MATHS 7027
MATHEMATICAL FOUNDATION OF DATA SCIENCE

ASSIGNMENT-3

NAME : ARPIT GARG
ID : A1784072
SEMESTER 2 , 2019
MASTER IN DATA SCIENCE

ASSIGNMENT-3

Answer-1

Matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

General Matrix $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

for Commute $AB = BA$

So first finding AB (LHS)

$$AB = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

So multiplying both matrix we get

$$\begin{bmatrix} a-c & b-d \\ -a+c & -b+d \end{bmatrix} \quad -(1)$$

Now finding BA (RHS)

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a-b & -a+b \\ c-d & -c+d \end{bmatrix} \quad (2)$$

Now, equating both terms (1) and (2).

$$\begin{bmatrix} a-c & b-d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} a-b & -a+b \\ c-d & -c+d \end{bmatrix} \quad -③$$

$$AB = BA$$

Now equating terms in eqn ③

$$\left.\begin{aligned} a-c &= a-b \quad -(i) \\ b-d &= -a+b \quad -(ii) \\ -a+c &= c-d \quad -(iii) \\ -b+d &= -c+d \quad -(iv) \end{aligned}\right] -④$$

Solving ④

Solving (i) $\quad \cancel{a}-c = \cancel{a}-b$

$$c = b$$

Solving (ii) $\quad b-d = -a+b$

$$-d = -a$$

$$d = a$$

Solving (iii) $\quad -a+\cancel{c} = \cancel{c}-d$

$$-a = -d$$

$$a = d$$

Solving (iv) $\quad -b+\cancel{d} = -c+d$

$$b = c$$

Now using the above results in Matrix B, we get

$$\boxed{B = \begin{bmatrix} a & b \\ b & d \end{bmatrix}}$$

Now, verifying the matrix B

$$AB = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a-b & b-a \\ -a+b & -b+a \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} a-b & -a+b \\ b-a & -b+a \end{bmatrix}$$

So we can see that matrix B is satisfying the commute condition

$$\underline{AB = BA}$$

So final Matrix B is $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

3

Answer - 2

$$x_1 + 4x_2 - 6x_3 - 3x_4 = 3$$
$$x_1 - x_2 \qquad + 2x_4 = -5$$
$$x_1 \qquad + x_3 + x_4 = 1$$
$$x_2 + x_3 + x_4 = 0$$

So, our augmented matrix will be

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 1 & -1 & 0 & 2 & -5 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \quad (i)$$

$$R_2 \to R_2 - R_1$$
$$R_3 \to R_3 - R_1$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & -5 & 6 & 5 & -8 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$R_2 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & -5 & 6 & 5 & -8 \end{array}\right]$$

$$R_1 \to R_1 - 4R_2$$
$$R_3 \to R_3 + 4R_2$$
$$R_4 \to R_4 + 5R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -10 & -7 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 11 & 8 & -2 \\ 0 & 0 & 11 & 10 & -8 \end{array}\right]$$

$R_3 \to R_3/11$
$R_4 \to R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -10 & -7 & 3 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 8/11 & -2/11 \\ 0 & 0 & 0 & 2 & 6 \end{array}\right]$$

$R_1 \to R_1 + 10R_3$
$R_2 \to R_2 - R_3$
$R_3 \to R_3/2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 3/11 & 13/11 \\ 0 & 1 & 0 & 3/11 & 2/11 \\ 0 & 0 & 1 & 8/11 & -2/11 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$R_1 \to R_1 - 3/_{11} R_4$
$R_2 \to R_2 - 3/_{11} R_4$
$R_3 \to R_3 - 8/_{11} R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & +2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

Now from this reduced echelon form, we can find x_1, x_2, x_3 & x_4

$x_1 = 2$
$x_2 = 1$
$x_3 = 2$
$x_4 = -3$

Answer 2b) Verifying

$$\begin{bmatrix} 1 & 4 & -6 & -3 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2+4-12+9 \\ 2-1-6+0 \\ 2+0+2-3 \\ 0+1+2-3 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 1 \\ 0 \end{bmatrix} \quad - \text{ⓔ}(2)$$

This is the same as in eq^n (1)
~~so~~ as eq^n (1) = eq^n (2)
our result verified

number

Answer - 3

$$x + y + z = 0$$
$$2x - 6y - 2z = 0$$
$$2x + z = 0$$

Augmented matrix will be

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -6 & -2 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -8 & -4 & 0 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 0 & -8 & -4 \\ 0 & -2 & -1 \end{array}\right]$$

