# Look Ma! No Interference!

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# Proofs in SAT solving: the good ol' times

**Resolution** [Davis, Putnam '60]

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*but also DRAT [Wetzler, Heule, Hunt ’14], DPR [Heule, Kiesl, Biere ’17]*

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**The real issue** non-monotonicity and global dependence a.k.a. **interference**

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Operate over constraints $\nabla(\sigma_1 :- T_1). \ldots \nabla(\sigma_n :- T_n).C$

$I \vDash \nabla(T :- \sigma).\varphi$ iff $I' \vDash \varphi$, where $I' = I \circ \sigma$ if $I \vDash T$, and $I' = I$ otherwise

If $C$ is SR over $F$ upon $\sigma$ then $F \vDash \nabla(\overline{C} :- \sigma).F \cup \{C\}$

Fixpoints in trimming are now gone! Lemmas can now safely derived! Deletions are now semantically irrelevant! All inferences have clearly defined dependencies!

# Accumulated formulas are not your friend

$C$ is SR over $F$ upon $\sigma$ if $F \cup \{\overline{C}\} \vDash (F \cup \{C\})|_\sigma$

If I had an assignment $I$ satisfying $F$, then check if it satisfies $C$.

If it does, then $I$ satisfies $F \cup \{C\}$; otherwise, $I \circ \sigma$ satisfies $F \cup \{C\}$.

If $\pi$ is a proof of $F \vdash G$ then, given any assignment $I$ satisfying $F$, I know how to construct $\mathrm{mut}(I)$ satisfying $G$

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... but subproofs still can't be merged *I must be missing something...*

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Dominance needs three accumulated formulas *please stop...*

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$$F \overset{}{\underset{\kappa}{\rightsquigarrow}} K \underset{\delta}{\rightsquigarrow} K \cup R \qquad K(x) \vDash O(x, \delta(x))$$

RUP and SR add clauses in $R(x)$

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If is dominance-redundant over $K, R, O$ upon $\sigma$, then $C$ is redundant on $K \cup R$

# Look who's hiding under dominance

$K(x), R(x), O(x, x')$ are CNF formulas        $O$ encodes a preorder

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If I had an assignment $I_0$ satisfying $K$,
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Why do we keep having global conditions on classical reasoning when *we know* classical logic is monotonic? [Martin Suda over lunch in 2017]

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$$I \vDash \varepsilon.C \quad \text{iff} \quad J \vDash C \text{ for all } J \text{ such that } I \otimes J \vDash \varepsilon$$

# What’s in the box? Programs!

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# What's in the box? Programs!

**Noop** $I \otimes J \vDash 1$ iff $I = J$

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**Assignment** $I \otimes J \vDash \langle \sigma \rangle$ iff $J = I \circ \sigma$

# What's in the box? Programs!

**Noop** $I \otimes J \vDash 1$ iff $I = J$

**Composition** $I \otimes J \vDash \varepsilon_1\, \varepsilon_2$ iff $\exists K,\ I \otimes K \vDash \varepsilon_1$ and $K \otimes J \vDash \varepsilon_2$

**Assignment** $I \otimes J \vDash \langle \sigma \rangle$ iff $J = I \circ \sigma$

**Choice** $I \otimes J \vDash \varepsilon_1 \sqcup \varepsilon_2$ iff $I \otimes J \vDash \varepsilon_1$ or $I \otimes J \vDash \varepsilon_2$

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**Branch** $I \otimes J \vDash \nabla(T : \varepsilon_1 \parallel \varepsilon_0)$ iff $I \otimes J \vDash (T?\, \varepsilon_1) \sqcup (\overline{T}?\, \varepsilon_0)$

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**Repeat** $I \otimes J \vDash \varepsilon^*$ iff $\exists I_i,\ I_0 = I,\ I_n = J,\ I_{i-1} \otimes I_i \vDash \varepsilon$

