Georgia Institute of Technology
George W. Woodruff School of Mechanical Engineering
ME6406 Machine Vision (Fall 2016)
Assignment #3: Due **Thursday Nov. 10, 2016**

Instruction: Solution should be in **electronic** format, either as a single MS-Word file or pdf file. This solution file should include any derivations, equations, results, figures and discussions. Put this solution file and all m-files into one single zipped file and submit electronically through T-Square. A confirmation email will be sent upon receipt of submission.

1. Camera Model and Calibration

(a) Camera Model. Write a program (CameraModel.m) to transform the feature points (denoted as ✳ in Fig. 1a) from the 3D world coordinates ($X_wY_wZ_w$) to the 2D undistorted image coordinates (uv). Use [$\mathbf{R}_x$(120°)], $\mathbf{T}$=[2 3 8]^T, f=1.2 and the 10 data in Table 1(a) to demonstrate your solutions. Determine and plot these 10 feature points on the uv plane. Save the (X_w, Y_w) and (u, v) values in camera_calibration_data.mat for Part (b).

(b) Camera Calibration. Write a program (CameraCalibration.m) to calibrate f, [$\mathbf{R}$] and $\mathbf{T}$. Use the data camera_calibration_data.mat obtained in Part (a) to compute f, [$\mathbf{R}$] and $\mathbf{T}$ to demonstrate your solutions.

(c) Stereo Vision. Write a program (CameraStereo.m) to compute Z (depth) from two parallel (identical) cameras (Fig. 1b). Transform the 10 features points in Table 1(b) from the 3D world coordinate ($X_wY_wZ_w$) to the 2D undistorted image coordinate (u_1v_1, u_2v_2) for Camera 1, 2 respectively (Fig. 1b):

Use [$\mathbf{R}_x$(180°)], $\mathbf{T}$=[2 3 8]^T, f=1.2 for Camera 1 and [$\mathbf{R}_x$(180°)], $\mathbf{T}$=[−2 3 8]^T, f=1.2 for Camera 2.

Calculate and plot the 10 feature points in the u_1v_1, u_2v_2 plane. Determine the depth of each point (Z) from the equation $Z=bf/d$ where b is the distance between two cameras; f is the focus length of these two cameras; and $d(=u_1-u_2)$ is the disparity.

Note: The (u, v) values should be rounded off to a reasonable number of decimal places, say 6; otherwise, numerical errors due to high number of decimals places could cause complex number matrix [$\mathbf{R}$].

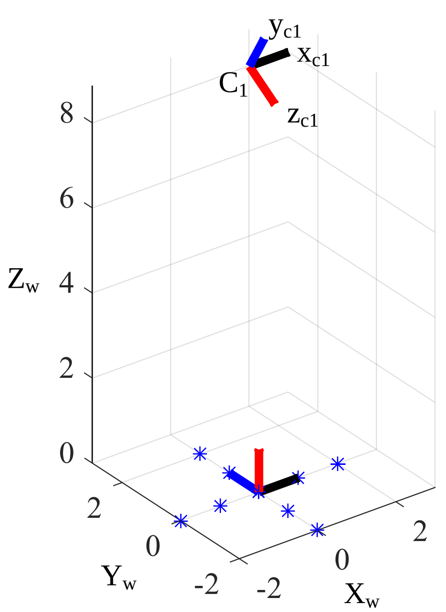

Fig. 1(a) Camera calibration

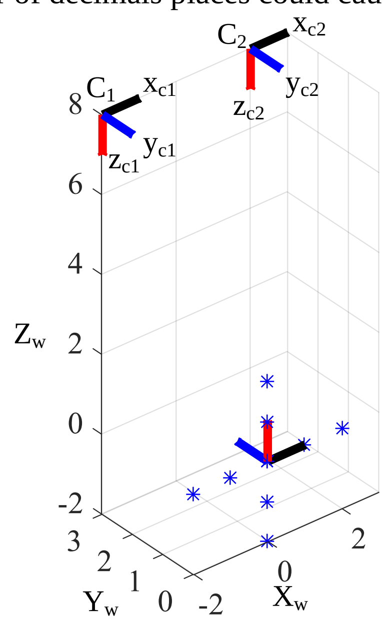

Fig. 1(b) Stereo Vision

Table 1(a) Camera calibration points

X_w	2	1	0	-1	-2	0	0	0	0	0
Y_w	0	0	0	0	0	2	1	0	-1	-2
Z_w	0	0	0	0	0	0	0	0	0	0

Table 1(b) Stereo vision points

X_w	0	0	0	0	0	0	0	0	0	0
Y_w	2	1	0	-1	-2	0	0	0	0	0
Z_w	0	0	0	0	0	2	1	0	-1	-2

2. Robot Eye-on-Hand Calibration

(a). A camera coordinate can be formulated by a rotation by an angle θ around an axis through the origin with direction cosines $\mathbf{n}=[n_1, n_2, n_3]^T$ followed by a translation $\mathbf{T}=[t_1, t_2, t_3]^T$ in the world coordinate (using Eq. 8 in [2]). Assume that there are a point $\mathbf{p}_W=[p_1, p_2, p_3]^T$ and a vector $\mathbf{v}_W=[v_1, v_2, v_3]^T$ in the world coordinate. Find the values of $\mathbf{p}_W$, $\mathbf{v}_W$ in the camera coordinate ($\mathbf{p}_c$, $\mathbf{v}_c$) with the following two cases:

Case 1: $\mathbf{n}=[0, 0, 1]^T$, $\theta=60°$, $\mathbf{T}=[0, 0, 1]^T$, $\mathbf{p}_W=[0, 0, 0]^T$, $\mathbf{v}_W=[0, 0, 1]^T$.

