Interpolants and interference

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TU Wien, JKU Linz (Austria)

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Proofs for SAT solving

SAT
solver

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SAT
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theorem provers
many industrial applications

[Davis, Putnam ’60]

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model checking

formal guarantees on
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[McMillan ’03]

Proofs for SAT solving

[Craig ’57]

abstraction of
information relevant
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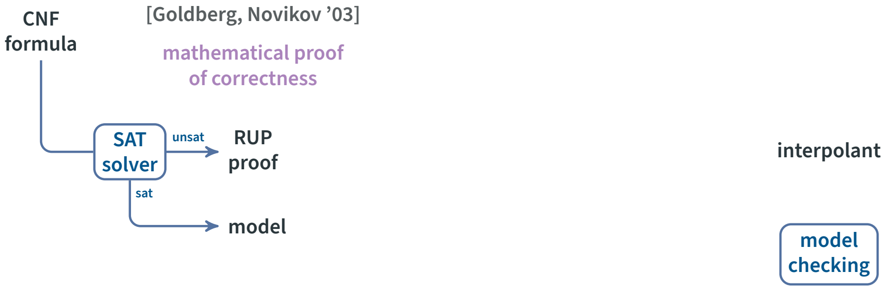
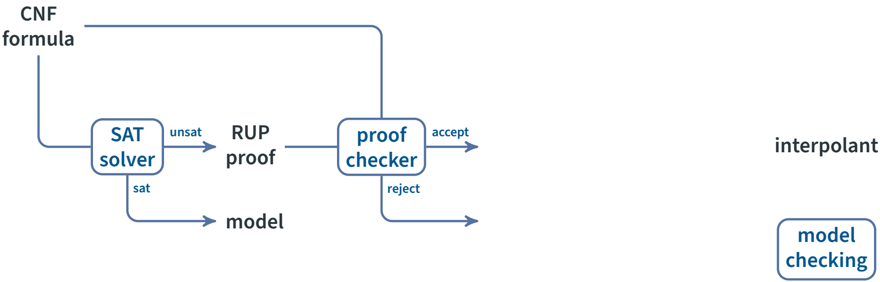


Proofs for SAT solving



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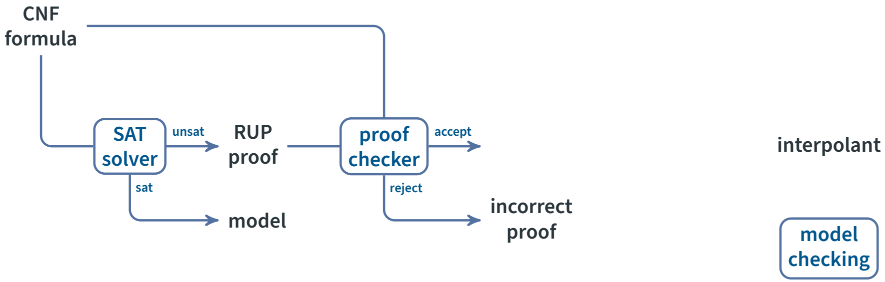

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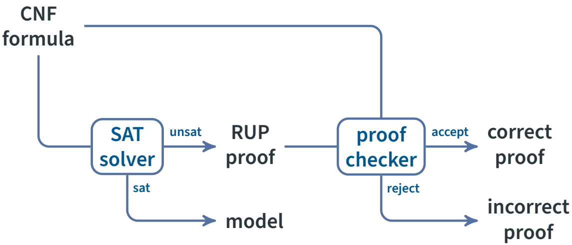

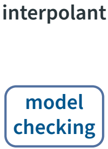

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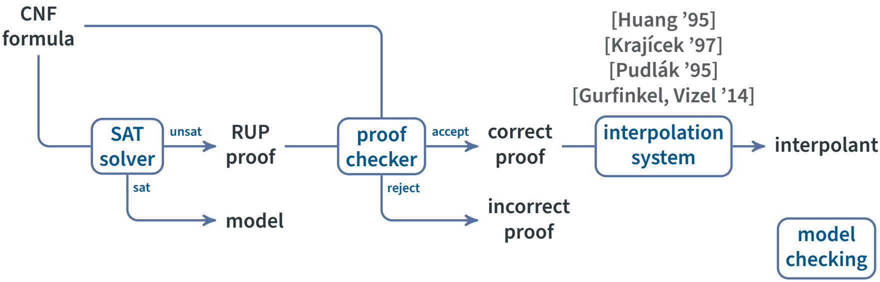

