Two Flavors of DRAT

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$\pi =$ `Despite the constant negative press covfefe`

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π is a correct resolution refutation of F.

Proof correctness depends on the proof system, not on implication or consistency!

otherwise “$\bot$” is always a correct proof

Proof checking

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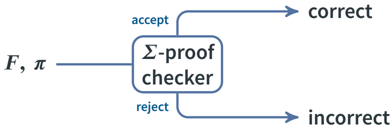

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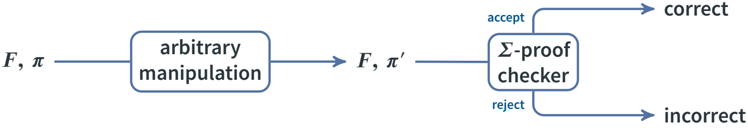

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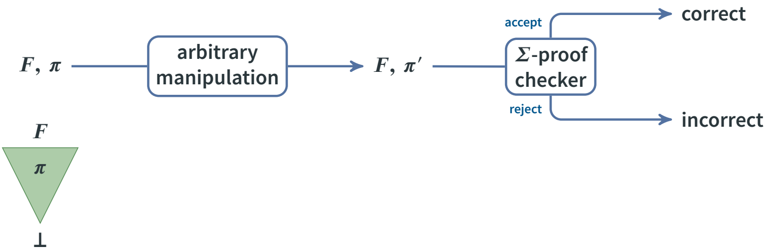

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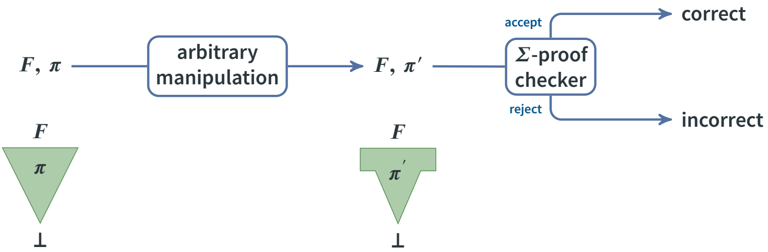

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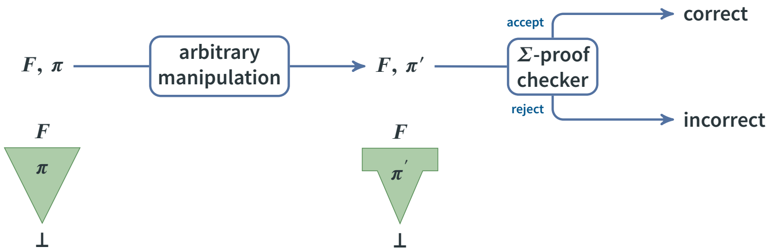

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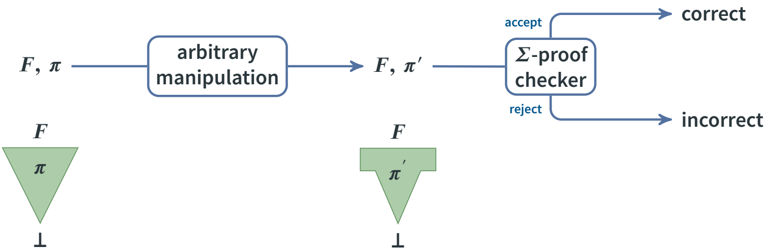

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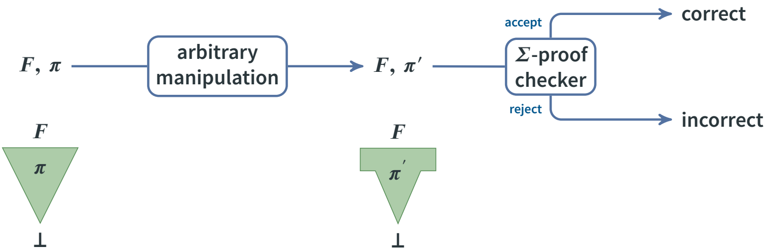

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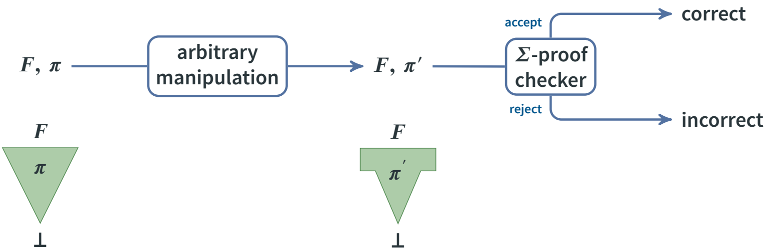

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Problem what should we do when a proof is rejected?