Case 2: $\mathbf{n}=[0, 0.5, 0.5]^T$, $\theta=25°$, $\mathbf{T}=[-1, -2, -5]^T$, $\mathbf{p}_W=[2, 1, 3]^T$, $\mathbf{v}_W=[0.5, 0.2, 0.3]^T$.

(b). Fig. 2a shows the setup for performing an eye-on-hand calibration where a stationary planar calibration board is viewed at 3 different locations by a camera mounted on a robot gripper. Fig. 2b shows the images in three camera image planes. The transformation matrices from *CW* to *Ci* can be determined by the camera calibration ([**H**ci] where *i*=1, 2, 3). The rigid body transformations of the robot gripper from Station 1 to 2 and 2 to 3 ([**H**g12] and [**H**g23]) are given by the robot controller. Write a MATLAB program for the eye-an-hand calibration. Using the given ([**H**c1] [**H**c2] [**H**c3]) data in '*hand_eye_data.mat*' to illustrate your solutions:

1) Compute ([**R**c12], **T**c12) and ([**R**c23], **T**c23).

2) Obtain the equivalent angle-axis representation (n, θ) for each of the rotation matrixes; [**R**c12], [**R**c23], [**R**g12] and [**R**g23]. Next, compute P_{c12}, P_{c23}, P_{g12} and P_{g23}. Check your solutions by computing $[\mathbf{R}_{g12}]$ and $[\mathbf{R}_{g23}]$ using Eqns. (8) and (10) in [2] and comparing with those given in the data file '*hand_eye_data.mat*'.

3) Use the procedure in [2] to compute $\mathbf{P}_{cg}$, $[\mathbf{R}_{cg}]$ and $\mathbf{T}_{cg}$.

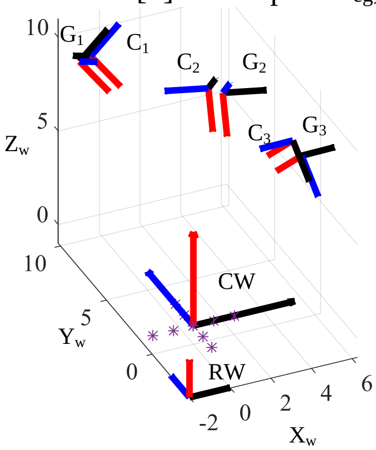

Fig. 2a

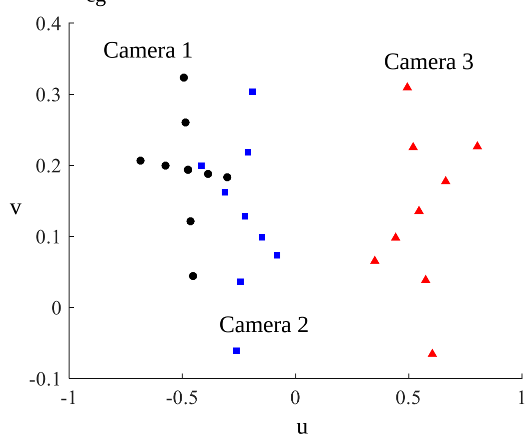

Fig. 2b

3. Ellipse-Circle Correspondence

Using a least squares fit, a circle captured by a camera (with focal length *f*=0.1) in the image frame has the following ellipse equation,

$$\frac{(u-1)^2}{4}+\frac{(v-1)^2}{16}=1 \tag{1}$$

Transform Eq. (1) to a standard conic form. If the radius of the circle is 1, find the plane equation (with respect to the camera frame) that contains the circle. Where is the center of the circle with respect to the camera frame?

Reference:

[1] Tsai, R. "A Versatile Camera Calibration Technique for High-accuracy 3D Machine Vision Metrology using Off-the-shelf TV Cameras and Lenses," *IEEE Trans. on Robotics and Automation*, Vol. 3, No.4, August 1987, Pg: 323- 344

[2] Tsai, R.Y. and R.K. Lenz, "A New Technique for Fully Autonomous and Efficient 3D Robotics Hand/Eye Calibration," *IEEE Trans. on Robotics and Automation*, VOL. 5, NO. 3, JUNE 1989.

[3] Qiang Ji, Mauro Costa, Robert Haralick, and Linda Shapiro, "An Integrated Linear Technique for Pose Estimation from Different Features," *Int. J. of Pattern Recognition and Artificial Intelligence*, Vol. 13, No. 5, 1999.