Towards a Semantics of Unsatisfiability Proofs with Inprocessing

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$$
\begin{array}{ccccc}
xy & xz & xt & yz & yt \\
zt & \bar{x}\,\bar{y}\,\bar{z} & \bar{x}\,\bar{y}\,\bar{t} & \bar{x}\,\bar{z}\,\bar{t} & \bar{y}\,\bar{z}\,\bar{t}
\end{array}
$$

Symmetry breaking clause $\bar{x}y$ can be added without affecting satisfiability

Inprocessing for SAT Solving

xy	xz	xt	yz	yt
zt	$\overline{x}\,\overline{y}\,\overline{z}$	$\overline{x}\,\overline{y}\,\overline{t}$	$\overline{x}\,\overline{z}\,\overline{t}$	$\overline{y}\,\overline{z}\,\overline{t}$

Symmetry breaking clause $\overline{x}y$ can be added without affecting satisfiability

In-/preprocessing techniques replace the formula by an equisatisfiable one

symmetry breaking

clause elimination

parity reasoning

bounded variable addition

cardinality resolution

How to **certify** the result of a SAT solver with inprocessing?

Satisfiability-preserving proof systems

How to certify the result of a SAT solver with inprocessing?

RUP cannot be used!

Theorem

If C is a RAT in F, then whenever F is satisfiable, so is $F \cup \{C\}$.

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Idea build a proof system with $\vDash_{\text{sat}}$ as an invariant

- If C can be inferred, then $F \vDash_{\text{sat}} C$.
- Rules must be simple and efficient to check.
- Inprocessing rules should be easy to express.

The RAT property satisfies these conditions!

The semantics of DRAT proofs

Clause C is a RAT in F

RAT

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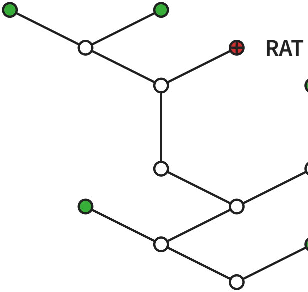

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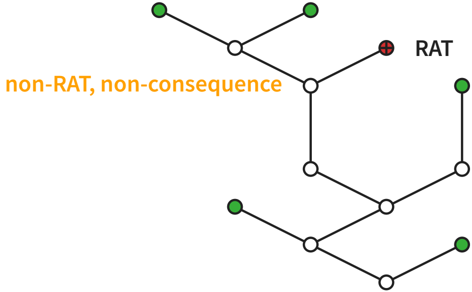

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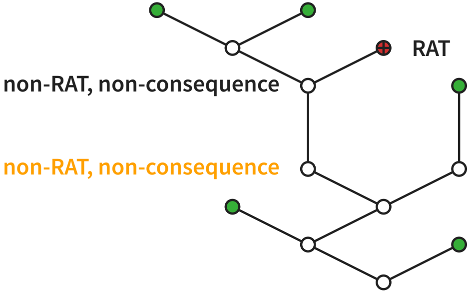

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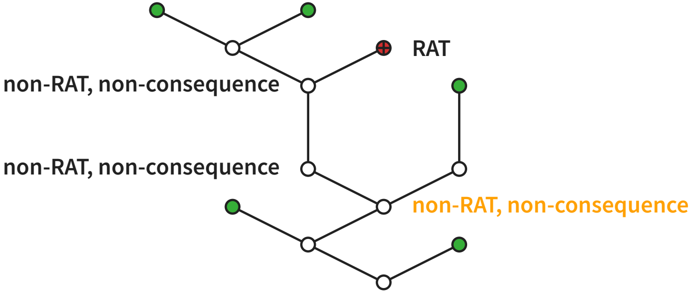

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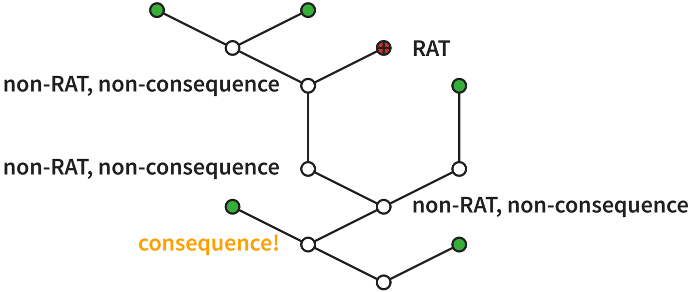

