MATHS 7027

MATHEMATICAL FOUNDATION OF DATA SCIENCE

ASSIGNMENT-4

NAME: ARPIT GARG

ID: A1784072

SEMESTE 2, 2019

MASTER IN DATA SCIENCE

Answer - 1 (a)

$$A = \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}$$

Eigenvalues:

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$|\lambda I - A| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - 17 & -12 \\ -8 & \lambda + 3 \end{vmatrix} = 0$$

$$(\lambda - 17)(\lambda + 3) - (-8)(-12) = 0$$

$$(\lambda - 17)(\lambda + 3) = 96$$

$$\lambda^2 + 3\lambda - 17\lambda - 51 - 96 = 0$$

$$\lambda^2 \cancel{+} - 14\lambda - 147 = 0$$

$$\frac{+14 \pm \sqrt{14^2 - 4 \times -147}}{2} = \frac{+14 \pm 28}{2}$$

$\lambda_1 = 21 \quad \lambda_2 = -7$

Eigenvectors :

$(\lambda I - A)x = 0$

for $\lambda_1 = 21$

$$\left[\begin{bmatrix} 21 & 0 \\ 0 & 21 \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}\right] \underset{\sim}{x} = 0$$

$$\begin{bmatrix} 4 & -12 \\ -8 & 24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented form

$$\left[\begin{array}{cc|c} 4 & -12 & 0 \\ -8 & 24 & 0 \end{array}\right]$$

$R_2 \to R_2 + 2R_1$

$$\left[\begin{array}{cc|c} 4 & -12 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$R_1 \to R_1/4$

$$\left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Let $y = t$

$\cancel{4x - 12t = 0}$

$x - 3t = 0$

$x = 3t$

So $\underset{\sim}{x} = \begin{bmatrix} 3t \\ t \end{bmatrix}$

$\underset{\sim}{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} t$

Eigen vector for $\lambda = 21$

for $\lambda_2 = -7$

$(\lambda I - A)\underset{\sim}{x} = 0$

$$\left[\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}\right]\underset{\sim}{x} = 0$$

$$\begin{bmatrix} -24 & -12 \\ -8 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented form

$$\left[\begin{array}{cc|c} -24 & -12 & 0 \\ -8 & -4 & 0 \end{array}\right]$$

$R_1 \to R_1 - 3R_2$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ -8 & -4 & 0 \end{array}\right]$$

$R_2 \to -R_2/4$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 2 & 1 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$R_2 \to R_2/2$

3

$$\left[\begin{array}{cc|c} 1 & 1/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Let $y = t$

$x = -1/2\, t$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2\, t \\ t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} t$$

4

# VERIFICATION

Answer 1 a) Check for eigenvalue and eigenvector: $A\vec{v} = \lambda\vec{v}$

Comput $A\vec{v}$

$$\begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 63 \\ 21 \end{bmatrix}$$

When 21 common

$$21 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\lambda = 21$

verified $A\vec{v} = \lambda\vec{v}$.

Compute $A\vec{v}$

$$\begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 7/2 \\ -7 \end{bmatrix}$$

$$7 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$A\vec{v} = \lambda\vec{v}$

verified

Answer 1 b)

$$A = \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}$$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda-8 & 0 & -36 \\ -3 & \lambda-20 & 9 \\ -6 & 0 & \lambda-2 \end{vmatrix} = 0$$

$$(\lambda-8)\left[(\lambda-20)(\lambda-2) - (9\times 0)\right]$$
$$-(-3)\left[0-0\right] + 6\left[9\times 0 - (-36)(\lambda-20)\right]$$

$$\lambda_1 = -10$$
$$\lambda_2 = 20 \qquad \lambda_3 = 20$$

When $\lambda_1 = -10$

$$(\lambda I - A)\underset{\sim}{w} = 0$$

$$\left[\begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}\right]\underset{\sim}{w} = 0$$

$$\left[\begin{array}{ccc|c} -18 & 0 & -36 & 0 \\ -3 & -30 & 9 & 0 \\ -6 & 0 & -12 & 0 \end{array}\right]$$

$R_1 \to R_1 / 18$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ -3 & -30 & 9 & 0 \\ -6 & 0 & -12 & 0 \end{array}\right] \quad R_2 \to R_2 + 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & -30 & 15 & 0 \\ -6 & 0 & -12 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 + 6R_1 \\ R_2 \to R_2 / -30 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -\tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$