In this talk

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Discussion which of the two criteria is more convenient?

DRAT proofs, in theory

DRAT proofs as instruction strings

DRAT proofs strings of introduction and deletion instructions

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i: xy, d: $xy\bar{z}$, i: x, d: y, i: $\bot$

xyz	$xy\bar{z}$	$x\bar{y}z$	$x\overline{yz}$
$\bar{x}yz$	$\bar{x}y\bar{z}$	$\overline{xy}z$	$\overline{xyz}$

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A DRAT proof modifies an initial formula F into the accumulated formulas F_j

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$F_0 = F$

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F_1

xyz	$xy\overline{z}$	$x\overline{y}z$	$x\overline{y}\overline{z}$
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xy			

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i: xy, d: $xy\overline{z}$, i: x, d: y, i: $\perp$

F_2

xyz $\quad x\overline{y}z$ $\quad x\overline{yz}$

$\overline{x}yz$ $\quad \overline{x}y\overline{z}$ $\quad \overline{xy}z$ $\quad \overline{xyz}$

xy

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F_3

xyz		$x\overline{y}z$	$x\overline{yz}$
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xy	x		

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F_4

xyz		$x\bar{y}z$	$x\overline{yz}$
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F_5

xyz		$x\bar{y}z$	$x\bar{y}\bar{z}$
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Observation

RAT introduction is non-monotonic: C is a RAT in $F \nRightarrow C$ is a RAT in $F \wedge G$

deletion may disable but also enable RAT inferences! *[Rebola-Pardo, Philipp '17]*

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Multiset semantics **required for efficient proof generation**

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Unit clause deletion **simpler (but not necessarily faster) proof checking**

Is this really needed?

Proof generation and multiset semantics

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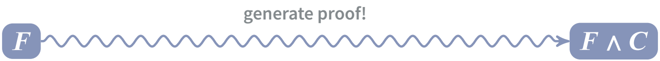

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what if C is not
a RUP/RAT in F?

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careful there!

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derived formula:
$F \wedge C$

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DRAT checkers assume this, but it was not specified in the definition

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$$
\begin{array}{cccc}
x_1 & \overline{x_1}x_2 & \overline{x_1}\,\overline{x_2}x_3 & \overline{x_1}\,\overline{x_3}x_4 \\
x_5x_6 & \overline{x_2}\,\overline{x_5}x_7 & \overline{x_1}\,\overline{x_5}x_6 & x_4\overline{x_5}\,\overline{x_6} \\
\overline{x_3}\,\overline{x_6}x_8 & x_3\overline{x_4}\,\overline{x_6} & x_5\overline{x_8} & \overline{x_3}x_9x_{10} \\
\overline{x_4}\,\overline{x_9}x_{10} & x_9\overline{x_{10}} & x_7\overline{x_9} & \overline{x_7}\,\overline{x_8}\,\overline{x_9}\,\overline{x_{10}}
\end{array}
$$

i: x_5, i: x_4, i: x_9, i: $\perp$

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\overline{x_3}\,\overline{x_6}\,x_8 & x_3\,\overline{x_4}\,\overline{x_6} & x_5\,\overline{x_8} & \overline{x_3}\,x_9\,x_{10} \\
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$x_1,\ x_2,\ x_3,\ x_4$

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$x_5 x_6$	$\overline{x}_2 \overline{x}_5 x_7$	$\overline{x}_1 \overline{x}_5 x_6$	$x_4 \overline{x}_5 \overline{x}_6$
$\overline{x}_3 \overline{x}_6 x_8$	$x_3 \overline{x}_4 \overline{x}_6$	$x_5 \overline{x}_8$	$\overline{x}_3 x_9 x_{10}$
$\overline{x}_4 \overline{x}_9 x_{10}$	$x_9 \overline{x}_{10}$	$x_7 \overline{x}_9$	$\overline{x}_7 \overline{x}_8 \overline{x}_9 \overline{x}_{10}$

