ASEN 5519 - Algorithmic Motion Planning
Fall 2020

Mini Project

Assigned November 15; Due November 23

INSTRUCTIONS:

1. For this project, you may use material outside of the class under the condition that appropriate citations will be provided in the solutions.
2. You may **not** use assistance in any form (e.g., brainstorming) from anybody, including fellow students.
3. The solutions must be **typed** and submitted on Gradescope.
4. The solutions must contain a cover page with your name, the following statement, and your signature:

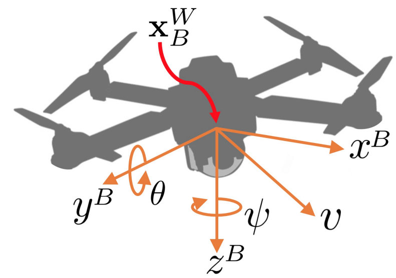
   > **HONOR CODE PLEDGE:** '*On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.*'

5. For Extra Credit 1 & 2, you must submit your code with line-by-line instructions on how to run your code.

Scenario (imaginary). The department of Aerospace Engineering Sciences at CU Boulder holds an annual autonomous aerial system competition in the Autonomous Systems Programming Evaluating and Networking (ASPEN) Lab. Due to your impressive knowledge of motion planning, you have been selected to participate in the competition. You are provided with a drone, whose schematics is shown below. The challenge is to design a safe navigation system that can take the vehicle through an obstacle course that extends from one side of the lab to another. Fortunately, ASPEN Lab is equipped with a motion capture system that can perfectly detect the state of the vehicle in a protected area that is 25-meter long, 10-meter wide, and 6-meter high. Hence, the autonomous navigation problem can be reduced to a motion planning problem.

A (simplified) motion model of the aerial system (vehicle) is given by

$$
\begin{aligned}
\dot{x} &= v\ \cos(\psi)\ \cos(\theta), \\
\dot{y} &= v\ \sin(\psi)\ \sin(\theta), \\
\dot{z} &= z \sin(\theta), \\
\dot{\psi} &= \omega \\
\dot{\theta} &= \alpha \\
\dot{v} &= a,
\end{aligned}
$$



where x, y and z correspond to the position of the vehicle with respect to a predefined reference frame, v is the forward velocity, and ψ and θ define the system's orientation around the z-axis and y-axis, respectively. The control inputs $u_1 = \omega$ and $u_2 = \alpha$ are the turning rates and $u_3 = a$ is the linear acceleration. For simplicity, you can model the shape of the drone as a rectangular box with the width and length of 0.6 meters and height of 0.3 meters. To ensure stability, you are advised to impose the following constraints to the states of the vehicle:

$$\theta \in [-\frac{\pi}{3}, \frac{\pi}{3}], \quad v \in [-1, 1], \quad u_1, u_2 \in [-\frac{\pi}{6}, \frac{\pi}{6}], \quad u_3 \in [-\frac{1}{2}, \frac{1}{2}].$$

To succeed in the competition, you need to design a motion planning algorithm that, given an initial state $(x_0, y_0, z_0, \psi_0, \theta_0, v_0)$, a destination set defined by $x_d \in [x_d^-, x_d^+]$, $y_d \in [y_d^-, y_d^+]$, $z_d \in [z_d^-, z_d^+]$, and $v_d \in [v_d^-, v_d^+]$, and the map of the obstacles, computes a valid trajectory that respects all the constraints and takes the drone from the initial state to a point in the destination set without colliding with an obstacle and leaving the protected area.

(a) What space will you choose for motion planning for this vehicle? Define the topology and dimension of this space and state your reasons for this choice.

(b) Describe the advantages and disadvantages of each method of motion planning listed below for your problem.

- i. Gradient descent planner with a potential function
- ii. Wave-front planner
- iii. Probabilistic roadmap planner
- iv. Randomized tree-based planner

(c) Design an appropriate motion planner for the drone. Present the planner with a detailed pseudocode and explain each line. Make sure you define all the necessary elements.

(d) Can you guarantee that your planner will find a trajectory for every given obstacle course? Justify your answer.

(e) The competition also keeps track of the time that each drone takes to complete the course. Are you be able to modify your algorithm to optimize for travel time? Justify your answer.

(f) You have just been notified that there are ropes hanging from the ceiling of the ASPEN Lab. The exact locations of the ropes are not given, but you are assured that they are located along a given straight line (vertical plane) across the lab. If the drone hits a rope, one of its motors will be damaged. In that case, the drone continues flying by turning off the damaged motor, which will cause a reduction in its speed and a positive or negative bias in its yaw depending on which motor is turned off. Describe the dynamics of the drone in this environment. (Hint: write the dynamics before and after the drone passes through the given cross section that the ropes are hanging)

(g) Design a motion planner for the drone in the ASPEN Lab with hanging ropes. First, describe your method in a simple language. Then, write a detailed pseudocode for it and provide an explanation for each line. Make sure all the components are well-defined.

(h) Can you guarantee that your algorithm in part (g) will find a solution if one exists? Justify your answer.

Extra Credit 1. Implement your planner in part (c) and plot three generated motion plans in the following environment:

- Initial condition: $x_0 = 1$, $y_0 = 1$, $z_0 = 1$, $\psi_0 = 0$, $\theta_0 = 0$, and $v = 0$.
- Destination set: $x_d \in [23, 24]$, $y_d \in [8, 9]$, $z_d \in [4, 5]$, and $v_d \in [-\frac{1}{20}, \frac{1}{20}]$
- Obstacles: four identical cube-shaped obstacles. The sides are 1-meter long and aligned with x-, y-, and z-axis, and the center points are:

$$\begin{aligned}
&\text{center of } O_1 : (4.5, 7.5, 1.5),\\
&\text{center of } O_2 : (9.5, 3.5, 4.5),\\
&\text{center of } O_3 : (14.5, 7.5, 1.5),\\
&\text{center of } O_4 : (19.5, 3.5, 4.5).
\end{aligned}$$

Extra Credit 2. Implement your planner in part (g) and plot three generated motion plans in the environment defined above with ropes hanging in the (cross section) plane $x = 12$.