Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

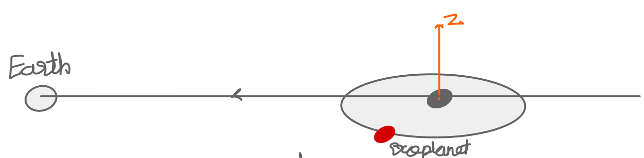
A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

Sol^n If we detect this Planet using transit method, then we can be sure that its inclination is around 90° for sure. But If we detect it By Radial Velocity method then we can found the inclination, If we have data of certain things

If transit



$i = 90°$

If Radial :- $K = M_p \sin i \left(\frac{2\pi G}{P M_*^2}\right)^{1/3}$

So we got 3 unknowns so planet must be detected by transit method

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

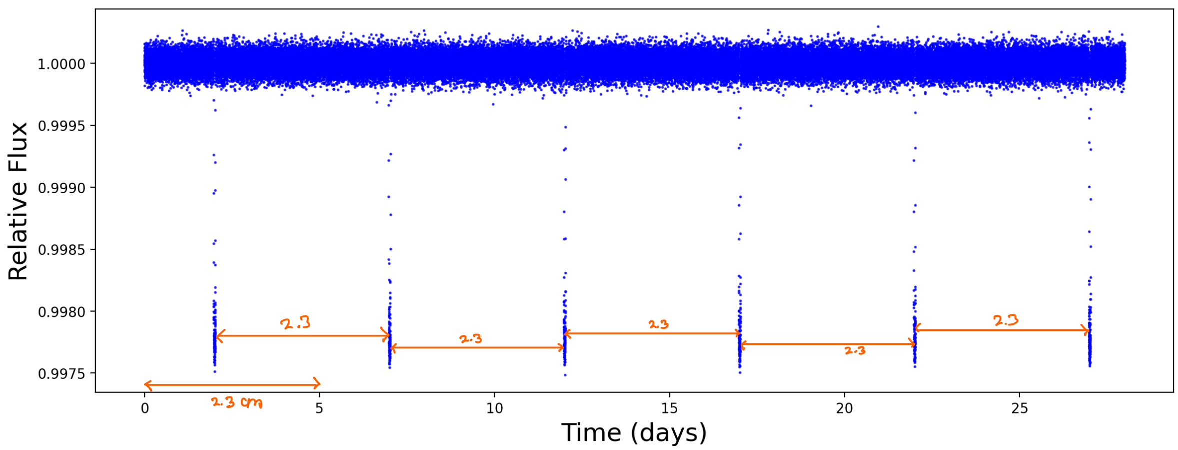


Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

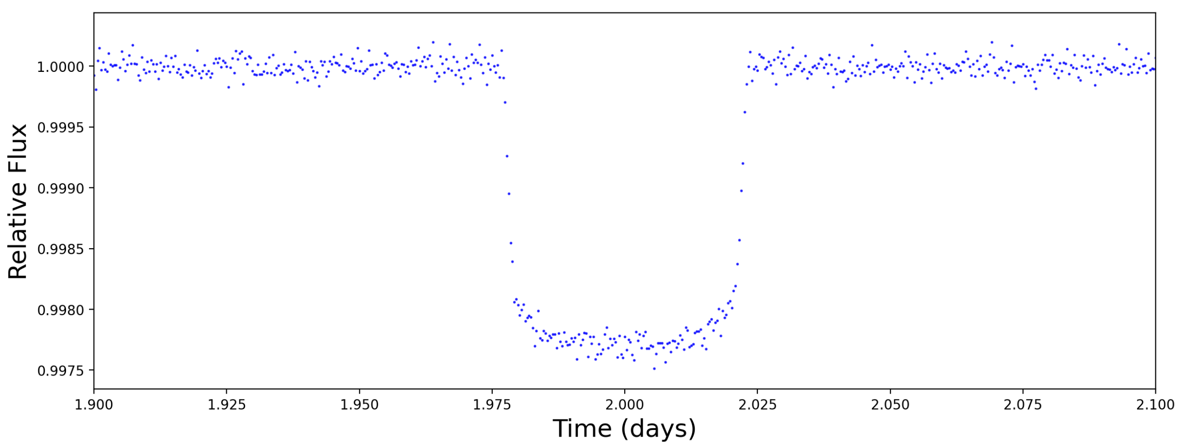


Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

So, I use rural scale in a Notes app to measure dist between each dip.

Avg of all dip is 2.3 cm i.e. 5 days

Period = 5 days

c) What is the radius of this planet?

So $z = \left(\frac{R_p}{R_*}\right)^2$ (z is the transit depth)
(as we see in the graph.