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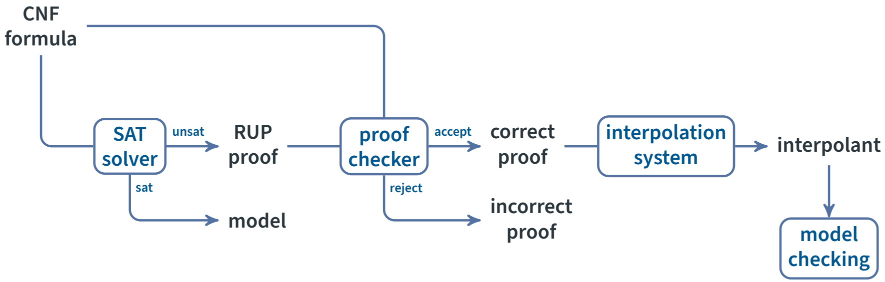

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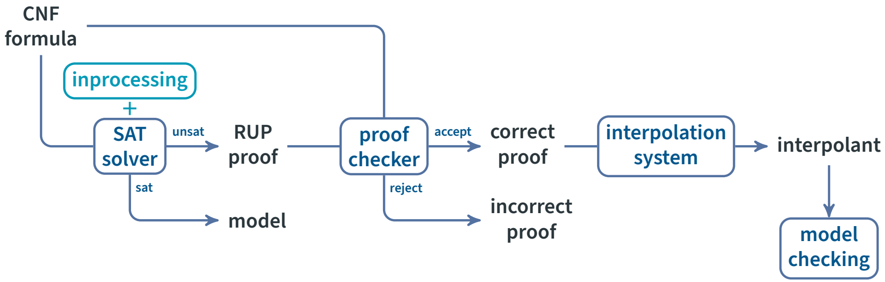

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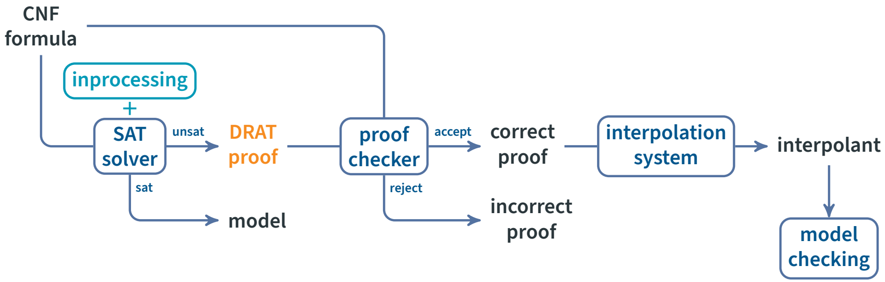

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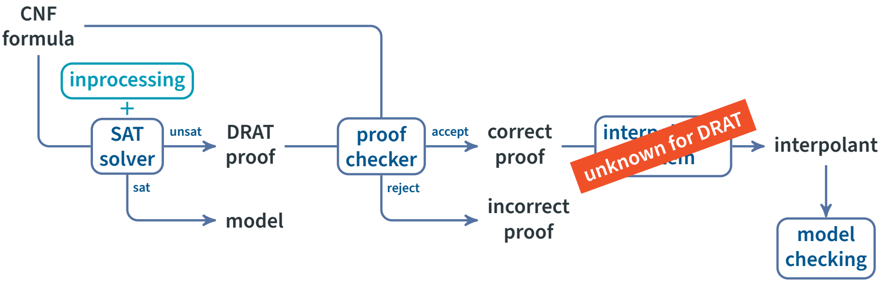

Inference-based proofs and recursive interpolation

Inference-based proofs

$$\otimes_z \dfrac{\otimes_y \dfrac{\overline{x}yzt \qquad \overline{xy}zt}{\overline{x}zt} \qquad \otimes_y \dfrac{\overline{x}y\overline{z}t \qquad \overline{xyz}t}{\overline{xz}t}}{\overline{x}t}$$

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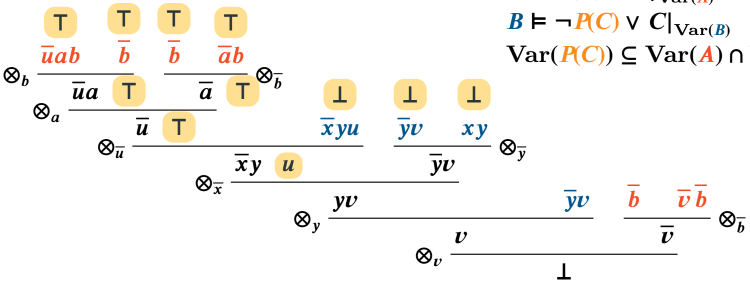