$R_3 \to R_3/2$; $R_2 \Leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -1/2 & 0 \\ 0 & -8 & -4 & 0 \end{array}\right]$$

$R_3 \to R_3 - 8R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

from row 1, we are getting

$$x + y + z = 0$$

from row 2, we are getting

$$y + \frac{1}{2} z = 0$$

No row for z, so let $z = t$

$$y = -\frac{1}{2} z$$

$$y = -\frac{1}{2} t$$

$$x + \left(-\frac{1}{2} t\right) + t = 0$$

$$x = -\frac{1}{2} t$$

$$(x, y, z) = \left(-\frac{1}{2} t, -\frac{1}{2} t, t\right)$$

Answer 4

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 8 & k \\ -4 & 3 \end{bmatrix}$$

Answer 4

a) $|AB|$

We need to find AB first

$$A\ (3\times 2) = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B\ (2\times 3) = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

Multiplication possible :

$$AB = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-4 & -2-6 & -1+0 \\ 1+2 & -2+3 & -1+0 \\ 2 & -4 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -8 & -1 \\ 3 & 1 & -1 \\ 2 & -4 & -2 \end{bmatrix}$$

Now finding determinant $|AB|$

$$|AB| = -3\begin{vmatrix} 1 & -1 \\ -4 & -2 \end{vmatrix} - 3\begin{vmatrix} -8 & -1 \\ -4 & -2 \end{vmatrix} + 2\begin{vmatrix} -8 & -1 \\ 1 & -1 \end{vmatrix}$$

$$-3(-2 - \cancel{+}4) \quad -3(16-4) + 2(8+1)$$

$$18 \quad -36 \quad +18$$

$$= 0$$

Answer 4-b) $|C^{-1}|$

We know that $C \cdot C^{-1} = I$

$$C = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$$

$$C \cdot C^{-1} = I$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10

Let $C^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$C \cdot C^{-1} = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$CC^{-1} = \begin{bmatrix} 4a + c & 4b + d \\ 2a + \cancel{2}c & 2b + d \end{bmatrix}$$

Equating it with identity matrix

$$\begin{bmatrix} 4a + c & 4b + d \\ 2a + c & 2b + d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$4a + c = 1 \quad (1)$$

$$4b + d = 0$$

$$b = -d/4 \quad (2)$$

$$2a + c = 0$$

$$a = -c/2 \quad (3)$$

$$2b + d = 1 \quad (4)$$

Using (1) and (3)

$$= 4 \times -\frac{c}{\cancel{2}} + c = 1$$

$$-2c + c = 1$$

$$-c = 1$$

$$c = -1$$

$a = -c/2$

$a = 1/2$

Using 2 and 4

$b = -d/4$

$2b + d = 1$

$2 \times -\frac{d}{4} + d = 1$

$-\frac{d}{2} + d = 1$

$\frac{d}{2} = 1$

$d = 2$

$b = -2/4$ $b = -1/2$

$a = 1/2$ $b = -1/2$ $c = -1$ $d = 2$

$$C^{-1} = \begin{bmatrix} 1/2 & -1/2 \\ -1 & 2 \end{bmatrix}$$

So $|C^{-1}|$ (determinant)

$(1/2 \times 2) - (-1/2 \times -1)$

$|C^{-1}| = 1 - 1/2 = \boxed{1/2}$

12

Verifying

$$C \cdot C^{-1} = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ -1 & 2 \end{bmatrix}$$

$$C \cdot C^{-1} = \begin{bmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{bmatrix}$$

$$C \cdot C^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C \cdot C^{-1} = I$$

Verified and $C^{-1} = \frac{1}{2}$

Answer 4 c)

$$D = \begin{bmatrix} 8 & x \\ -4 & 3 \end{bmatrix}$$

So for the matrix to be invertible $|D| \neq 0$

So,

$$(8 \times 3 - (-4 \times x)) \neq 0$$

$$24 + 4x \neq 0$$

$$x \neq -6$$

So $\{x \in \mathbb{R} \mid x \neq -6\}$

or

So, set of x is $x \in \{R - (-6)\}$

or

$R \setminus \{-6\}$

for all values of real number except $-6 \rightarrow D$ is invertible

14