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$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$

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i: x_5, i: x_4, i: x_9, i: $\perp$

$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8$

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$\overline{x_3 x_6} x_8$	$x_3 \overline{x_4 x_6}$	$x_5 \overline{x_8}$	$\overline{x_3} x_9 x_{10}$
$\overline{x_4 x_9} x_{10}$	$x_9 \overline{x_{10}}$	$x_7 \overline{x_9}$	$\overline{x_7 x_8 x_9 x_{10}}$

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$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10},\ \overline{x_{10}}$

literals implied by unit propagation

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For efficiency, DRAT checkers keep track of literals implied by unit propagation.

x_1	$\overline{x_1}x_2$	$\overline{x_1}\,\overline{x_2}x_3$	$\overline{x_1}\,\overline{x_3}x_4$
x_5x_6	$\overline{x_2}\,\overline{x_5}x_7$	$\overline{x_1}\,\overline{x_5}x_6$	$x_4\overline{x_5}\,\overline{x_6}$
$\overline{x_3}\,\overline{x_6}x_8$	$x_3\overline{x_4}\,\overline{x_6}$	$x_5\overline{x_8}$	$\overline{x_3}x_9x_{10}$
$\overline{x_4}\,\overline{x_9}x_{10}$	$x_9\overline{x_{10}}$	$x_7\overline{x_9}$	$\overline{x_7}\,\overline{x_8}\,\overline{x_9}\,\overline{x_{10}}$

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x_1, x_3, x_4, x_5, x_6, x_8

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Solution DRAT checkers ignore unit clause deletions [Heule '16]

Two flavors of DRAT

DRAT checkers are checking proofs with respect to a different proof system.

- Multiset semantics is justified by constraints in proof generation
  checking if a clause occurs in the formula is expensive
- Ignoring unit clause deletions is justified by constraints in proof checking
  No efficient unit propagation without two-watched literal schema

Multiset semantics should be included in the DRAT specification.
Should ignoring unit clause deletions be?

Specified DRAT
  Original definition + multiset semantics

Operational DRAT
  Original definition + multiset semantics + ignoring unit clause deletions

Operational DRAT as a proof system

Is operational DRAT sound?

Remember: deletions may change whether RATs can be infered

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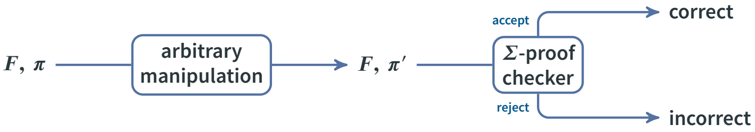

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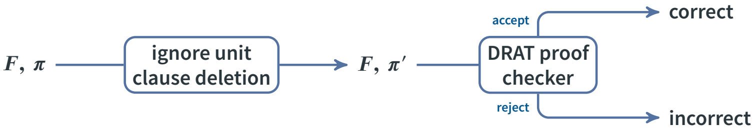

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Is operational DRAT sound? **Yes!**

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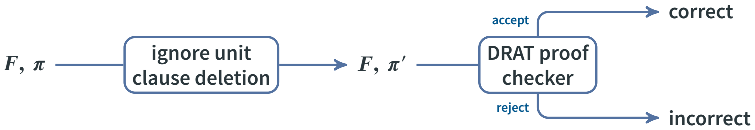

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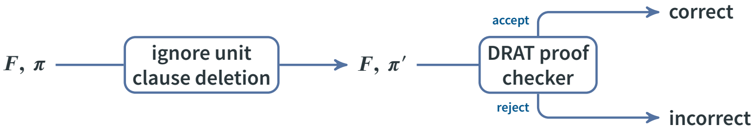

Is operational DRAT stronger or weaker than specified DRAT?

