RAT Elimination

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LPAR-23
Adrián's flat
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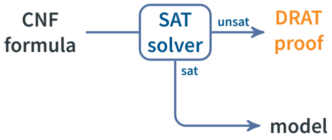
SAT solving and DRAT proofs

CNF
formula

SAT solving and DRAT proofs



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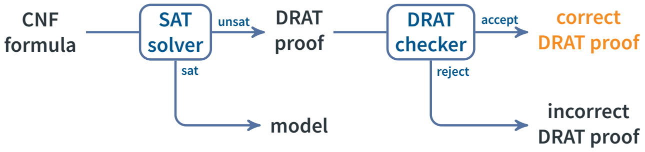

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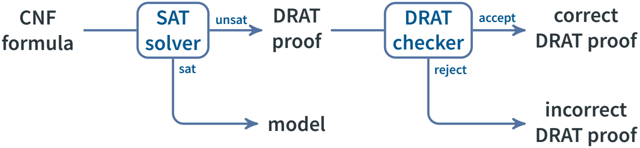

DRAT proofs [Heule, Hunt, Wetzler ’13]

a list of clause introductions and deletions over the premise formula

$$\begin{array}{rl} \text{i:} & x_1 x_2 x_3 \\ \text{i:} & \overline{x_3} x_4 \\ \text{d:} & x_1 x_2 x_3 \\ & \vdots \\ \text{i:} & \square \end{array}$$

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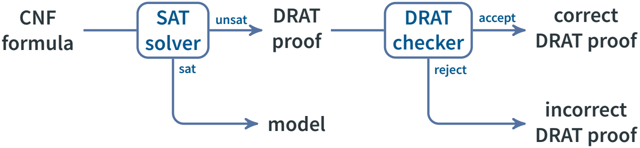

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i: $x_1 x_2 x_3$
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d: $x_1 x_2 x_3$
$\vdots$
i: $\square$

A clause C can be added to a formula F whenever C is a resolution asymmetric tautology (RAT) upon some literal l over F.

DRAT proofs vs DRUP proofs

Reverse unit propagation [Goldberg, Novikov '03]

A clause C is a RUP over F if unit propagation over $F \wedge \neg C$ reaches a conflict.

in that case, $F \models C$

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	DRUP proofs	DRAT proofs
proof complexity	$\simeq$ DAG-like resolution	$\simeq$ extended resolution

[Kiesl, Rebola-Pardo, Heule '18]

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proof complexity	$\simeq$ DAG-like resolution	$\simeq$ extended resolution
soundness	truth-preserving	satisfiability-preserving
practical expressivity	captures CDCL	also captures inprocessing

[Beame, Kautz, Sabharwal '04] [Heule, Hunt, Wetzler '15]

[Philipp, Rebola-Pardo '17]

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soundness	truth-preserving	satisfiability-preserving
practical expressivity	captures CDCL	also captures inprocessing
interpolation	easy, polynomial interpolation	no interpolation, possibly super-polynomial

[Gurfinkel, Vizel '14] [Bonet, Pitassi, Raz '97]

RAT elimination and clause isolation

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Any clause isolation procedure yields a RAT elimination procedure:

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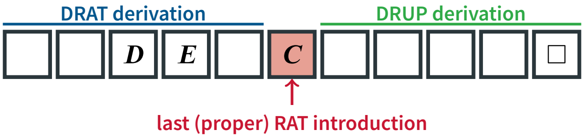

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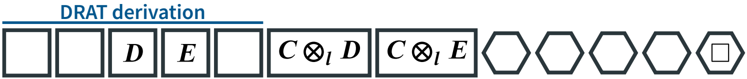

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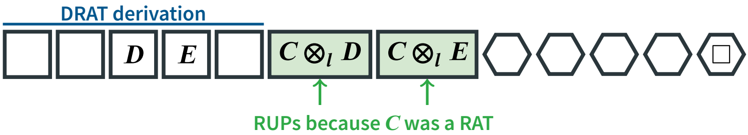

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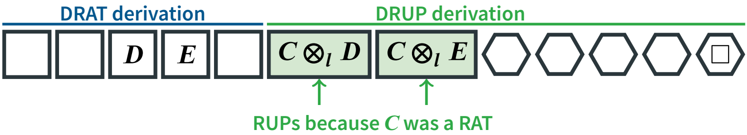

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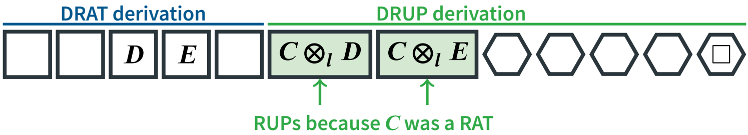

Contributions

We present an algorithm for clause isolation in subsumption + resolution proofs.

