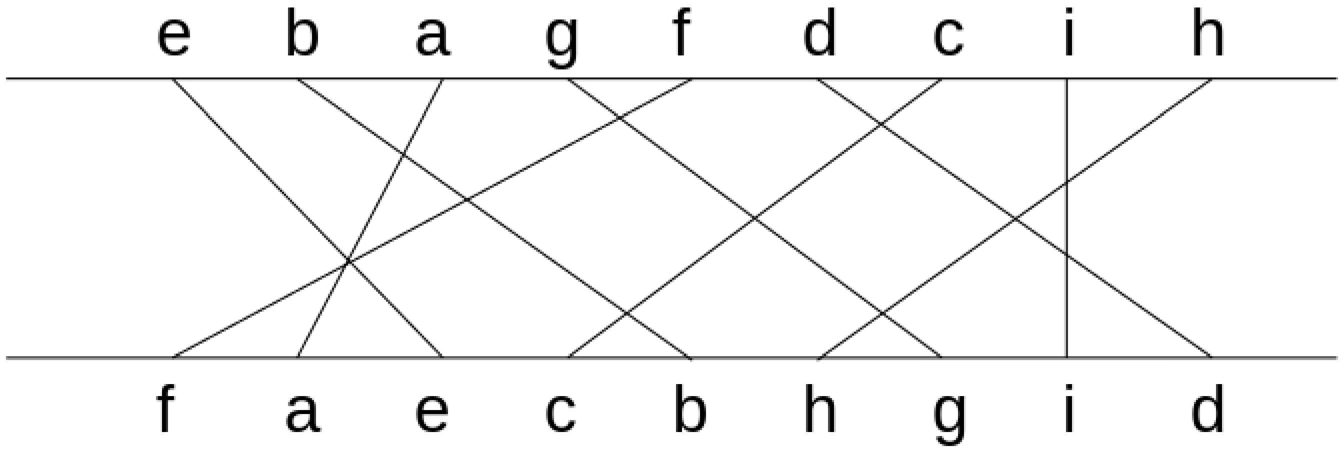
Case 1



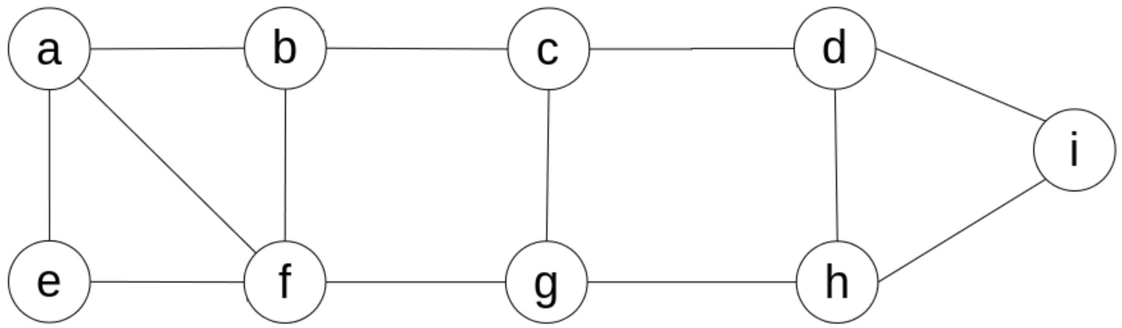
Case 2

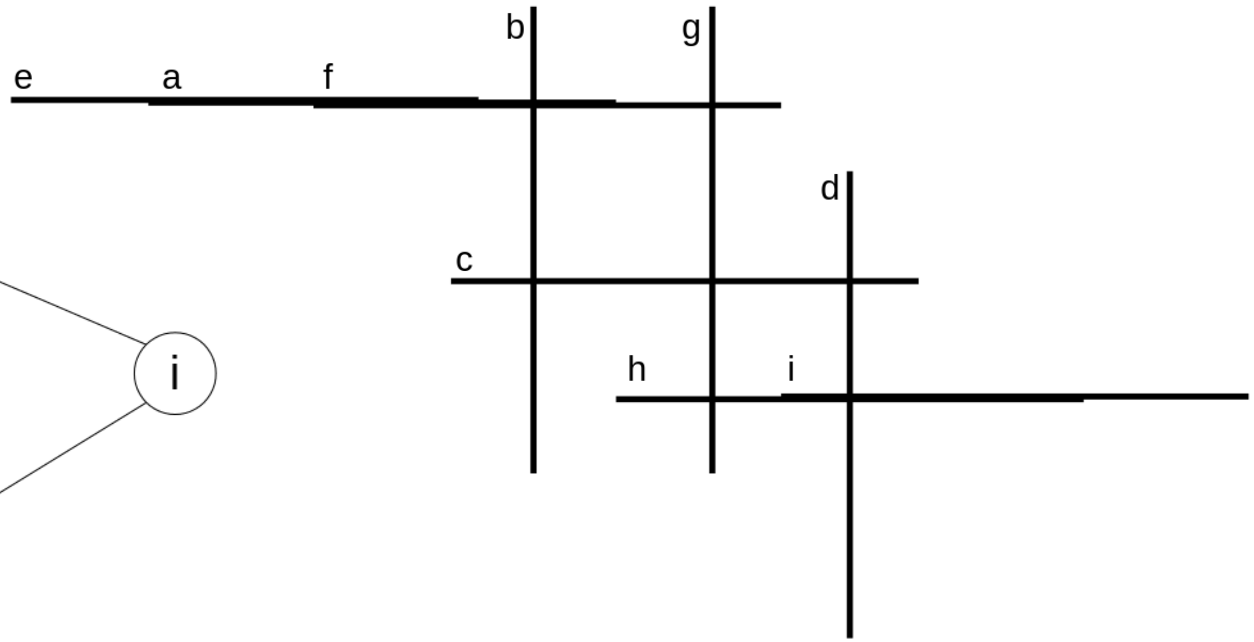


Example of a B_0-VPG



Example of a B_0-VPG





Conclusion and Future Work

- Devise a concrete algorithm to construct B_0-VPG Diagram for 2-Connected Outerplanar Permutation Graphs
- Since we are using only AT-Freeness and restriction of induced cycles to only C_3 and C_4, we may be able to target a bigger subclass of 2-Connected Outerplanar Graphs
- It will be interesting to further expand the idea and categorize Outerplanar Graphs which are also B_0-VPG graphs

References

1. E. Cohen, M. C. Golumbic, W. T. Trotter, and R. Wang, “Posets and vpg graphs,” Order, vol. 33, pp. 39–49, 2016.
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Thank You!