$x_2 = \frac{1}{2} t$

$x_1 = -2t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ \tfrac{1}{2} t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ \tfrac{1}{2} \\ 1 \end{bmatrix} t$$

When $\lambda_2 = 20$

$(\lambda I - A)\underset{\sim}{x} = 0$

$$\left[\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}\right]\underset{\sim}{x} = 0$$

$$\left[\begin{array}{ccc|c} 12 & 0 & -36 & 0 \\ -3 & 0 & 9 & 0 \\ -6 & 0 & 18 & 0 \end{array}\right] \quad R_1 \to R_1/12$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ -3 & 0 & 9 & 0 \\ -6 & 0 & 18 & 0 \end{array}\right] \quad R_2 \to R_2 + 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ -6 & 0 & 18 & 0 \end{array}\right] \quad R_3 \to R_3 + 6R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t \quad x_2 = s$

$x_1 = 3t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3t \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} s$$

# VERIFICATION

Answer 1 b)

$$A \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}$$

Check for eigenvalue and eigenvector

$$A\vec{v} = \lambda\vec{v}$$

$$\begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -16+36 \\ -6+10-9 \\ -12+2 \end{bmatrix} = \begin{bmatrix} 20 \\ -5 \\ -10 \end{bmatrix}$$

$$= -10 \begin{bmatrix} -2 \\ 1/2 \\ 1 \end{bmatrix}$$

$A\vec{v} = \lambda\vec{v}$

(verified!

$$\begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 24+36 & 0 \\ 9-9 & 20 \\ 18+2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 60 & 0 \\ 0 & 20 \\ 20 & 0 \end{bmatrix} = 20 \begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A\vec{v} = \lambda\vec{v}$$

verified

Answer 2:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix}$$

Eigenvalues:

$$|\lambda I - A| = 0$$

$$\lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda-1 & 0 & 0 \\ 4 & \lambda+7 & -5 \\ +8 & +10 & \lambda-8 \end{vmatrix} = 0$$

$$(\lambda-1)\left((\lambda+7)(\lambda-8)+50\right)$$

$$-4(0-0)+8(0-0)=0$$

$$(\lambda-1)(\lambda+7)(\lambda-8)+50\lambda-50=0$$

$$(\lambda^2+7\lambda-\lambda-7)(\lambda-8)+50\lambda-50=0$$

$$(\lambda^2+6\lambda-7)(\lambda-8)+50\lambda-50=0$$

$$\lambda^3-8\lambda^2+6\lambda^2-48\lambda-7\lambda+56+50\lambda-$$

$$\lambda^3-2\lambda^2-5\lambda+6=0$$

$$\lambda_1=1 \quad \lambda_2=-2 \quad \lambda_3=3$$

Eigenvectors:

$\lambda_1 = 1$

$(\lambda I - A)\underset{\sim}{x} = 0$

$$\left( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix} \right) \underset{\sim}{x} = \underset{\sim}{0}$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 4 & 8 & -5 & 0 \\ 8 & 10 & -7 & 0 \end{array} \right] \quad R_3 \to R_3 - 2R_2$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 4 & 8 & -5 & 0 \\ 0 & -6 & 3 & 0 \end{array} \right] \quad R_3 \to R_3 / 3$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 4 & 8 & -5 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] \quad R_3 \leftrightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 4 & 8 & -5 & 0 \end{array} \right] \quad \begin{matrix} R_3 \to R_3 + 4R_1 \\ R_1 \leftrightarrow R_3 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 4 & 8 & -5 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_2 \to R_2/-2$$

$$\left[\begin{array}{ccc|c} 4 & 8 & -5 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_1 \to R_1/4$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -5/4 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad R_1 \to R_1 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/4 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$

$x_2 - \frac{1}{2}t = 0$

$x_2 = \frac{1}{2}t$

$x_1 = \frac{1}{4}t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{4}t \\ \frac{1}{2}t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 1/4 \\ 1/2 \\ \vdots \end{bmatrix} t$$

for $\lambda = -2$

$$\left[\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix}\right] = 0$$

$$\left[\begin{array}{ccc|c} -3 & 0 & 0 & 0 \\ 4 & 5 & -5 & 0 \\ 8 & 10 & -10 & 0 \end{array}\right] \quad R_1 \to R_1 / -3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 4 & 5 & -5 & 0 \\ 8 & 10 & -10 & 0 \end{array}\right] \quad R_2 \to R_2 - 4R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 5 & -5 & 0 \\ 8 & 10 & -10 & 0 \end{array}\right] \quad R_3 \to R_3 - 8R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 10 & -10 & 0 \end{array}\right] \quad R_3 \to R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 \to R_2 / 5$$