We refine this algorithm for RAT elimination in DRAT proofs.

This can be used to generate Craig interpolants from DRAT proofs.

Distributivity for resolution and subsumption

Resolution rule

$$l\ \frac{C \qquad D}{C \otimes_l D} \quad = C\setminus\{l\} \vee D\setminus\{\bar{l}\}$$

if $l \in C$ and $\bar{l} \in D$

Subsumption rule

$$\sqsubseteq\ \frac{C}{C \vee D}$$

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Using distributivity for clause isolation

if a RAT clause C upon a literal l is *actually* used to derive $\square$, then every time C is introduced its l literal will eventually be eliminated by resolution.

apply distributivity all the way to the top

Clause isolation through distributivity

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we isolate $C = xy\overline{z}$ upon x in a refutation of $F = \{xz,\ xy\overline{z},\ \overline{y},\ \overline{xz},\ z\}$

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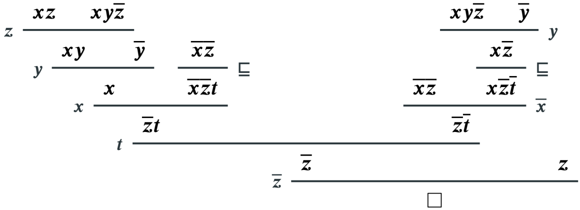

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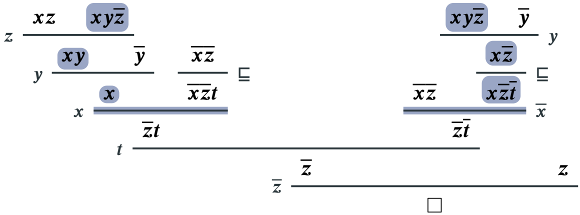

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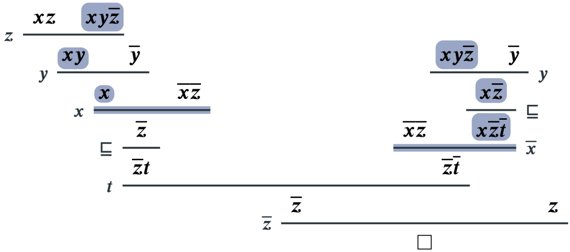

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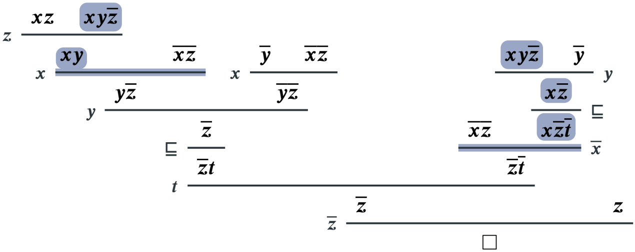

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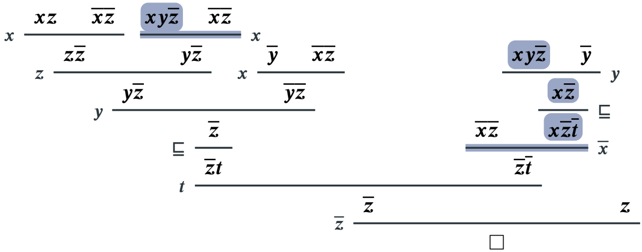

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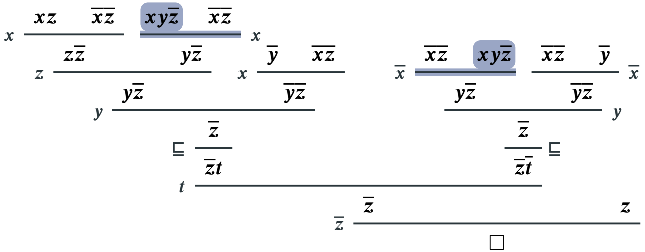

