MATHS 7027
MATHEMATICAL FOUNDATION OF DATA SCIENCE

ASSIGNMENT-05

NAME: ARPIT GARG
ID: A1784072
SEMESTE 2, 2019
MASTER IN DATA SCIENCE

Assignment-5

Answer 1 a)

$Pr(P^c)$

P^c = Total number of people doesn't own IP.
= 124

T = Total number of people
= 224

$$Pr(P^c) = \frac{P^c}{T} = \frac{124}{224} \approx 0.553$$

Answer 1 b)

$Pr(L \cdot G)$

LG denote the event they are a Labor/Greens

$LG = 105$
$T = 224$

$$Pr(L \cdot G) = \frac{105}{224} = 0.468$$

Answer 1 c)

$$Pr(P^c | LN)$$

$$P(LN) = \frac{105}{224}$$

$$P(LN \cap P^c) = P(LN \cap P) - P(LN)$$

$$P(LN \cap P^c) = \frac{105}{224} - \frac{58}{224}$$

$$= \frac{47}{224}$$

$$Pr(P^c | LN) = \frac{P(LN \cap P^c)}{P(LN)}$$

$$= \frac{47}{224} \Big/ \frac{105}{224}$$

$$= \frac{47}{105} = 0.447$$

Answer 1 d)

$$Pr(P|LG)$$

$$= \frac{P(LG \cap P)}{P(LG)}$$

$$= \frac{\frac{42}{224}}{\frac{105}{224}}$$

$$= \frac{42}{105} = 0.400$$

Answer 1 e)

$$Pr(O|P^c)$$

$$P(O) = P(O \cap P) + P(O \cap P^c)$$

$$P(O \cap P^c) = \frac{14}{224}$$

$$Pr(O|P^c) = \frac{P(O \cap P^c)}{P(P^c)} = \frac{14/224}{124/224}$$

$$P(O|P^c) = \frac{14}{124} = 0.1129$$

Answer 2 a)

$P_r(\text{Placebo}) = \frac{400}{1000} = 0.4$

$P_r(\text{Antibiotic}) = \frac{600}{1000} = 0.6$

$P(\text{Recovered} / P) = 0.30$

P = Randomly selected participant

$P(\text{Recovered}^C / P) = 1 - P(\text{Recovered})$

$= 1 - 0.30$

$= 0.7$

A = randomly selected participant takes antibiotic

$P(\text{Recovered} / A) = 0.75$

$P(\text{Recovered}^C / A) = 1 - 0.75$

$= 0.25$

(i) Person did not recover $= P(R^c)$

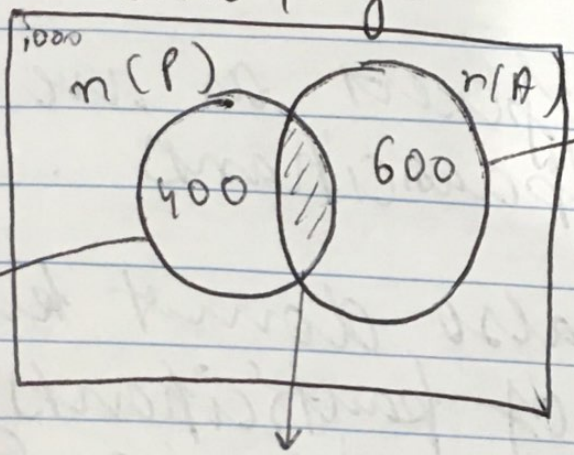
$= P(R^c/P)\,P(P) + P(R^c/A)\,P(A)$

$= 0.7 \times 0.4 + 0.25 \times 0.6$

$= 0.43$

(ii)

Venn Diagram



Recovered (25%)
= 150

recovered
70%
= 280

not recoverd = 430

Answer 2 b)

8% participants so $\frac{8}{100} \times 400 = 32$

21% participants so $\frac{21}{100} \times 600 = 126$

Total = 126 + 32 = 158

so $P = \frac{126}{158} = 0.797$

Answer 2 c i) Person taking second antibiotic recovered in three days

A_1 = patient takes second medicine

$$P(A_1) = \frac{100}{1000} = 0.1$$

$$P(A_1^c) = 1 - P(A_1)$$
$$= 1 - 0.1$$
$$= 0.9$$

$$P(R \mid A_1) = 0.78$$

$$P(R \mid A_1^c) = \text{Unknown}$$

Data is insufficient as we do not know no. of participants.