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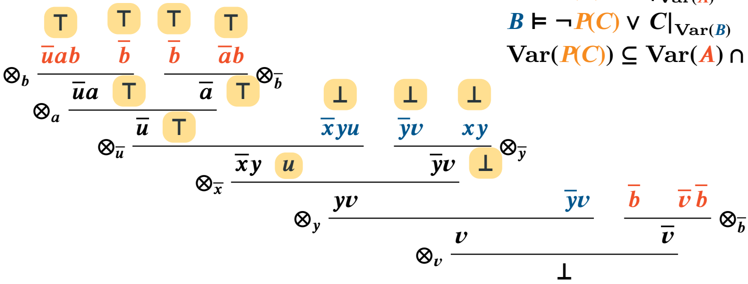

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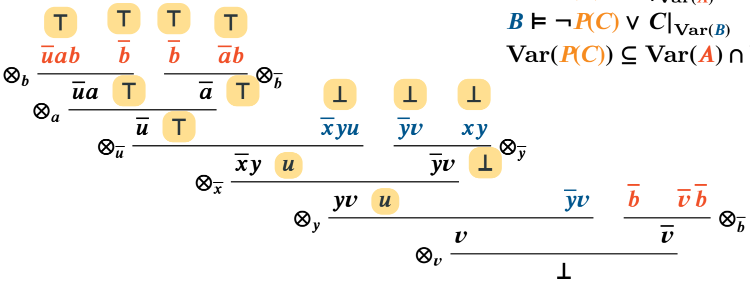

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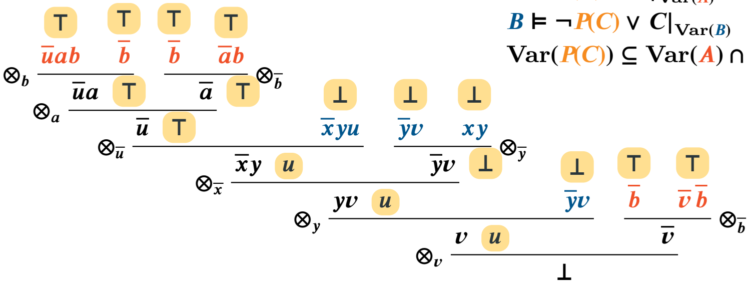

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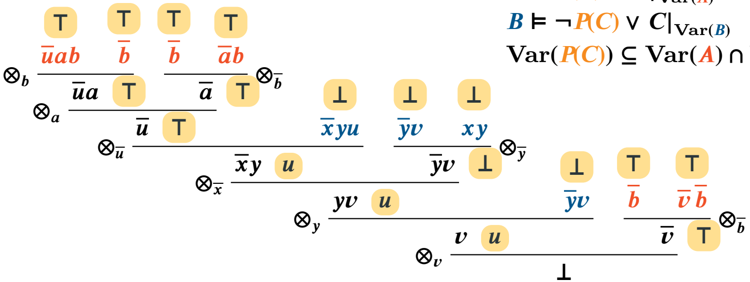

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$$\otimes_v\dfrac{\otimes_y\dfrac{\otimes_{\bar{x}}\dfrac{\otimes_{\bar{u}}\dfrac{\otimes_a\dfrac{\otimes_b\dfrac{\overset{\top}{\bar{u}ab} \quad \overset{\top}{\bar{b}}}{\bar{u}a \;\; \top} \quad \dfrac{\overset{\top}{\bar{b}} \quad \overset{\top}{\bar{a}b}}{\bar{a} \;\; \top}\otimes_{\bar{b}}}{\bar{u} \;\; \top} \quad \overset{\bot}{\bar{x}yu}}{\bar{x}y \;\; u} \quad \dfrac{\overset{\bot}{\bar{y}v} \quad \overset{\bot}{xy}}{\bar{y}v \;\; \bot}\otimes_{\bar{y}}}{yv \;\; u} \quad \overset{\bot}{\bar{y}v}}{v \;\; u} \quad \dfrac{\overset{\top}{\bar{b}} \quad \overset{\top}{\bar{v}b}}{\bar{v} \;\; \top}\otimes_{\bar{b}}}{\bot \;\; \boxed{u \vee v}}$$

Interference and non-monotonicity

Interference-based proofs

i:$\overline{x}\,y$
i:$\overline{z}t$
d:$\overline{x}\,yzt$
d:$x\,y\overline{z}t$
i:$\overline{x}t$

Interference and non-monotonicity

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Interference and non-monotonicity

Interference-based proofs

[Reckhow '75; Kullman '99]