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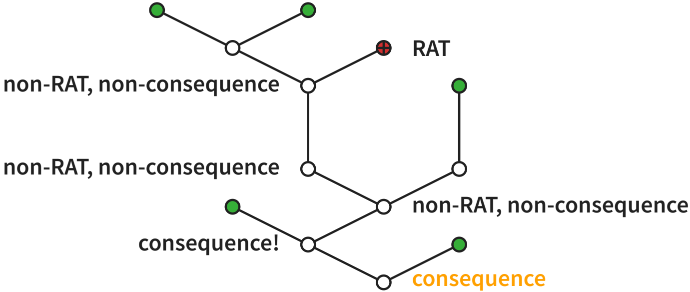

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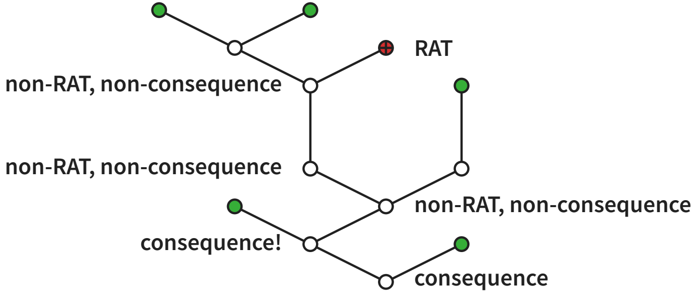

Some invariant seems to be preserved...

Question what is the invariant preserved along a DRAT proof?

if derived clauses are not consequences, then what are they?

DRAT proofs

RUP Introduction

Most derived clauses are simply consequences of previously introduced clauses.

Definition

A clause C is a reverse unit propagation (RUP) clause in a CNF formula F if boolean constraint propagation over $F \cup \overline{C}$ leads to contradiction.

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Characterization

C is a RUP in F if and only if $\Box \in F$, or the least fixed point of ala_F contains complementary clauses, where:

$$\mathrm{ala}_F(C) = C \cup \{\bar{l} \mid \text{for some } D \in F,\ l \in D \text{ and } D \setminus \{l\} \subseteq C\}$$

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C is a RUP in F if and only if it can be proved from F through an SSSR chain:

$$\cfrac{\cfrac{\cfrac{C_0}{D_0}\,\text{sub} \qquad C_1}{D_1}\,\text{ssr}}{\begin{array}{c}\ddots \\ \cfrac{D_{n-1} \qquad C_n}{D_n}\,\text{ssr}\end{array}} = C$$

RAT Introduction and Clause Deletion

Introducing a RAT clause preserves satisfiability

Definition

A clause C is a resolution asymmetric tautology (RAT) in a CNF formula F upon a literal $l \in C$ if every resolvent $C \otimes_l D$ for $D \in F$ is a RUP in F.

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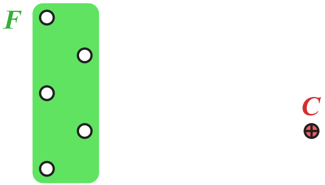

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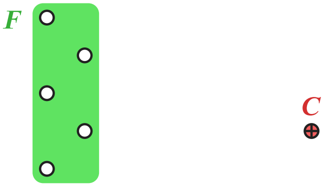

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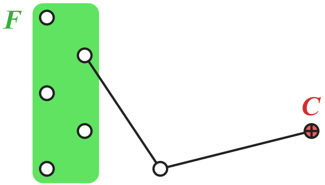

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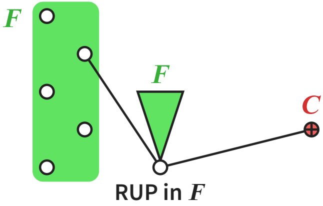