$R_2 \to R_2/5$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$

$x_2 = x_3$

$x_2 = t$

$x_1 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} t$$

for $\lambda = +3$

$(\lambda I - A)x = 0$

$$\left[\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix}\right] = 0$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 4 & 10 & -5 & 0 \\ 8 & 10 & -5 & 0 \end{array}\right] \quad R_1 \to R_1/2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 4 & 10 & -5 & 0 \\ 8 & 10 & -8 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 8R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 10 & -5 & 0 \\ 0 & 10 & -5 & 0 \end{array}\right] \quad R_3 \to R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 10 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 \to R_2 / 10$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = t$

$x_2 = \frac{t}{2}$

$x_1 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t/2 \\ t \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1/2 \\ 1 \end{bmatrix} t$$

Answer 2 (cont...)

The diagonal matrix

(the diagonal entries are the eigenvalues - $\lambda_1, \lambda_2, \lambda_3$):

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

The matrix with eigenvectors as colums

$$P = \begin{bmatrix} 1/4 & 0 & 0 \\ 1/2 & 1 & \frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Now we need to find $P^{-1}$

$$\left[\begin{array}{ccc|ccc} 1/4 & 0 & 0 & 1 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - 4R_1$

$$\left[\begin{array}{ccc|ccc} 1/4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 & 1 \end{array}\right]$$

$R_1 \to 4R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 1/2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 1/2 & -2 & 1 & 0 \\ 0 & 0 & 1/2 & -2 & -1 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_3 \quad ; \; R_3 \to 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -4 & -2 & 2 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

$$A = P D P^{-1}$$

$$\begin{array}{ccc} P & D & P^{-1} \\ \begin{bmatrix} 1/4 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} & \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & -1 \\ -4 & -2 & 2 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1/4 & 0 & 0 \\ \frac{1}{2} & -2 & 3/2 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -16 & 8 \end{bmatrix} = A$$

$$P D P^{-1} = A$$

Hence verified

Answer - 3

PCA for X :

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} X_{ij}$$

Difference :

$$X' = X - 1_{n \times 1} \bar{x}$$

Covariance matrix :

$$C = \frac{1}{n-1} (X')^T X'$$

Now considering for Y

$$Y = \begin{bmatrix} X \\ \bar{X} \end{bmatrix}$$

where $\bar{X}$ is the mean

When we calculate the mean of Y which is equal to Y'

$$Y' = Y - 1_{n\times 1}\bar{y}$$

So we get the last ~~column~~ row as 0 so which is basically equal to X'.

Because when we are calculating the mean of X and $\bar{X}$ we get the same mean as $\bar{X}$ is the mean of X only.

So, when we calculate the difference of $Y - 1_{n\times 1}\bar{y} = Y'$ we get the last row as zeros.

So, when we calculate the covariance $C = \frac{1}{n-1}(Y')^T Y'$ so

we have one row of zero and one column of zero.

Since the difference is unchanged the principale components (and their eigenvalues) remain the same.

Considering the example:

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Step-1 Calculate mean:

$$\bar{\underset{\sim}{x}} = \left[\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right]$$

Step-2 Calculate difference $\bar{X}$ from mean

$$X' = X - 1_{n\times 1}\,\bar{x}$$

$$X' = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

Step-3 $C = \dfrac{1}{n-1}(X')^T X'$

$$C = \frac{1}{4-1}\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{bmatrix}$$

Now considering $Y$

$$Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Step-1: $\bar{Y} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Step 2: Calculate difference $\bar{Y}$ from mean

$$Y' = Y - 1_{n\times 1}\bar{Y}$$

$$Y' = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -3/2 & 1/2 \\ -3/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -3/2 \\ 0 & 0 & 0 \end{bmatrix} = X'$$

Step-3 $C = \frac{1}{n-1}(Y')^T Y'$

$$C = \frac{1}{4-1}\begin{bmatrix} \frac{1}{2} & 1/2 & -3/2 & 1/2 & 0 \\ 1/2 & -3/2 & 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 1/2 & -3/2 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 1/2 & 1/2 \\ \frac{1}{2} & -3/2 & 1/2 \\ -2/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

So they are equal to
$x'$.
So the difference is unchanged
the principale components
(and their eigenvalues) remain
the same.