ER

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Interference and non-monotonicity

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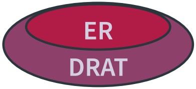
[Wetzler, Heule, Hunt ’14]



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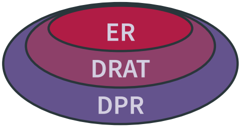
Interference and non-monotonicity

[Heule, Kiesl, Biere ’17]



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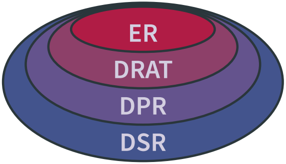
[Buss, Thapen '19]



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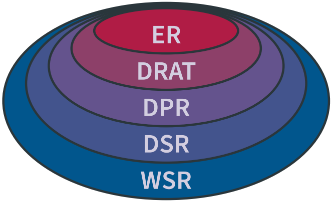
[Rebola-Pardo '22]



Interference and non-monotonicity

Interference-based proofs

[Kiesl, Rebola-Pardo, Heule ’18; Heule, Biere ’18]

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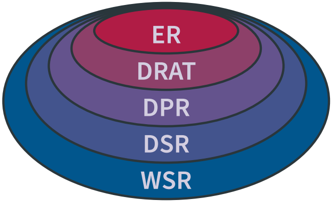

all these have the same proof complexity

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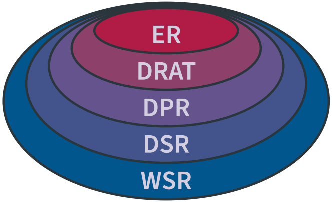

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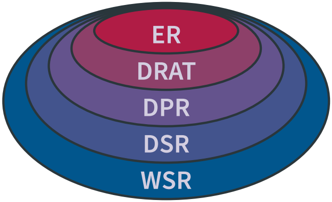

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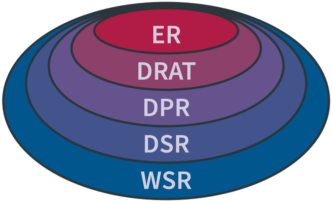

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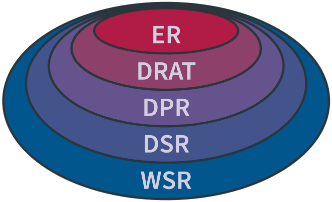

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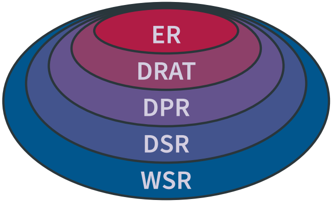

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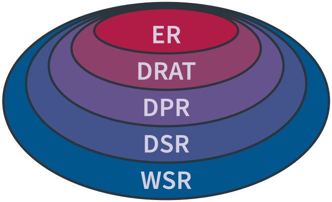

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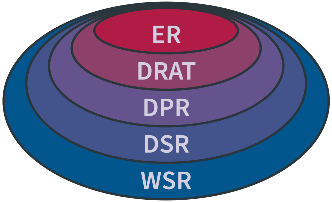

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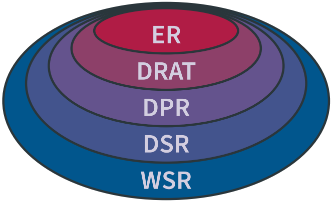

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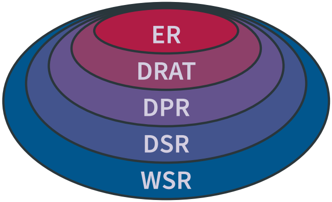

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unless...

Mutation logic

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Intuition interference encodes reasoning without loss of generality

C holds; otherwise we apply σ and get a similar problem where it does

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Mutation resolution an inference-based proof system over mutated clauses that fully encodes all interference-based proof systems

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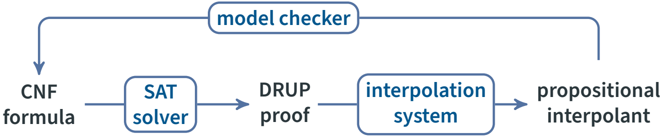
Interpolants in mutation logic?