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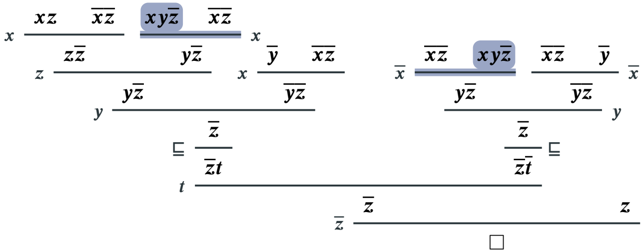

Problems with this approach

- Exponential blow-up in proof size
  probably unavoidable *[Bonet, Pitassi, Raz ’97]*
- Many redundant inferences and tautologies
  we might be able to do something about this...

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A characterization of RUPs [Philipp, Rebola-Pardo '17]

A clause D is a RUP over F if there exists a **RUP chain** deriving D from F:

$$\sqsubseteq\ \frac{E_0}{A_0} \qquad k_1\ \frac{A_0 \qquad E_1}{A_1} \qquad k_2\ \frac{A_1 \qquad E_2}{A_2} \qquad \ddots \qquad k_n\ \frac{A_{n-1} \qquad E_n}{A_n} \quad = D$$

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$$\sqsubseteq\ \frac{C \otimes_l E_0}{C \otimes_l A_0} \qquad k_1\ \frac{C \otimes_l A_0 \qquad C \otimes_l E_1}{C \otimes_l A_1} \qquad k_2\ \frac{C \otimes_l A_1 \qquad C \otimes_l E_2}{C \otimes_l A_2} \qquad \ddots \qquad k_n\ \frac{C \otimes_l A_{n-1} \qquad C \otimes_l E_n}{C \otimes_l A_n} \quad = C \otimes_l D$$

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Issues

- This generates n recursive calls to distributivity
- Many clauses after distributivity might be tautologies

Specializing distributivity for RUP chains

$$
{\scriptstyle v}\,\dfrac{{\scriptstyle u}\,\dfrac{{\scriptstyle t}\,\dfrac{{\scriptstyle z}\,\dfrac{{\scriptstyle y}\,\dfrac{{\scriptstyle \sqsubseteq}\,\dfrac{xy}{xyztuvw} \qquad \bar{y}z}{xztuvw} \qquad \bar{z}uv}{xtuvw} \qquad x\bar{t}v}{xuvw} \qquad \bar{u}v}{xvw} \qquad \bar{v}w}{xw}
$$

Specializing distributivity for RUP chains

$$v\,\dfrac{u\,\dfrac{t\,\dfrac{z\,\dfrac{y\,\dfrac{\sqsubseteq\,\dfrac{xy}{xyztuvw}\qquad \bar{y}z}{xztuvw}\qquad \bar{z}uv}{xtuvw}\qquad x\bar{t}v}{xuvw}\qquad \bar{u}v}{xvw}\qquad \bar{v}w}{xw}$$

We distribute over $C = \bar{x}y\bar{t}u$ upon $\bar{x}$

Specializing distributivity for RUP chains

$$\sqsubseteq \; \frac{y\bar{t}u}{\qquad\qquad} \qquad = C \otimes_{\bar{x}} xy$$

$$y\; \frac{yzt\bar{t}uvw \qquad y\bar{y}z\bar{t}u}{\qquad\qquad} \qquad = C \otimes_{\bar{x}} \bar{y}z$$

$$z\; \frac{yzt\bar{t}uvw \qquad y\bar{z}\bar{t}uv}{\qquad\qquad} \qquad = C \otimes_{\bar{x}} \bar{z}uv$$

$$t\; \frac{yt\bar{t}uvw \qquad y\bar{t}uv}{\qquad\qquad} \qquad = C \otimes_{\bar{x}} x\bar{t}v$$

$$u\; \frac{y\bar{t}uvw \qquad y\bar{t}u\bar{u}v}{\qquad\qquad} \qquad = C \otimes_{\bar{x}} \bar{u}v$$

$$v\; \frac{y\bar{t}uvw \qquad y\bar{t}u\bar{v}w}{y\bar{t}uw} \qquad = C \otimes_{\bar{x}} \bar{v}w$$