$z = 1 - 0.9975$

$\therefore R_p = \sqrt{z} \times R_*$ ⟶ (By que ① $R_* = 0.2 R_\odot$)

$R_p = \sqrt{0.0025} \times 0.2 R_\odot$

$= 0.05 \times 0.2 R_\odot$

$\boxed{R_p = 0.01 R_\odot}$

So radius of planet is 1 % of Radius of sun.

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

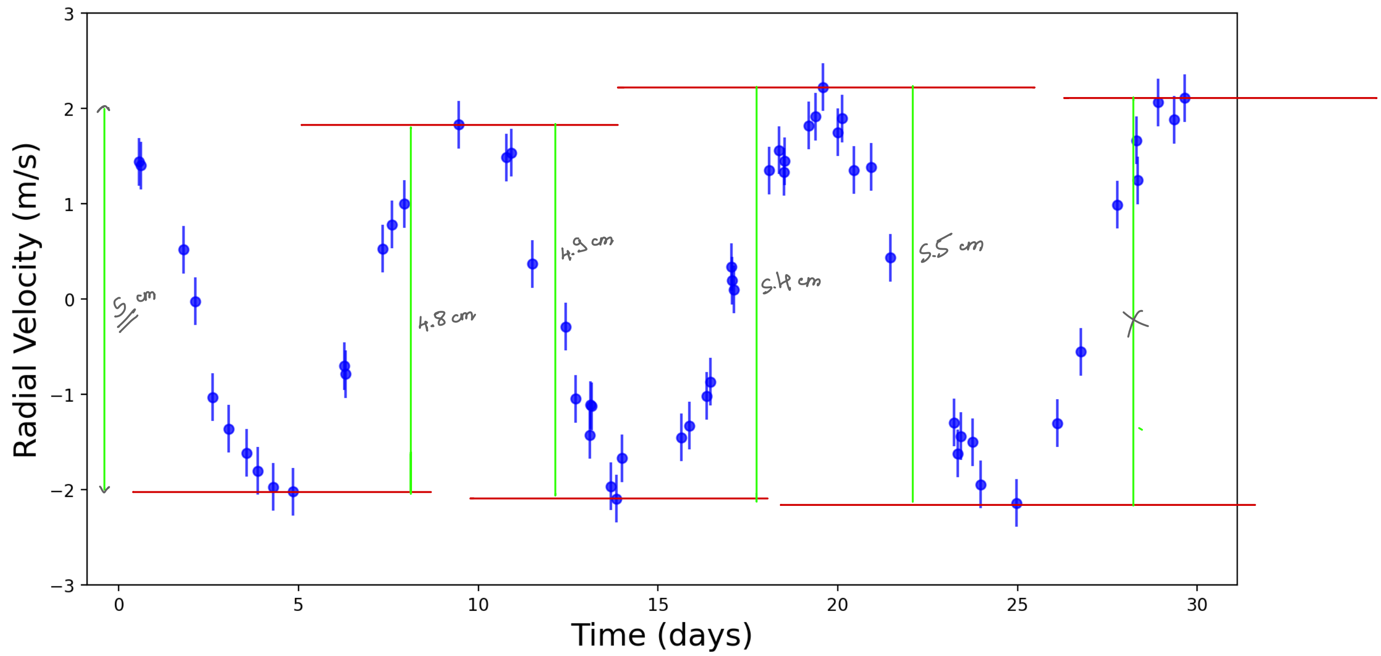


Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

Sol. By using given graph and rular scale from mynotes app average of Amplitude of Sinosudoidal wave is....

$$2K = \frac{4.8 + 4.9 + 5.4 + 5.5}{4} = 5.15\text{ cm}$$

$$\therefore 2K = 5.15\text{ cm} \quad \text{ie, } K = 2.575\text{ cm}$$

as $5\text{ cm} = 4\text{ m/s}$

$1\text{ cm} = 4/5\text{ m/s}$

$$\therefore K = 2.575 \times \frac{4}{5}\text{ m/s}$$

$$\therefore K = 2.06\text{ m/s}$$

Made with Goodnotes

e) What is the mass of this planet?

Now,
$$K = M_p \sin(i) \left(\frac{2\pi G}{P M_*^2} \right)^{1/3}$$

So we have, $M_p \longrightarrow$ (Mass of Planet)

$M_* = 0.2\, M_\odot$ (Stellar Mass)

$P = 5$ Days / 432×10^3 sec (Period)

$K = 2.06$ m/s (Semi-Amplitude)

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$ (Gravitational const)

We assume $i = 90°$ $\therefore$ $\sin i = 1$

$$\therefore \quad M_p = K \times \left(\frac{P M_*^2}{2\pi G} \right)^{1/3} = 2.06 \times \left(\frac{432 \times 10^3 \times 0.2^2 M_\odot^2}{2\pi G} \right)^{1/3}$$

$M_p = 1.1258 \times 10^{25}$

at $i = 90°$

$M_\oplus = 3 \times 10^{-6} M_\odot$

$M_\oplus = 5.972 \times 10^{24}$ kg.

