#ASLI ENGINEERING

Resizing a Hash Table

SWIPE

BY

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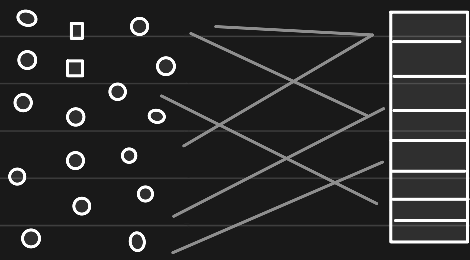
Resizing a Hash Table

load Factor $\alpha = \frac{\# keys}{\# slots}$



Performance of hash table degrades as load factor increases

and hence, at a certain threshold, to maintain the performance of our hash table we have to resize.

When should we resize?

Resize when load factor is "too high"

* Resizing takes time

and it depends on the underlying structure and algorithm

We cannot be too aggressive, nor too lineant

↓ we would waste too much time doing resizing

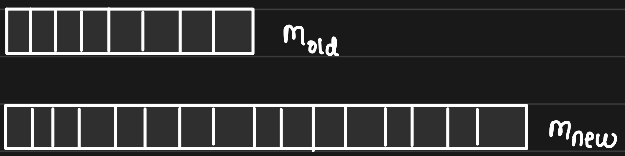
↓ performance will degrade

A common decision,

resize when $\alpha = 0.5$ → array is half filled

But we can make this logic as complex as possible

How to resize?



1. create new array - len = m_{new}
2. copy elements from old to the new array
3. delete old array

* allocating new array and moving keys takes time

If you are implementing it in C language, the most common way to do this is 'realloc'

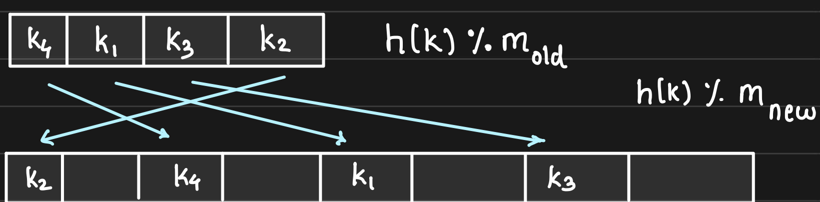
Note: Time taken to resize is $t \propto$ keys

A typical strategy to resize is to always double the array

↳ during insert if we trigger resize

we re-allocate a new array of 2x size and rehash keys

* we may not be able to use 'realloc' call, as we would need to rehash keys as per the new size (mod m)



Why do we always double?

It is a common practice that whenever we need to resize a hash table, we always double

growing by a constant factor

1. Say, we always increase by 1

			Operation
insert a	$\boxed{a}$	allocate array and insert a	1
insert b	$\boxed{a}\boxed{b}$	allocate array of size 2 Copy 1 element, insert b	2
insert c	$\boxed{a}\boxed{b}\boxed{}$	allocate array of size 3 Copy 2 elements, insert c	3

⋮

insert z $\boxed{a}\boxed{b}\boxed{c}\boxed{d}\boxed{e}\boxed{f}\boxed{g}\boxed{h}\boxed{i}\cdots\boxed{y}\boxed{z}$

- reallocate array of size 'n'
- copy all (n-1) elements from old to new
- insert n^{th} element

'n' operations

$$\text{Total ops} = 1+2+\cdots+n = \frac{n(n-1)}{2} = O(n^2)$$

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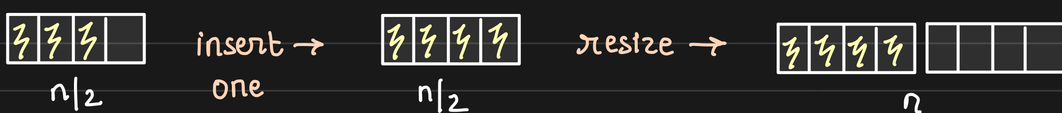
2. Say, we double everytime