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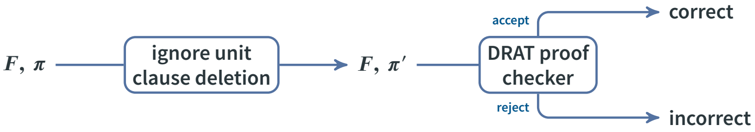

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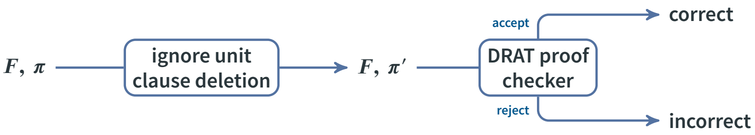

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Operational DRAT as a proof system

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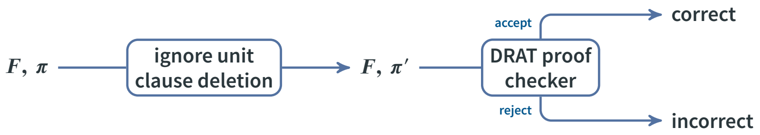

Is operational DRAT stronger or weaker than specified DRAT? Neither!

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Discussion

What flavor should be used?

Discussion should operational DRAT or specified DRAT be used?

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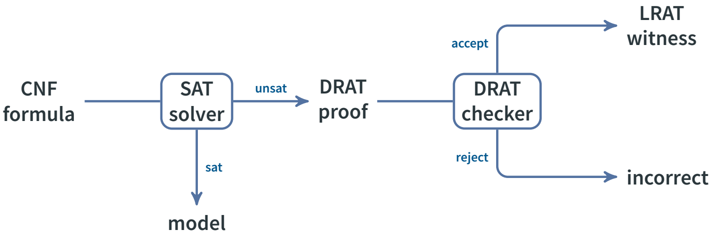

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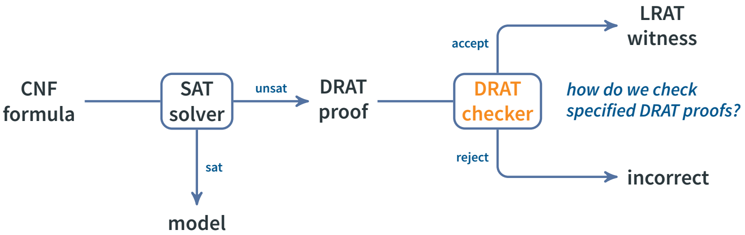

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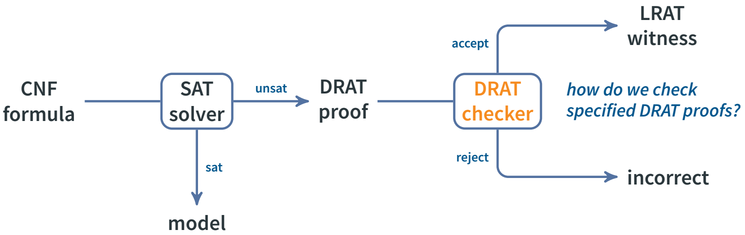

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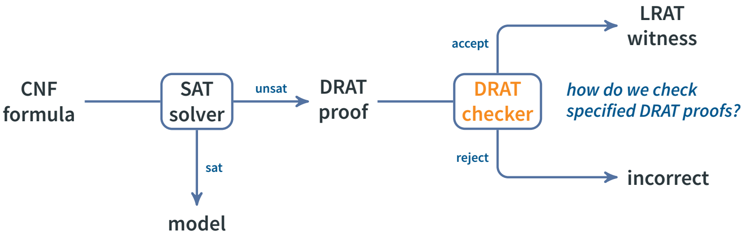

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Two-watched literal invariants are hard to maintain

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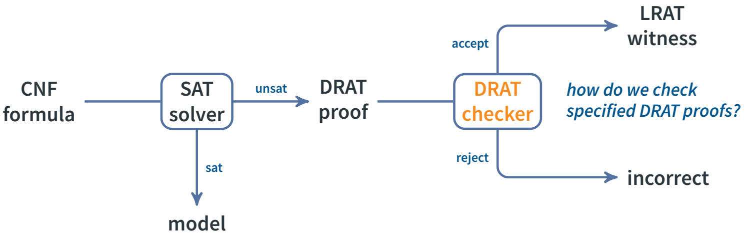

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Two-watched literal invariants are hard to maintain

Under review first specified DRAT checker [RP, Cruz-Filipe]