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**Repeat** $I \otimes J \vDash \varepsilon^*$ iff $\exists I_i,\ I_0 = I,\ I_n = J,\ I_{i-1} \otimes I_i \vDash \varepsilon$

**Loop** $I \otimes J \vDash \Box(T : \varepsilon)$ iff $I \otimes J \vDash (\overline{T}?\varepsilon)^*\, T?$

**Rendezvous** $I \otimes J \vDash \Diamond(V : \varepsilon_1 \parallel \varepsilon_0)$ iff $\exists J_1, J_0,\ I \otimes J_i \vDash \varepsilon_i$ and $J = J_1 +_V J_0$

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**Solve** $I \otimes J \vDash [R]$ iff $I \otimes J \vDash R$

# What's in the box? Programs!

**Noop** $I \otimes J \models 1$ iff $I = J$

**Composition** $I \otimes J \models \varepsilon_1\, \varepsilon_2$ iff $\exists K,\ I \otimes K \models \varepsilon_1$ and $K \otimes J \models \varepsilon_2$

**Assignment** $I \otimes J \models \langle \sigma \rangle$ iff $J = I \circ \sigma$

**Choice** $I \otimes J \models \varepsilon_1 \sqcup \varepsilon_2$ iff $I \otimes J \models \varepsilon_1$ or $I \otimes J \models \varepsilon_2$

**Test** $I \otimes J \models T?$ iff $I = J$ and $I \models T$

**Branch** $I \otimes J \models \nabla(T : \varepsilon_1 \parallel \varepsilon_0)$ iff $I \otimes J \models (T?\, \varepsilon_1) \sqcup (\overline{T}?\, \varepsilon_0)$

**Repeat** $I \otimes J \models \varepsilon^*$ iff $\exists I_i,\ I_0 = I,\ I_n = J,\ I_{i-1} \otimes I_i \models \varepsilon$

**Loop** $I \otimes J \models \square(T : \varepsilon)$ iff $I \otimes J \models (\overline{T}?\varepsilon)^*\, T?$

**Rendezvous** $I \otimes J \models \lozenge(V : \varepsilon_1 \parallel \varepsilon_0)$ iff $\exists J_1, J_0,\ I \otimes J_i \models \varepsilon_i$ and $J = J_1 +_V J_0$

**Solve** $I \otimes J \models [R]$ iff $I \otimes J \models R$

**Havoc** $I \otimes J \models \forall V$ iff $I \otimes J \models \lozenge(V : [\top] \parallel 1)$

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Solve $I \otimes J \vDash [R]$ iff $I \otimes J \vDash R$

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Not even a new thing! [Fischer, Ladner '79] [Balbiani, Herzig, Troquard '13]

Propositional dynamic logic (PDL) defines modalities for each program

# What's in the box? Programs!

Noop $I \otimes J \vDash \mathbf{1}$ iff $I = J$

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Not even a new thing! [Fischer, Ladner '79] [Balbiani, Herzig, Troquard '13]

Propositional dynamic logic (PDL) defines modalities for each program

Necessitation law (a.k.a. I can apply programs to a proof)

If $F \vDash G$ holds, then $\varepsilon.F \vDash \varepsilon.G$ holds too

# What can I do with this?

**Proving unsatisfiability** $F$ is unsatisfiable if $F \vdash \varepsilon.\bot$ and $\varepsilon.\bot \vdash \bot$

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**Parametric proofs** If I have proven $F \vdash G$, then I can prove $F|_\sigma \vdash G|_\sigma$ as $\langle\sigma\rangle.F \vdash \langle\sigma\rangle.G$

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**Refinements** $\varepsilon$ refines $\delta$ if $\varepsilon \vdash [R_\varepsilon]$ and $[R_\delta] \vDash \delta$ and $R_\varepsilon \vdash R_\delta$

# What can I do with this?

**Proving unsatisfiability** $F$ is unsatisfiable if $F \vdash \varepsilon.\bot$ and $\varepsilon.\bot \vdash \bot$

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An appetizer: [Rebola-Pardo ’25, SYNASC]

# Trying to merge proofs...

all unsat, over the same variables

$F_1$ $F_2$ $F_3$

# Trying to merge proofs...

$u_1 \vee F_1$ $\qquad$ $u_2 \vee F_2$ $\qquad$ $u_3 \vee F_3$

# Trying to merge proofs...

still unsat!