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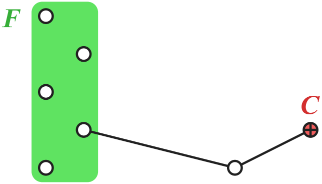

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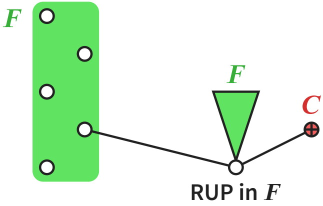

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Clause deletion preserves satisfiability for arbitrary clauses

- Deletion information is needed for efficiency in BCP
- Arbitrary clause deletion can be expressed

DRAT proofs

Delete Resolution Asymmetric Tautology (DRAT) proof system

$$F \Rightarrow_{\text{DRAT}} G \quad \begin{cases} G = F \cup \{C\} & \begin{cases} C \text{ is a RUP in } F \\ C \text{ is a RAT in } F \end{cases} \\ G = F \setminus \{C\} \end{cases}$$

A DRAT proof of G from F is a sequence of DRAT inferences:

$$F = F_0 \Rightarrow_{\text{DRAT}} F_1 \Rightarrow_{\text{DRAT}} \cdots \Rightarrow_{\text{DRAT}} F_{n-1} \Rightarrow_{\text{DRAT}} F_n = G$$

RATs as Definitions

Defining a Definition

Clauses containing variable x can be regarded as a definition:

$$x \lor C_1 \qquad \dots \qquad x \lor C_n \qquad \overline{x} \lor D_1 \qquad \dots \qquad \overline{x} \lor D_m$$

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$$x = \begin{cases} 1 & \text{if } \overline{C_1} \\ \vdots & \\ 1 & \text{if } \overline{C_n} \\ 0 & \text{if } \overline{D_1} \\ \vdots & \\ 0 & \text{if } \overline{D_m} \\ \text{don't care} & \text{otherwise} \end{cases}$$

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Definition refinement to add rule $x \leftarrow C_{n+1}$, we require:

$$\overline{C_{n+1}} \wedge \overline{D_i} \text{ is unsatisfiable for } 1 \leq i \leq m$$

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$$F \models (x \vee \overline{C_{n+1}}) \otimes_x (\overline{x} \vee \overline{D_i}) \text{ for } 1 \leq i \leq m$$

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The Semantics of Definitions

Question when is a definition correct from the semantic perspective?

- A definition changes the interpretation to force a truth value under some conditions.
- After the change, the interpretation still satisfies the original constraints.

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Mutation rule $l \coloneq\!\!- \; Q$

Interpretation mutation

$$I \lhd (l \coloneq\!\!- \; Q) = \begin{cases} I + l & \text{if } I \models Q \\ I & \text{if } I \not\models Q \end{cases}$$

Definition

$l \coloneq\!\!- \; Q$ is a definition refinement of F if, whenever $I \models F$ holds, then $I \lhd (l \coloneq\!\!- \; Q) \models F$ holds as well.

Definition Refinements are Resolution Consequences

Definition

A clause C is a resolution consequence in F upon l if every resolvent $C \otimes_l D$ for $D \in F$ is a consequence of F.

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Theorem

$C \vee l$ is a RC in F upon l if and only if $l :- \overline{C}$ is a definition refinement.

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Example z is defined as true when x holds, and false when y holds

correct only if $\overline{x} \wedge \overline{y}$ is disallowed

$$\overline{x}\,\overline{y} \xrightarrow{z \,:\!-\, \overline{x}} zx \xrightarrow{\overline{z} \,:\!-\, \overline{y}} \overline{z}y$$