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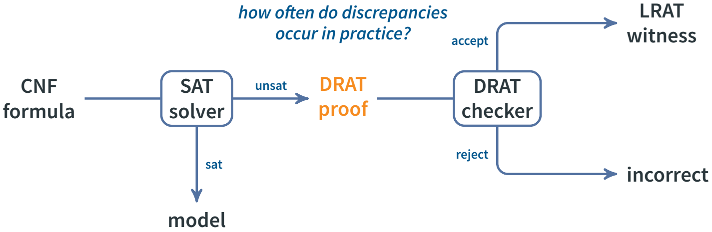

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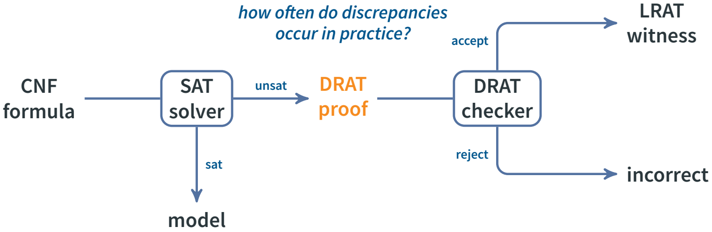

Potentially often 95% DRAT proofs contain unit deletions

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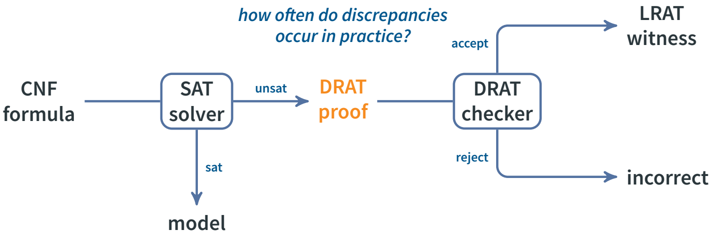

Potentially often 95% DRAT proofs contain unit deletions

Under review 59% discrepancies [RP, Cruz-Filipe]

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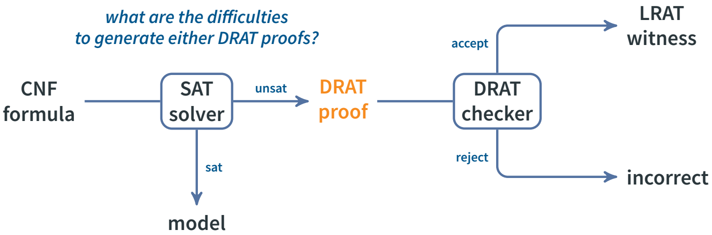

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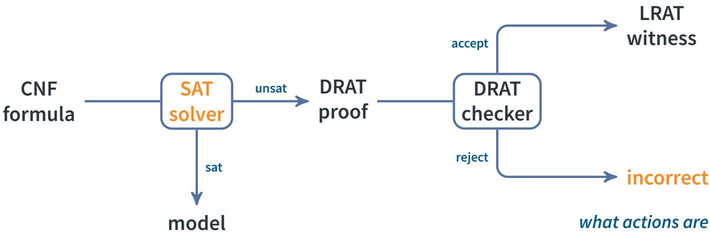
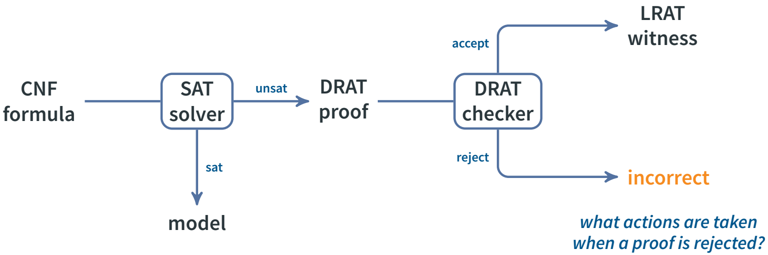


what actions are taken when a proof is rejected?

Solver debugging could lead to huge waste of time

What flavor should be used?

Discussion should operational DRAT or specified DRAT be used?



Solver debugging could lead to huge waste of time

Disqualifying solvers proofs rejected by DRAT-trim may be correct!