$u_1 \vee F_1$ $\quad$ $u_2 \vee F_2$ $\quad$ $u_3 \vee F_3$ $\quad$ $\overline{u_1} \vee \overline{u_2} \vee \overline{u_3}$

# Trying to merge proofs...

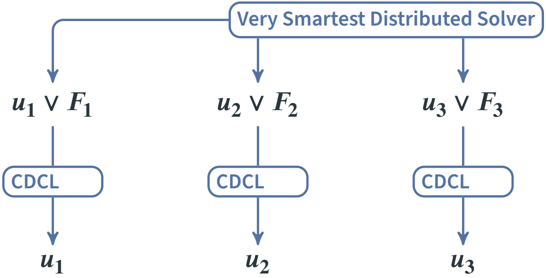
$u_1 \vee F_1$ $\quad$ $u_2 \vee F_2$ $\quad$ $u_3 \vee F_3$ $\quad$ $\overline{u_1} \vee \overline{u_2} \vee \overline{u_3}$

Very Smartest Distributed Solver

$u_1 \vee F_1$ $\quad$ $u_2 \vee F_2$ $\quad$ $u_3 \vee F_3$

# Trying to merge proofs...

$u_1 \lor F_1$ $\quad$ $u_2 \lor F_2$ $\quad$ $u_3 \lor F_3$ $\quad$ $\overline{u_1} \lor \overline{u_2} \lor \overline{u_3}$



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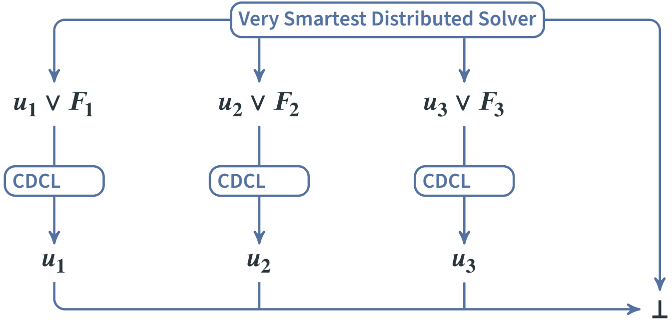

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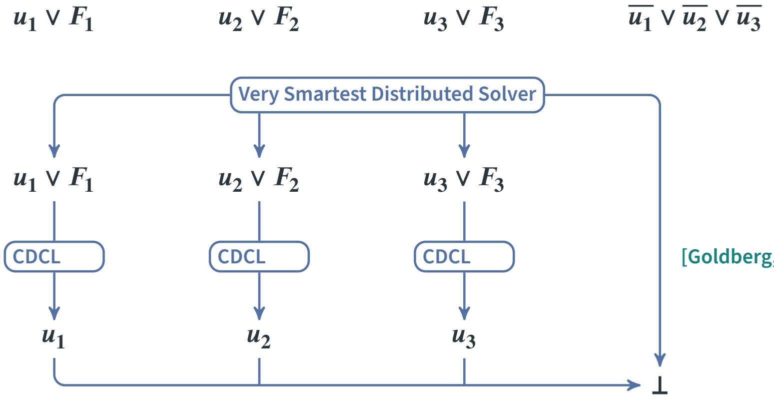


[Goldberg, Novikov '03]

$$\pi_1 : u_1 \vee F_1 \vdash u_1$$
$$\pi_2 : u_2 \vee F_2 \vdash u_2$$
$$\pi_3 : u_3 \vee F_3 \vdash u_3$$