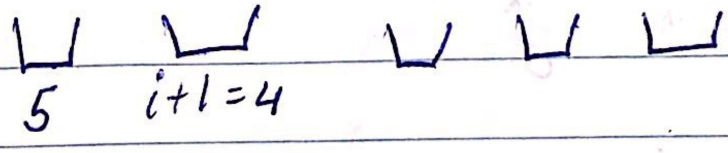
Answer - 4

a) Different seating arrangements possible for the directors

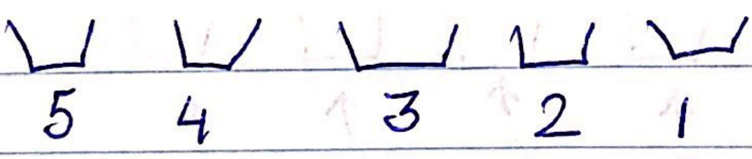
Directors = 5
Let's assume i, 5 chairs are there:


for i, 5 options are available any one of the director can sit there.

If one is already seated for i+1 i.e. next ~~see~~ seat 4 options are available.



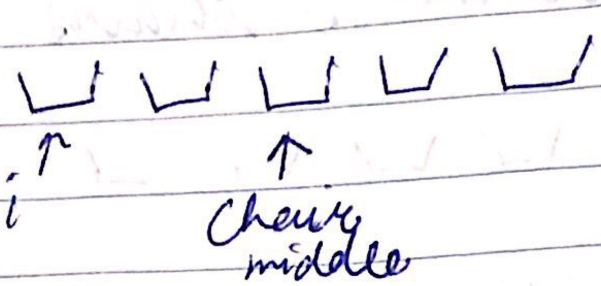

Similarly for all seats there will be



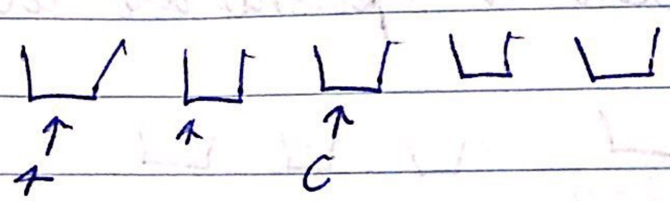

$\Rightarrow 5! = 120$ possible arrangements

Answer 4b)

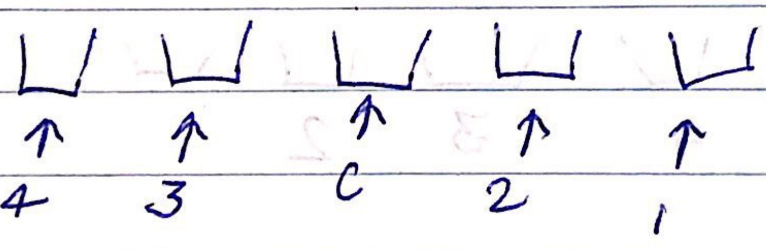
If one of the five directors was already nominated to chair the meeting and must sit in the middle




So now 4 people or directors left so for first $i \rightarrow 4$ options are available




So now for $i+1$, 3 options are available Similarly for all other so,

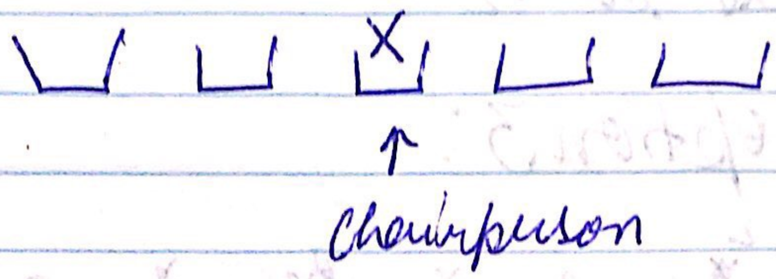



$4! = 24$ possible arrangements

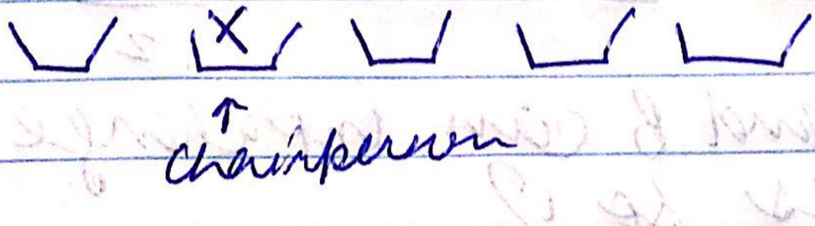
Answer 4c)