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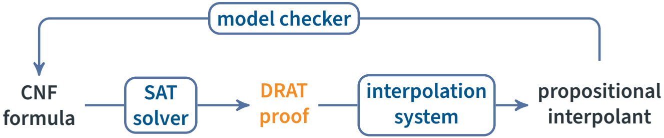

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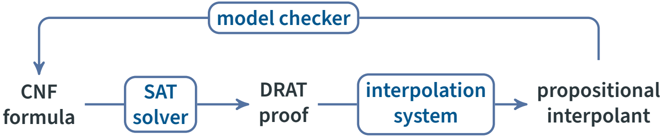

Universally mutated CNF $\nabla(\sigma_1 :- Q_1) \dots \nabla(\sigma_n :- Q_n).F$

DRAT operates naturally in this fragment

linearly as expressive as full mutation propositional logic

satisfiability problem reduces linearly to SAT

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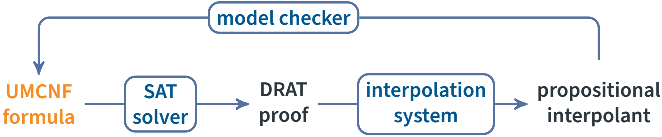

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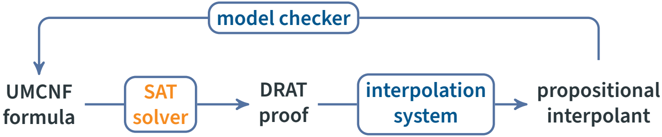

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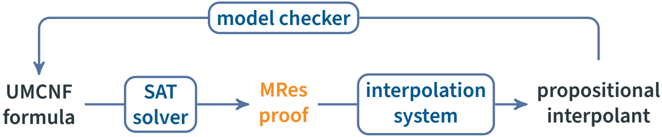

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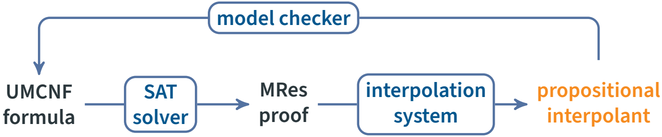

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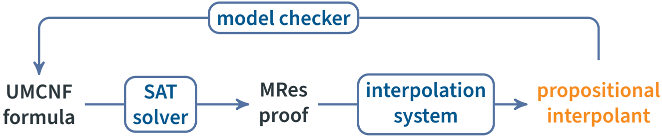

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Propositional interpolants from MRes proofs

- ER p-simulates DRAT [Kiesl, Rebola-Pardo, Heule ’18]
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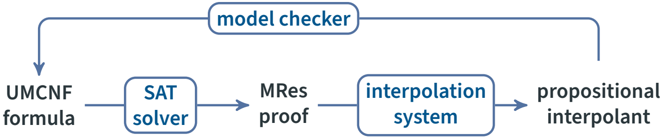

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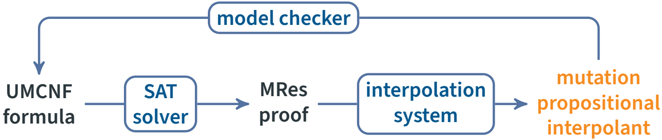

Universally mutated CNF $\nabla(\sigma_1 :- Q_1)\ldots\nabla(\sigma_n :- Q_n).F$

DRAT operates naturally in this fragment

linearly as expressive as full mutation propositional logic

satisfiability problem reduces linearly to SAT

Propositional interpolants from MRes proofs

- ER p-simulates DRAT [Kiesl, Rebola-Pardo, Heule '18]
- EF p-simulates ER [Reckhow '75]
- RSA is insecure $\Rightarrow$ EF lacks feasible interpolation [Krajícek, Pudlák '98]

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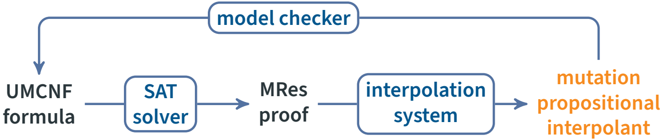

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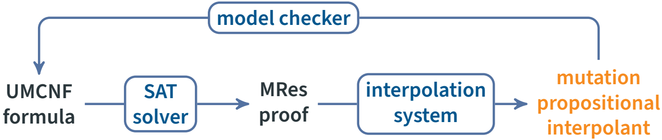

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Mutation-propositional interpolants from MRes proofs

What are the variables of a mutation formula?

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Semantic variables

A variable x is a semantic variable of a formula φ whenever we can find two models I^+ and I^- differing only on their truth value on x such that $I^+ \vDash \varphi$ and $I^- \nvDash \varphi$. $\Rightarrow$ semantic interpolants

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- Finding if x is a variable of φ is NP-complete

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Questions

- Does this affect the unfeasible interpolation [Krajícek, Pudlák '98]?
- Do other similar variations on the notion of interpolant exist?
- Does this shed any light on the connection between feasible interpolation and cryptography?