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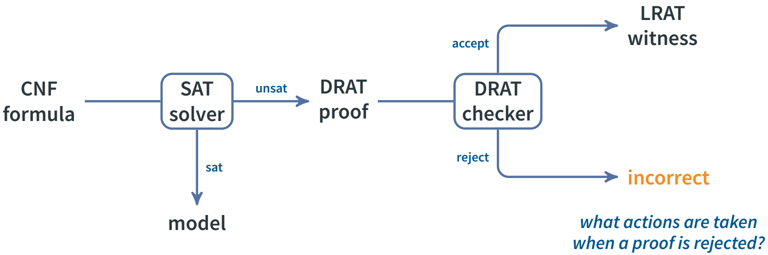

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Future work verifying incorrectness results

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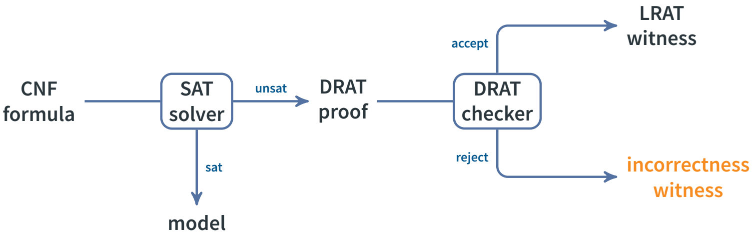

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Backup slides

Proof checking and unit clause deletion

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\begin{array}{cccc}
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\end{array}
$$

i: x_5, i: x_4, i: x_9, i: $\perp$

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\end{array}$$

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literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

x_1	$\overline{x_1} x_2$	$\overline{x_1}\,\overline{x_2} x_3$	$\overline{x_1}\,\overline{x_3} x_4$
$x_5 x_6$	$\overline{x_2}\,\overline{x_5} x_7$	$\overline{x_1}\,\overline{x_5} x_6$	$x_4 \overline{x_5}\,\overline{x_6}$
$\overline{x_3}\,\overline{x_6} x_8$	$x_3 \overline{x_4}\,\overline{x_6}$	$x_5 \overline{x_8}$	$\overline{x_3} x_9 x_{10}$
$\overline{x_4}\,\overline{x_9} x_{10}$	$x_9 \overline{x_{10}}$	$x_7 \overline{x_9}$	$\overline{x_7}\,\overline{x_8}\,\overline{x_9}\,\overline{x_{10}}$

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\bot$

x_1, x_3, x_4, x_5, x_6, x_8

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$$
\begin{array}{cccc}
x_1 & \overline{x}_1 x_2 & \overline{x}_1 \overline{x}_2 x_3 & \overline{x}_1 \overline{x}_3 x_4 \\
x_5 x_6 & \overline{x}_2 \overline{x}_5 x_7 & \overline{x}_1 \overline{x}_5 x_6 & x_4 \overline{x}_5 \overline{x}_6 \\
\overline{x}_3 \overline{x}_6 x_8 & x_3 \overline{x}_4 \overline{x}_6 & x_5 \overline{x}_8 & \overline{x}_3 x_9 x_{10} \\
\overline{x}_4 \overline{x}_9 x_{10} & x_9 \overline{x}_{10} & x_7 \overline{x}_9 & \overline{x}_7 \overline{x}_8 \overline{x}_9 \overline{x}_{10}
\end{array}
$$

i: x_5, d: $\overline{x}_1 x_2$, i: x_4, i: x_9, i: $\perp$

x_1, x_3, x_4, x_5, x_6, x_8

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$\overline{x_2}\ \ \overline{x_5}\ \ x_7$

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\perp$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$\overline{x_2}\ \ \overline{x_5}\ \ x_7$

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\bot$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$$\overline{x_2} \quad \overline{x_5} \quad x_7$$

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\perp$

x_1, x_3, x_4, x_5, x_6, x_8

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$\overline{x_2}\ \ \overline{x_5}\ \ x_7$

two-watched literal invariant is broken!

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\perp$

x_1, x_3, x_4, x_5, x_6, x_8

literals implied by unit propagation

Proof checking and unit clause deletion

For efficiency, DRAT checkers keep track of literals implied by unit propagation.

Two-watched literal schema

if one watched literal is assigned to false, then the other watched literal must be assigned to true

$\overline{x_2}\ \overline{x_5}\ x_7$

i: x_5, d: $\overline{x_1} x_2$, i: x_4, i: x_9, i: $\perp$

x_1, x_3, x_4, x_5, x_6, x_8

literals implied by unit propagation

Solution DRAT checkers ignore unit clause deletions

clauses whose literals are all falsified except for one satisfied literal