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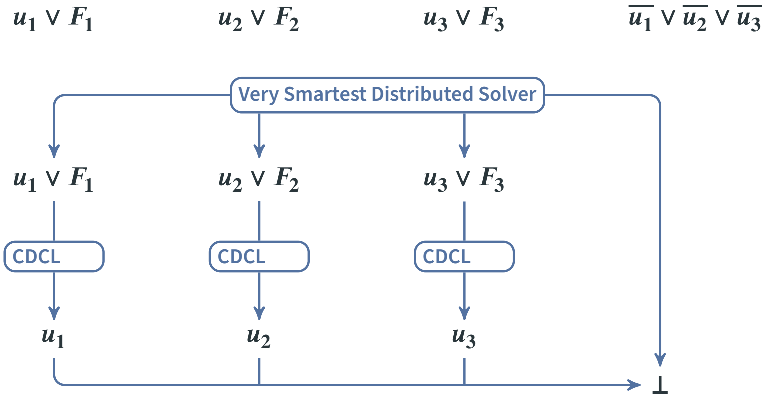


$\pi_1 : u_1 \lor F_1 \vdash u_1$

$\pi_2 : u_2 \lor F_2 \vdash u_2$

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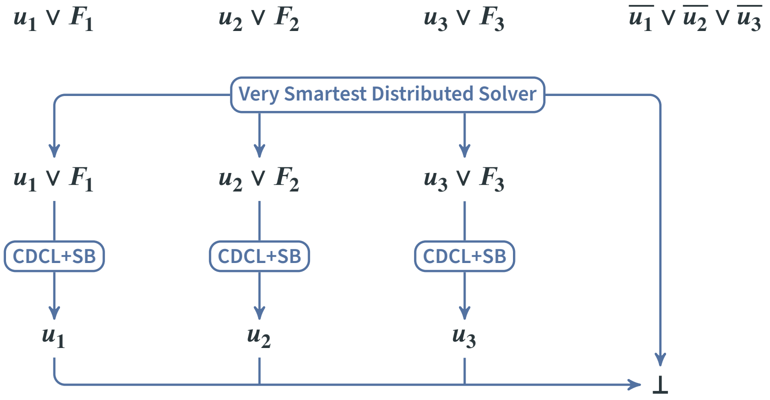
$\pi : u_1 \land u_2 \land u_3 \land (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3}) \vdash \perp$

# Trying to merge proofs...




$$\pi_1 : u_1 \vee F_1 \vdash u_1$$
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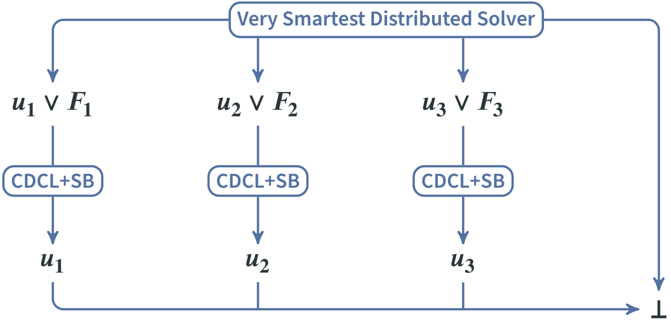
# Trying to merge proofs...