assume, we resize when array is completely filled

* upon filling the last spot

when we insert $n/2$ elements in array of size $n/2$



let's go from $n \rightarrow 2n$

To get to the next resize, we need to add $n/2$ elements

First $n/2 - 1$ takes $O(1)$, the final element

- $O(1)$ to insert

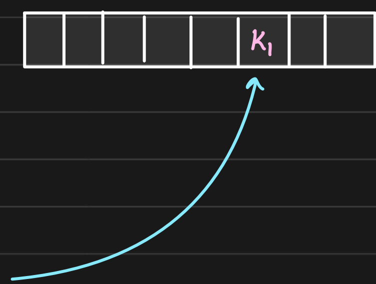
- $O(2n)$ for resize

- $O(n)$ for copying elements

Total operations $= \frac{n}{2} - 1 + 1 + 2n + n = \frac{7n}{2} = O(n)$

Why always a power of 2?

The underlying array of the hash table has the length = power of 2



Because it is efficient, but how?

Hash table works on hash function

key $k_1 \longrightarrow f \longrightarrow i \longrightarrow i \,\%\, m \longrightarrow$ index

MOD operation is super-important as it bounds the integer output from hash function to a range that fits in array

But, MOD is really expensive

internally it does division, and captures the remainder

can we do it faster?

Say, $m = 4$

hash key	mod (m)	index
1	% 4	1
2	% 4	2
3	% 4	3
4	% 4	0
5	% 4	1
6	% 4	2
7	% 4	3

Let's use bitwise AND to get the same output

Say, $m = 4 = 2^2$, what if we AND it with $2^2 - 1 = 3$

$1 \% 4 = 1$ $\quad 1 \,\&\, 3$

$$\begin{array}{r} 0\,0\,0\,1 \\ \oplus\; 0\,0\,1\,1 \\ \hline 0\,0\,0\,1 \end{array}$$

$2 \% 4 = 2$ $\quad 2 \,\&\, 3$

$$\begin{array}{r} 0\,0\,1\,0 \\ \oplus\; 0\,0\,1\,1 \\ \hline 0\,0\,1\,0 \end{array}$$

$3 \% 4 = 3$ $\quad 3 \,\&\, 3$

$$\begin{array}{r} 0\,0\,1\,1 \\ \oplus\; 0\,0\,1\,1 \\ \hline 0\,0\,1\,1 \end{array}$$

$4 \% 4 = 0$ $\quad 4 \,\&\, 3$

$$\begin{array}{r} 0\,1\,0\,0 \\ \oplus\; 0\,0\,1\,1 \\ \hline 0\,0\,0\,0 \end{array}$$

$5 \% 4 =$ $\quad 5 \,\&\, 3$

$$\begin{array}{r} 0\,1\,0\,1 \\ \oplus\; 0\,0\,1\,1 \\ \hline 0\,0\,0\,1 \end{array}$$

MOD m = AND$(m-1)$, $m = 2^k$

AND operation is significantly faster than division and hence performance gain!

- $2^k - 1$ has lower bits 1 and higher 0
- AND acts as a filter
  - ↳ zeros will keep result bounded
  - ↳ ones with percolate set bits

Shrinking a Hash Table

If keys are deleted from the hash table, then it does not make sense to keep them bloated

Hence, we shrink the table

* we grow when $\alpha \to 1/2$

if table has n slots, it would have $n/2$ elements (max)

$n=16$, $e=7$

if we shrink, it would have $n/2$ slots

$n=8$, $e=7$

← Poor performance, immediate resize as load factor = 1

hence, it can have at max $n/4$ elements

$n=8$, $e=3$

← consistent performance

But, one more insertion will cause a resize

Hence, we need to shrink when # elements are such that few insertions after shrink would not cause a resize

$n=16$, $e=2$

hence we would shrink when there are $n/8$ elements

Summary

- resizing is important to maintain performance
- resizing is costly
- hash table is always doubled
- hash table has 2^n slots for faster compute
- grow when $\alpha \rightarrow 1/2$
- shrink when $\alpha \rightarrow 1/8$