We distribute over $C = \bar{x}y\bar{t}u$ upon $\bar{x}$

Specializing distributivity for RUP chains

$$
v\,\dfrac{u\,\dfrac{t\,\dfrac{z\,\dfrac{y\,\dfrac{\sqsubseteq\,\dfrac{y\bar{t}u \qquad = C \otimes_{\bar{x}} xy}{yzt\bar{t}uvw} \qquad y\bar{y}z\bar{t}u \qquad = C \otimes_{\bar{x}} \bar{y}z}{yzt\bar{t}uvw} \qquad y\bar{z}\bar{t}uv \qquad = C \otimes_{\bar{x}} \bar{z}uv}{yt\bar{t}uvw} \qquad y\bar{t}uv \qquad = C \otimes_{\bar{x}} x\bar{t}v}{y\bar{t}uvw} \qquad y\bar{t}u\bar{u}v \qquad = C \otimes_{\bar{x}} \bar{u}v}{y\bar{t}uvw} \qquad y\bar{t}u\bar{v}w \qquad = C \otimes_{\bar{x}} \bar{v}w}{y\bar{t}uw}
$$

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RUP chain refinements

Specializing distributivity for RUP chains

$$
\sqsubseteq\ \dfrac{y\bar{t}u \qquad = C \otimes_{\bar{x}} xy}{y\ \dfrac{yzt\bar{t}uvw \qquad y\bar{y}z\bar{t}u \qquad = C \otimes_{\bar{x}} \bar{y}z}{z\ \dfrac{yzt\bar{t}uvw \qquad y\bar{z}\bar{t}uv \qquad = C \otimes_{\bar{x}} \bar{z}uv}{t\ \dfrac{yt\bar{t}uvw \qquad y\bar{t}uv \qquad = C \otimes_{\bar{x}} x\bar{t}v}{u\ \dfrac{y\bar{t}uvw \qquad y\bar{t}u\bar{u}v \qquad = C \otimes_{\bar{x}} \bar{u}v}{v\ \dfrac{y\bar{t}uvw \qquad y\bar{t}u\bar{v}w \qquad = C \otimes_{\bar{x}} \bar{v}w}{y\bar{t}uw}}}}}}
$$

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RUP chain refinements

- Drop redundant resolutions that do not change the main proof path

Specializing distributivity for RUP chains

$$\sqsubseteq \; \frac{y\bar{t}u}{yzt\bar{t}uvw} \quad = C \otimes_{\overline{x}} xy$$

$$z \; \frac{y\bar{z}\bar{t}uv}{} \quad = C \otimes_{\overline{x}} \bar{z}uv$$

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$$v \; \frac{y\bar{t}u\bar{v}w}{y\bar{t}uw} \quad = C \otimes_{\overline{x}} \bar{v}w$$

We distribute over $C = \overline{x}y\bar{t}u$ upon $\overline{x}$

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$$
\begin{array}{lll}
\sqsubseteq & \dfrac{y\bar{t}u}{yzt\bar{t}uvw \qquad y\bar{z}\bar{t}uv} & = C \otimes_{\bar{x}} xy \\
z & \dfrac{\qquad}{yt\bar{t}uvw \qquad y\bar{t}uv} & = C \otimes_{\bar{x}} \bar{z}uv \\
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$$

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RUP chain refinements

- Drop redundant resolutions that do not change the main proof path
- Restore premises where resolution does not have an effect

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We distribute over $C = \overline{x}y\bar{t}u$ upon $\overline{x}$

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RUP chain refinements

- Drop redundant resolutions that do not change the main proof path
- Restore premises where resolution does not have an effect
- Cut the chain off from the bottommost tautology

Specializing distributivity for RUP chains

$$\frac{}{y\bar{t}uv} \quad = C \otimes_{\bar{x}} x\bar{t}v$$

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An algorithm for RUP distributivity starting from the bottom:

- if $k_i, \overline{k_i} \notin C$, then use $C \otimes_l E_i$ as premise
- if $k_i \in C \cup \{\bar{l}\}$, then use E_i as premise
- if $\overline{k_i} \in C \setminus \{l\}$, then cut the chain short

Conclusion

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- The exponential blow-up in proof size is probably unavoidable. Does it still outperform running the SAT solver without inprocessing techniques (hence, directly generating a DRUP proof)?
- Can we generate Craig interpolants (or formulas that work like interpolants for model checking purposes) directly from DRAT proofs without transforming them into DRUP?