

MATHS 7027 Mathematical Foundations of Data Science

Assignment 4

Due: 4:59pm Tuesday 8 October 2019 via Canvas
(PDF only)

CHECKLIST

- ☐ Have you shown all of your working for every mathematical question?
- ☐ Are your questions submitted in order?
- ☐ Is the writing in your submission legible and clearly structured?
- ☐ Is your submission a single, correctly oriented, pdf file?
- ☐ If after the deadline, but within 24 hours, have you contacted us via email (if applying for an extension) and submitted your assignment online via Canvas?
- ☐ Have you checked that the assignment submitted is the correct one, as we cannot accept other submissions after the due date?

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1. Find the eigenvalues and eigenvectors of the following matrices. Make sure you check your answer, this will be worth one mark.

(a)

$$A = \begin{bmatrix} 17 & 12 \\ 8 & -3 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 8 & 0 & 36 \\ 3 & 20 & -9 \\ 6 & 0 & 2 \end{bmatrix}$$

2. If possible, diagonalise the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & -7 & 5 \\ -8 & -10 & 8 \end{bmatrix}$$

Make sure to check your answer (e.g. by verifying that $P^{-1}AP = D$), this will be worth one mark.

3. Consider $X \in \mathbb{R}^{n \times p}$ for which we have performed a principal component analysis (PCA), that is let $\bar{\mathbf{x}}_j = \frac{1}{n} \sum_{i=1}^n X_{ij}$, $X' = X - \mathbf{1}_{n \times 1} \bar{\mathbf{x}}$ and $C = \frac{1}{n-1} (X')^T X'$ as usual. Suppose we are given another row of data which happens to be equal to $\bar{\mathbf{x}}$ so that we now have the data $Y \in \mathbb{R}^{(n+1) \times p}$ which we can write as

$$Y = \begin{bmatrix} X \\ \bar{\mathbf{x}} \end{bmatrix}.$$

How does the PCA of Y differ from that of X ?

(Note: A related problem from the week 8 Practice Questions may be helpful here.)

4. At a company board meeting the five directors sit at the front facing the employees.
 - (a) How many different seating arrangements are possible for the directors?
 - (b) Suppose one of the five directors was nominated in advance to chair the meeting and must sit in the middle, how many different seating arrangements are there now?
 - (c) Two of the directors, neither of which is the chair person, don't get along and will sit on opposite sides of the chair. How many seating arrangements are there now?