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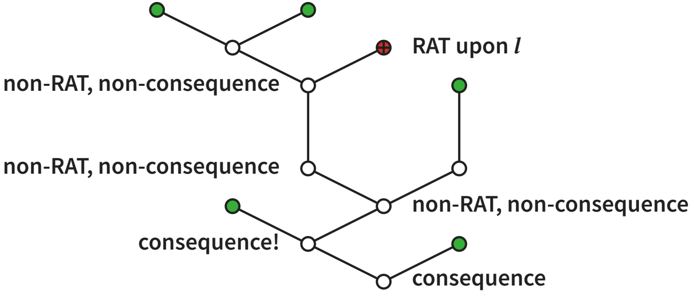
$$I = \{x, \overline{y}, z\} \qquad I' = \{x, \overline{y}, z\} \qquad I'' = \{x, \overline{y}, \overline{z}\}$$

Definitional Formulas

Resolution Consequences as Proof Invariants



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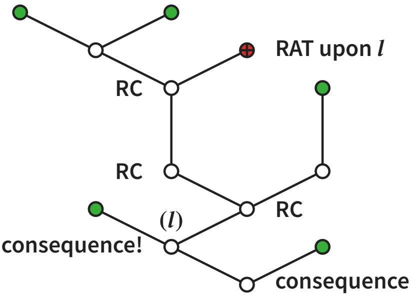

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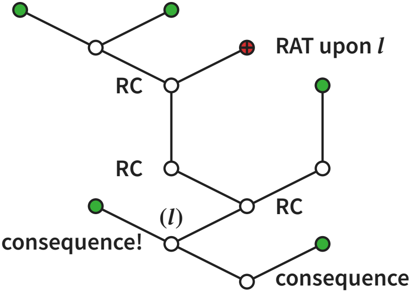

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Question which definitions are required for each derived clause?

Definitional formulas

Formula trees

$$\begin{array}{cc} xyz & \overline{x}yz \\ x\overline{y}z & \overline{x}\,\overline{y}z \end{array} \xrightarrow{\quad w \coloneq \overline{y} \quad} \begin{array}{c} wy \\ wz \end{array} \xrightarrow{\quad \overline{w} \coloneq \overline{z} \quad} \overline{w}z$$

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$$I \models \begin{matrix} xyz & \overline{x}yz \\ x\overline{y}z & \overline{x}\,\overline{y}z \end{matrix} \xrightarrow{\;w \coloneq \overline{y}\;} \begin{matrix} wy \\ wz \end{matrix} \xrightarrow{\;\overline{w} \coloneq \overline{z}\;} \overline{w}z$$

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$$I \lhd (w \coloneq \overline{y}) \quad \models \quad \boxed{\begin{array}{cc} xyz & \overline{x}yz \\ x\overline{y}z & \overline{x}\,\overline{y}z \end{array} \xrightarrow{\;w \,\coloneq\, \overline{y}\;} \begin{array}{c} wy \\ wz \end{array}} \xrightarrow{\;\overline{w} \,\coloneq\, \overline{z}\;} \overline{w}z$$

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Stratification derived clauses can be split based on which definitions they depend upon

Example $z = wz \otimes_w \bar{w}z$

Definitional formulas

Formula trees

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Stratification derived clauses can be split based on which definitions they depend upon

Example $z = wz \otimes_w \bar{w}z$

Definitional formulas

Formula trees

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Equivalence- and definitionality-preserving inferences

- Consequence introduction
  simulates RUP introduction
- Definition extension
  simulates RAT introduction
- Clause upgrade
  applied on resolvents upon pivot literals

Definitions and deletion

Clause deletion can make correct definitions incorrect:

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Question is there any invariant throughout DRAT proofs stronger than $\vDash_{\text{sat}}$?

Theorem G is DRAT-derivable from F if and only if $F \vDash_{\text{sat}} G$

no stronger invariant within propositional logic and unrestricted deletion

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Possible workarounds

- Restrict deletion to realistic situations
- Construe a DRAT proof as a derivation on DQBF clauses.
- Change mutation rules upon deletion.

Conclusion

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Future work

can we generate resolution/RUP proofs from a DRAT proof? yes!

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In the absence of deletion, proofs correspond to derivations on definitional formulas. In particular, resolution consequences correspond exactly to a form of permissive definition.

Future work

can we generate resolution/RUP proofs from a DRAT proof? yes!

Actual future work

can we find a semantics stronger than satisfiability preservation for DRAT proofs?