Answer 2 c

ii) Here we also don't know the number of participants who use both placebo and antibiotic so can't find the probability

Answer 4 a)

X can 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$P(X=1) = \frac{1}{6} \times {}^4C_0 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{1}{96}$$

$$P(X=2) = \frac{1}{6} \times {}^4C_1 \times \left(\frac{1}{2}\right)^4 + \frac{1}{6} \times {}^4C_0 \left(\frac{1}{2}\right)^4$$

$$= \frac{5}{96}$$

$P(X=3) = P($ 3 die and 2 heads $)$

$$= \frac{1}{6} \times {}^4C_2 \times \left(\frac{1}{2}\right)^4 + \frac{1}{6} \times {}^4C_1 + \frac{1}{6} {}^4C_0 \left(\frac{1}{2}\right)^4$$

$$= \frac{11}{96}$$

$$P(X=4) = \frac{1}{6} \times 2 \times \frac{1}{16} + \frac{1}{6} \times 6 \times \frac{1}{16} +$$

$$\frac{1}{6} \times 4 \times \frac{1}{16} + \frac{1}{6} \times 1 \times \frac{1}{16}$$

$$= \frac{5}{32}$$

$$P(X=5) = \frac{16}{96} \Rightarrow \frac{1}{6}\left(\frac{1}{16} + \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}\right)$$

$P(X=6) = 16/96 \Rightarrow \frac{1}{6}\left(\frac{1}{16} + \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}\right)$

$P(X=7) = 15/96 \Rightarrow \frac{1}{6}\left(\frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}\right)$

$P(X=8) = 11/96 \Rightarrow \frac{1}{6}(3/8 + 1/4 + 1/16)$

$P(X=9) = 5/96 \Rightarrow \frac{1}{6} \times \left(\frac{1}{4} + \frac{1}{16}\right)$

$P(X=10) = 1/96 \Rightarrow \frac{1}{6} \times \frac{1}{24} = \frac{1}{96}$

Answer 4 b)

b) $E(X) = \frac{1}{96} \times 1 + \frac{5}{96} \times 2 + \frac{11}{96} \times 3 + \frac{5}{32} \times 4$

$$+ \frac{1}{6} \times 5 + \frac{1}{6} \times 6 + \frac{5}{32} \times 7 +$$

$$\frac{11}{96} \times 8 + \frac{5}{96} \times 9 + \frac{1}{96} \times 10$$

$$= 5.5$$

Answer 4(c)

$$Var(X) = \frac{1}{96} \times 1^2 + \frac{5}{96} \times 2^2 + \frac{11}{96} \times 3^2$$

$$+ \frac{5}{32} \times 4^2 + \frac{1}{6} \times 5^2 + \frac{1}{6} \times 6^2 +$$

$$\frac{5}{32} \times 7^2 + \frac{11}{96} \times 8^2 + \frac{5}{96} \times 9^2$$

$$+ \frac{1}{96} \times 10^2 - (5.5)^2$$

$$= 34.167 - 30.25$$

$$= 3.917$$

Answer-3

A-3 Let A be the event 3 rolled on a dice

$$P(A) = 1/6$$

$$P(3|\text{All blue}) = \frac{P(AB|3)\,P(3)}{\sum P(AB|i)\,P(i)}$$

$$= \frac{P(AB|3)\,P(3)}{P(AB|1)\,P(1) + P(AB|2)\,P(2) + P(AB|3)\,P(3) + \cdots P(AB|6)\,P(6)}$$

$$= \frac{\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{1}{6}}{\frac{4}{10} \times \frac{1}{6} + \frac{4}{10} \times \frac{3}{9} \times \frac{1}{6} + \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{1}{6} + \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{1}{1}}$$

$P(AB|5)\,P(5) = 0$

$P(AB|6)\,P(6) = 0$

we only have 4 blue balls so 5 and 6 not possible

$$= \frac{1/180}{\frac{1}{15}\left(\frac{504 + 168 + 42 + 6}{504}\right)}$$

$$A = \frac{504}{180 \times 48} = 0.0583$$

Considering that balls are not replaced.