MA 202 | Numerical Methods

Assignment 1

INSTRUCTIONS

- Assignment problems must be submitted by **midnight of June 18**
- You are strongly encouraged to write a computer program in Matlab to solve the equations. If this is not possible (for instance, because you do not have a PC/laptop), you should write a pseudocode and workout iterations manually as much as possible.
- **Please submit all documents (including writeup, report, Matlab program files) in a single ZIP file. Name the zip file as: Assignmentproblemnumber_Rollnumber.zip. For example, if your roll number is 18110110 and you have opted for problem A2 in the assignment, name your submission as A2_18110110.zip.**

Note: You need to solve either problem A1 OR problem A2 OR problem A3. Problem A4 is compulsory for all students.

Problem A1

The amount of a uniformly distributed radioactive contaminant contained in closed reactor is measured by its concentration c (becquerel/liter or Bq/L). The contaminant decreases at a decay rate proportional to its concentration – that is

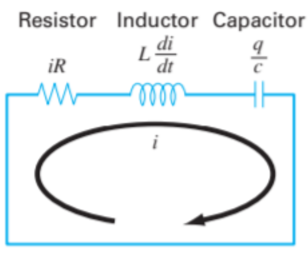
$$decay\, rate = -kc$$

where, k is a constant with units of day^{-1}. Formulate a mathematical model to describe the change in the concentration of the contaminant.

a) Use Euler's method to solve this equation from t = 0 to 1 d with k=0.2d^{-1}. Employ a step size of Δ t = 0.1. The concentration at t = 0 is 10 Bq/L.
b) Plot the solution on a semilog graph (i.e., ln c versus t) and determine the slope. Interpret your results.

Problem A2

An RLC circuit consists of three elements: a resistor (R), and inductor (L) and a capacitor (C).



The flow of current across each element induces a voltage drop. Kirchhoff's second voltage law states that the algebraic sum of these voltage drops around a closed circuit is zero:

$$iR + L\frac{di}{dt} + \frac{q}{C} = 0$$

where, i = current, R = resistance, L = inductance, t = time, q = charge and C = capacitance. In addition, the current is related to charge as in

$$\frac{dq}{dt} = i$$

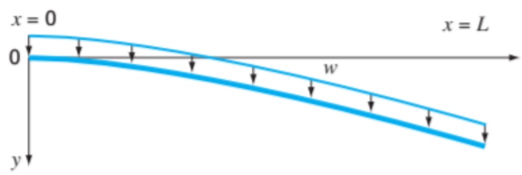
a) If the initial values are i(0) = 0 and q(0) = 1 C, use Euler's method to solve this pair of differential equations from t = 0 to 0.1 s using a step size of Δt = 0.01 s. Employ the following parameters for your calculation: R = 200 Ω, L = 5 H and C = 10^{-4} F.
b) Develop a plot of i and q versus t.

Problem A3

As depicted, the downward deflection y (m) of a cantilever beam with a uniform load w (kg/m) can be computed as

$$y = \frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$$

where, x = distance (m), E = the modulus of elasticity = 2 × 10^{11} Pa, I = moment of inertia = 3.25 × 10^{-4} m^4, w = 10,000 N/m and L = length = 4 m.



This equation can be differentiated to yield the slope of the downward deflection as a function of x:

$$\frac{dy}{dx} = \frac{w}{24EI}(4x^3 - 12Lx^2 + 12L^2x)$$

If y = 0 at x = 0, use this equation with Euler's method (Δ x = 0.125 m) to compute the deflection from x = 0 to L. Develop a plot of your results along with the analytical solution computed with the first equation.

Problem A4

Use zero- through third order Taylor series expansions to predict f(3) for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

Using a base point at x=1. Compute the true percent relative error ε_t for each approximation.