MATHS 7027 Mathematical Foundations of Data Science
Assignment 1
Due: 11am Tuesday 13 August 2017 via Canvas
(PDF only)

1. Consider the functions

$$f(x) = 1 + \frac{1}{x}; \quad g(x) = \frac{x+1}{x+2}.$$

   (a) State the domains of f and g.

   (b) Find: $f \circ g$; $g \circ f$; $f \circ f$; $g \circ g$.

   (c) For each function, find its inverse function, or explain why the inverse doesn't exist.

   (d) Find the range of the function(s) for which the inverse exists.

2. (a) If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 6x + 9$, find a function f such that $f \circ g = h$.

   (b) If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function (g) such that $f \circ g = h$.

3. Evaluate the following:

   (a)
   $$\sum_{i=2}^{97} \left(\frac{1}{i} - \frac{1}{i+1} \right)$$

   (b)
   $$\sum_{i=1}^{m} \left(\sum_{j=1}^{n} (i+j) \right)$$

   (c)
   $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left(\frac{i}{n} \right)^2$$

   (Simplify, then use the fact that $\lim_{n\to\infty} \frac{1}{n^c} = 0$, $\forall c > 0$.)

4. Test whether the following series converges:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n}$$