

# COL100

## Introduction to Computer Science

### Assignment 3

## 1 More Treasure Hunt (4 Marks)

### *Gradescope Submission Link*

After a successful treasure hunt in their previous adventure, the adventurers are gearing up for another challenge. This time, they have the same map as before, with a PIN code representing the directions and instructions for the journey.

However, just as they were about to embark on their new quest, a cosmic ray struck the map and scrambled the PIN code. Strangely, it also revealed the actual coordinates of the treasure. The group hurriedly made their way to the treasure's location, but they now need to unlock it by entering the correct original PIN.

The cosmic ray can damage the pin in following ways:

1. **Digit Reversal:** The digits of the PIN would be reversed. Eg: If the original PIN was **2134**, the cosmic ray would make the PIN as **4312**. The damage number of this ray is **1**.
2. **Pairwise Reversal:** This would swap the first two bits and the last two bits. Eg: If the original PIN was **2134**, the final PIN would be **1243**. The damage number of this ray is **2**.
3. **Forward Cyclic Movement:** This would move the PIN digits in cyclic order by 1 move. Eg: If the original PIN was **2134**, the final PIN would become **4213**. Essentially, the digit at the back moves to the front. The damage number of this ray is **3**.
4. **Reverse Cyclic Movement:** This would move the PIN digits in reverse cyclic order by 1 move. Eg: If the original PIN was **2134**, the final PIN would become **1342**. Essentially, the digit at the front moves to the back. The damage number of this ray is **4**.

Note that the TA making the assignment is not making up the cosmic ray theory. In practice, the bits can change by cosmic ray hitting the memory. Find out more here.

You are given the following information:

- The incorrect PIN of the game.
- The number of instructions followed by each instruction.

- The final coordinates of the treasure.

You need to return the correct PIN and the damage number of the cosmic ray.

### 1.1 Constraint on Input :

All entries are int.

### 1.2 Sample Input

```
4123
5
3 20
1 40
2 10
4 40
2 40
0 -30
```

### 1.3 Sample Output

```
3214
1
```

### 1.4 Explanation

Damage type 1 on 3214 caused the PIN to become 4123. Also, the PIN 3214 matches with the instructions to make the final position as (0, -30).

## 2 Different Types of Query (4 Marks)

#### *Gradescope Submission Link*

You will be given q Queries. For this problem we define 4 types of query.

**Query 1 :** 1 a b c

In this query you have to find the smallest whole number n for which the expression  $an^2 + bn + c$  is non-prime.

**Query 2 :** 2 a b

In this query you have to find count of triangular numbers in the range [a,b].

**Query 3 :** 3 a b

In this you have to find count of numbers in the range [a,b] which are divisible by 2, 3 or 5.

**Query 4 :** 4 a b

In this you have to find 2 smallest numbers say  $s_1$  and  $s_2$  in range [a,b] such that  $s_1$  has minimum set bits counts in the range [a,b] and  $s_2$  has maximum set bits count in the range [a,b]

## 2.1 Constraint on Input :

**Query 1 constraint :** a,b,c in range [-10000,10000].

**Query 2 constraint :** a,b in range  $[0,10^{12}]$ .

**Query 3 constraint :** a,b in range  $[0,10^{12}]$ .

**Query 4 constraint :** a,b in range  $[0,10^{12}]$ .

## 2.2 Definition :

**Set bits** in binary number is represented by 1.

**Triangular Number** is defined mathematically as  $T_n = \frac{n(n+1)}{2}$  for  $n=1,2,3,\dots$

Ex : 1st 3 triangular numbers are 1,3 and 6.

**Note :** These 4 Queries are identified by their number and query number is the first integer present in that query .For example suppose you have got a query of form 1 2 3 5 then the first integer **1** represent that it is **Query 1** , Similarly 3 4 5 query represent **Query 3**.

## 2.3 Sample Input

```
4
1 1 1 1
3 3 7
2 1 10
4 1 5
```

## 2.4 Sample Output

0  
4  
4  
1 3

## 2.5 Explanation :

First input 4 represent number of queries that will be asked.

Next input 1 1 1 1 represent **Query 1** , so we have to find smallest whole number  $n$  for which  $n^2 + n + 1$  is non-prime. For  $n = 0$  , the expression will give  $0+0+1=1$  , as 1 is non-prime . So smallest whole number for which  $n^2 + n + 1$  is non-prime (not prime) is  $n=0$ . Hence output is 0.

Next input 3 3 7 represent **Query 3** , so we have to find number in the set  $\{3,4,5,6,7\}$  which are divisible by either 2 , 3 or 5. Such numbers are  $\{3,4,5,6\}$ . Hence there are 4 such numbers in the range  $[3,7]$  which are divisible by either 2, 3 or 5. Hence output is 4.

Next input 2 1 10 represent **Query 2** , so we have to find count of triangular number in the range  $[1,10]$ . As we know that  $\{1,3,6,10\}$  are triangular numbers which belongs in the range  $[1,10]$ . Hence output is 4.

Next input 4 1 5 represent **Query 4** , so we have to find smallest numbers in range  $[1,5]$  one with lowest set bits count and one with largest set bits count. We can see that in binary representation :  $1 = 1$  ,  $2 = 10$  ,  $3 = 11$  ,  $4 = 100$  and  $5 = 101$ .  $\{1,2,4\}$  has lowest set bits count which is 1 in the range  $[1,5]$  , Similarly  $\{3,5\}$  has highest set bits count of 2 in the range  $[1,5]$ .

So smallest number with minimum set bits count is 1 in the range  $[1,5]$  and smallest number with maximum set bits is 3 in the range  $[1,5]$ . Hence output is 1 3.

## 3 Summation Operation on Infinite Grid (Optional)

**Star Question Submission Link** You are given an Infinite Grid where natural numbers are arranged in spiral fashion. You will be given 2 pairs ((row,column)) each representing a particular cell on that grid and you have to assume that these two cells represent the diagonal point of a rectangle.

You have to find the sum of all the numbers in that rectangle. You should output the  $Sum\%(10^9 + 7)$ .

1	2	9	10	25	.....	...
4	3	8	11	24	.....	...
5	6	7	12	23	.....	...
16	15	14	13	22	.....	...
17	18	19	20	21	.....	...
...	...	...	...	...	.....	...
...	...	...	...	...		
...	...	...	...	...		
....	...	....	...	...	.....	...

Figure 1: Infinite Grid

### 3.1 Constraint on Input :

row , column are in range  $[1,10^9]$

### 3.2 Sample Input I

```
1 1
2 2
```

### 3.3 Sample Output I

```
10
```

### 3.4 Explanation I :

The rectangle whose diagonal points are (1,1) and (2,2).

1	2
4	3

Figure 2: Rectangle whose diagonal points are (1,1) and (2,2)

$$Sum = (1 + 2 + 3 + 4) \%(10^9 + 7) = 10 \%(10^9 + 7) = 10.$$

### 3.5 Sample Input II

5 4  
2 3

### 3.6 Sample Output II

104

### 3.7 Explanation II :

The rectangle whose diagonal points are (5,4) and (2,3).

$$Sum = (8 + 11 + 7 + 12 + 14 + 13 + 19 + 20) \%(10^9 + 7) = 104 \%(10^9 + 7) = 104.$$

8	11
7	12
14	13
19	20

Figure 3: Rectangle whose diagonal points are  $(5,4)$  and  $(2,3)$