$M_p = \frac{11.258}{5.972} M_\oplus$

$$\boxed{M_p = 1.88\, M_\oplus}$$

and $R_p = 0.01\, R_\odot$

$R_\oplus = 9.15 \times 10^{-3} R_\odot$

$$\boxed{R_p = 1.09\, R_\oplus}$$

Made with Goodnotes

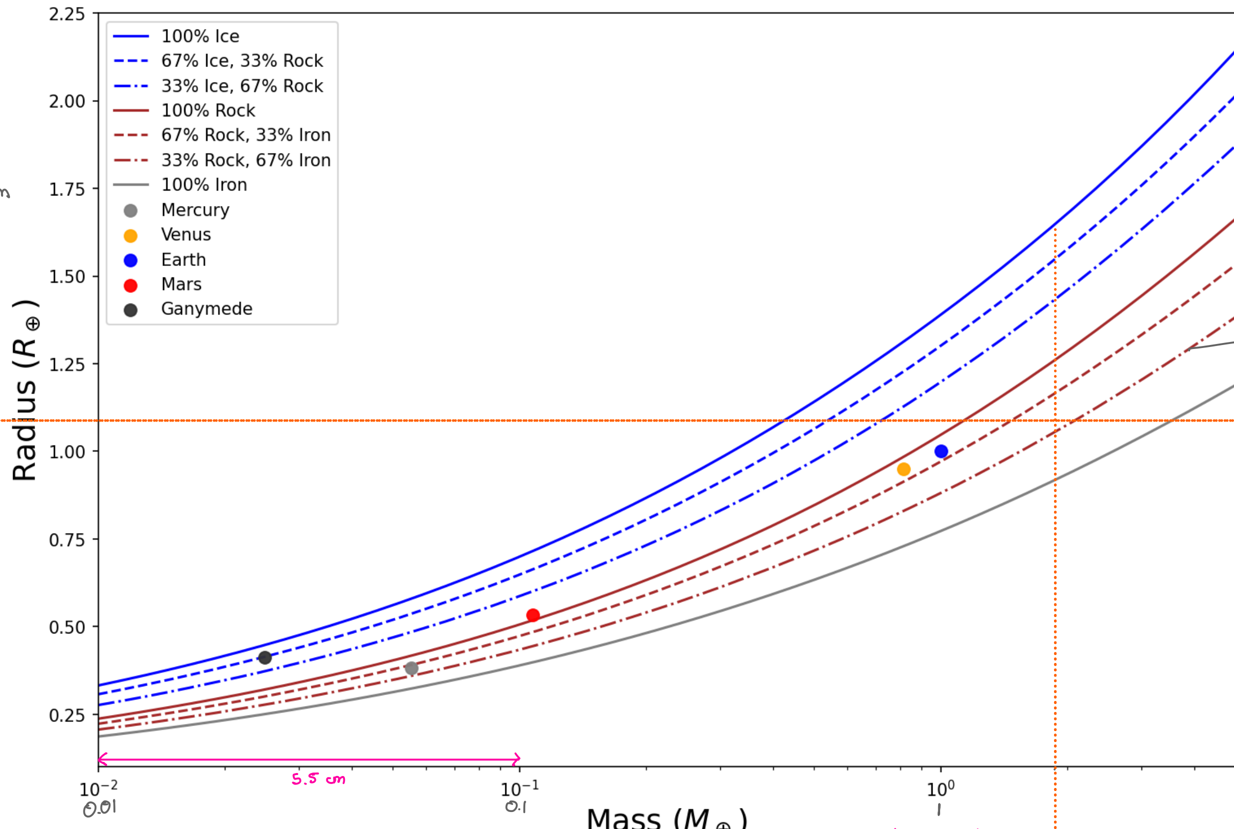
f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

$1R_\oplus = 4.6$ cm

$\therefore 1.09R_\oplus = 6.104$ cm

Rp



33% rock 67% Iron

Here the scale is not linear ie, it is logarithmic i.g ∴

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

Scale

10^{-1} at -5.5cm

10^{-2} at -11cm

$D = 5.5 \times \log_{10}(x)$

Position of x

checks.

$x = 10^{-2}$

$D = -11$ ✓

correct

So at $1.88\ M_\oplus$ scale $= 1.5$ cm

So, By scales and calculations.

We draw 2 lines wrt Earth and, the most very near to 33% rock & 67% Iron curve