ASEN 5519 - Algorithmic Motion Planning
Fall 2020
Homework 2

Assigned September 22; Due October 6

Exercise 1. Define a semi-algebraic set that models the shape in Figure 1.

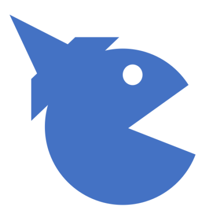

Figure 1: Birthday Pac-man.

Exercise 2. Consider the following Euler representation of rotation:

$$R(\alpha, \beta, \gamma) = R_z(\gamma) R_y(\beta) R_z(\alpha).$$

(a) Determine matrix $R(\alpha, \beta, \gamma)$.

(b) Show that $R(\alpha, \beta, \gamma) = R(\alpha - \pi, -\beta, \gamma - \pi)$.

(c) Given a rotation matrix R', determine α, β, and γ in terms of elements of R'.

Exercise 3. Consider the 3-link manipulator in Figure 2. The links $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ are identical.

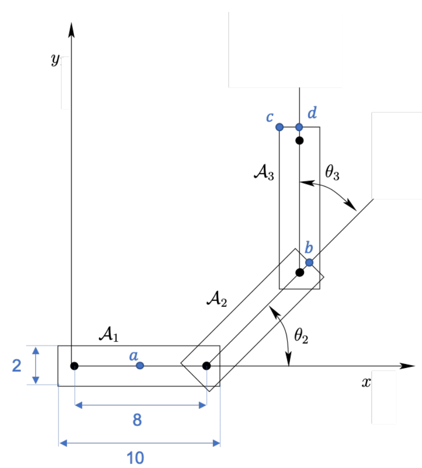

Figure 2: 3-link manipulator.

(a) For the configuration $(\theta_1, \theta_2, \theta_3) = (\pi/4, \pi/2, -\pi/6)$, determine the locations of points a, b, and c.

(b) Find the configuration(s) of the robot when point d is at $(0, 4)$.

Exercise 4. Express the configuration spaces of the following systems in terms of a Cartesian product of simpler spaces (such as $\mathbb{R}^n$, $\mathbb{S}^n$, etc.) and determine their dimensions. Justify your answer.

(a) Two trains on two train tracks.

(b) A spacecraft that can translate and rotate in 2D.

(c) Two mobile robots rotating and translating in the plane.

(d) Two translating and rotating planar mobile robots connected rigidly by a bar.

(e) A cylindrical rod that can translate and rotate in 3D. (Hint: if the rod is rotated about its central axis, it is assumed that the rod's position and orientation are not changed in any detectable way.)

(f) A spacecraft that can translate and rotate in 3D and is equipped with a 3-link robot arm (revolute joints only).

(g) The manipulator in Figure 3.

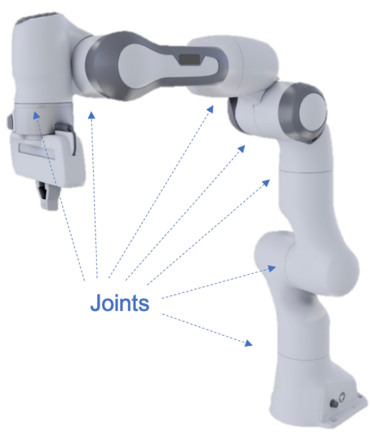

Figure 3: Robotic manipulator with revolute joints

Exercise 5. Consider workspace $W \subseteq \mathbb{R}^2$ with one obstacle in the shape of a triangle with vertices $(0, 0)$, $(0, 2)$, and $(1, 2)$.

(a) Determine the coordinates of all the vertices of the C-space obstacle for a robot with the same shape as the obstacle that is limited to translational motions in W.

(b) Plot the C-space obstacle for the same robot with the additional capability of rotational motion in W. (Hint: use a discretization of the additional parameter and the implementation of the algorithm used in part (a).) Use a view angle that clearly displays the full dimensionality and shape of the obstacle.

Exercise 6. Consider workspace $W \subseteq \mathbb{R}^n$ with convex obstacles. Show that the C-space obstacles are also convex for a convex robot with transitional motion in W.

Exercise 7. Implement a kinematic model for a planar 3-link manipulator with the following capabilities:

- The user can specify the length of each link as well as the configuration, in which case the output is the visual display of the robot configuration as well as the exact location of the end point on the final link.
- The user can specify the length of each link as well as the desired location of the end point, in which case the output is the visual display of the robot and the numeric values of the angles.

Illustrate the performance of your implementation on two examples: one on forward kinematics and one on inverse kinematics.

Exercise 8. Implement a C-space constructor for a 2-link manipulator in a 2-dimensional workspace with polygonal obstacles (hint: use a fine grid). Assume the base of the robot is at the origin of the workspace. The program should have the following capabilities:

- The user can specify the length of each link, number of obstacles, and vertices of each obstacle.
- The program displays the C-space with its obstacles.

Illustrate the performance of your implementation for a robot with link lengths of 1 in the following three workspaces:

(a) a workspace with a triangular obstacle with vertices $(0.25, 0.25)$, $(0, 0.75)$, and $(-0.25, 0.25)$.

(b) a workspace with two large rectangular obstacles with vertices:

$$\begin{aligned} O_1 : &\quad (-0.25, 1.1),\ (-0.25, 2),\ (0.25, 2),\ \text{and}\ \ (0.25, 1.1), \\ O_2 : &\quad (-2, -2),\ (-2, -1.8),\ (2, -1.8),\ \text{and}\ \ (2, -2). \end{aligned}$$

(c) a workspace with two obstacles: O_1 from part (b) and

$$O_2' : \quad (-2, -0.5),\ (-2, -0.3),\ (2, -0.3),\ \text{and}\ \ (2, -0.5).$$

Plot both the workspace and C-space for each case.