3 choices are available.
Consider two person A and B which donot sit together

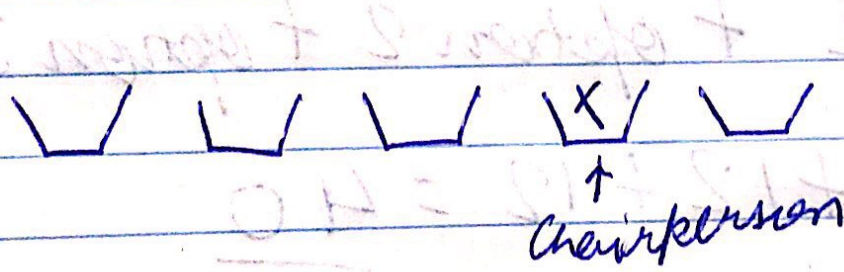
Option 1 :




Option 2 :



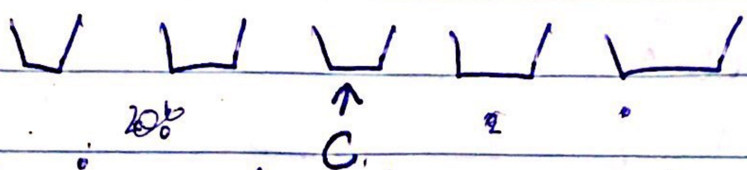

option 3 :



# Option 1:

Answer 4c) When chairperson is in the middle.

Two of the directors, neither of which is the chairperson don't get along and will sit on opposite sides of the chair




C = Chairperson.

Considering chairperson is in the middle.
Let X and Y don't get along =

Total no. of possible arrangements for 4 people – when they are sitting together.

Total no. of possible arrangements when chairperson is in middle
$= 4!$

When they two are sitting together =
$2 \cdot 2! \cdot 2!$

$= 4! - 2 \cdot 2! \cdot 2!$
$= 16$

Considering option 2:

A X B 2 1 × 2

A X 2 B 1 × 2

A X 2 1 B × 2

12

× 2 as A and B can interchange places so 12.

Considering option 3:

1 2 A X B × 2

1 A 2 X B × 2

A 1 2 X B × 2

12

× 2 as A and B can interchange places so 12.

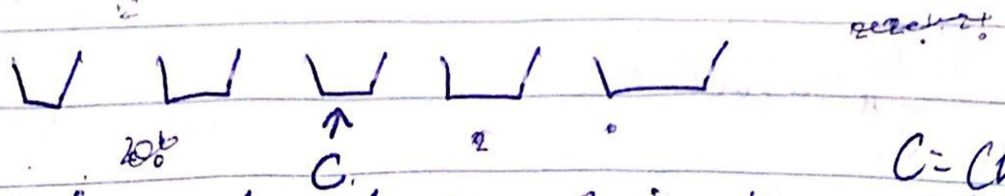
Option 1 + option 2 + option 3 =

16 + 12 + 12 = 40

Options a: # If assuming continuation of (4b

Answer 4c) When chairperson is the middle

Two of the directors, neither of which is the chairperson don't get along and will sit on opposite sides of the chair




C= Chairperson

Considering chairperson is in the middle.

Let X and Y don't get along =

Total no. of possible arrangements for 4 people - when they are sitting together.

Total no. of possible arrangements when chairperson is in middle = $4!$

When they two are sitting together = $2 \cdot 2! \cdot 2!$

$= 4! - 2 \cdot 2! \cdot 2!$

$= \underline{16}$ # assuming chairperson is always in middle.

In two cases when we are assuming Chairperson is fixed in middle then 16 is the answer.

But when we are not assuming the chairperson to be fixed in middle then answer is 40.