$\sigma_1 = \{x_1 \leftrightarrow y_1\}$ $\sigma_2 = \{x_2 \leftrightarrow y_2\}$ $\sigma_3 = \{x_3 \leftrightarrow y_3\}$

$u_1 \vee F_1$ $u_2 \vee F_2$ $u_3 \vee F_3$ $\overline{u_1} \vee \overline{u_2} \vee \overline{u_3}$




$\pi_1 : u_1 \vee F_1 \vdash u_1$

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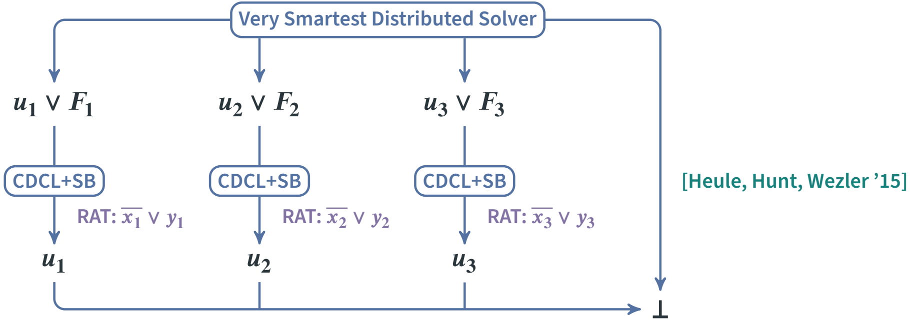
# Trying to merge proofs...

$\sigma_1 = \{x_1 \leftrightarrow y_1\}$ $\sigma_2 = \{x_2 \leftrightarrow y_2\}$ $\sigma_3 = \{x_3 \leftrightarrow y_3\}$

$u_1 \lor F_1$ $u_2 \lor F_2$ $u_3 \lor F_3$ $\overline{u_1} \lor \overline{u_2} \lor \overline{u_3}$




$$\pi_1 : u_1 \lor F_1 \vdash u_1$$
$$\pi_2 : u_2 \lor F_2 \vdash u_2$$
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# Trying to merge proofs...

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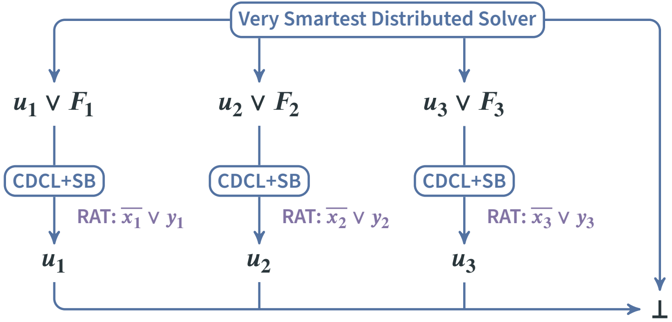


[Järvisalo, Heule, Biere ’12]

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$\pi_2 : u_2 \vee F_2 \vdash u_2$

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$\pi : u_1 \wedge u$ NOPE $\vee \overline{u_3}) \vdash \perp$

# … and failing because of interference

**What are DRAT proofs really doing?**

$\pi : F \vdash G$ proves that for each $I \vDash F$ we have $\mathrm{mut}(I) \vDash G$

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$\mathrm{mut}$ is a sequence of operations like if $I \vDash T$, then $I := I \circ \sigma$ [RP, Suda '18]

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**What are DRAT proofs really doing?**

$\pi : F \vdash G$ proves that for each $I \models F$ we have $\mathrm{mut}(I) \models G$

$\mathrm{mut}$ is a sequence of operations like if $I \models T$, then $I := I \circ \sigma$ [RP, Suda '18]

*each SR addition of $C$ upon $\sigma$ introduces a new operation with $T = \overline{C}$*

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# … and failing because of interference

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what we need is this!

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Interference-based proof systems force a single concurrent $\mathrm{mut}$ prefix…

*… because of the accumulated formula*

# RUP-like inferences in dynamic logic

Unit propagation (BCP) a *blazingly* fast sound but incomplete algorithm for inconsistency in CNF formulas [Zhang, Madigan, Moskewicz, Malik ’01]

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**Can we have this for dynamic formulas?** yes, but not built on consistency.

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$F \wedge \overline{\varepsilon.C} \vDash \bot$ iff $F \vDash \varepsilon.C$ I don’t have diamonds!

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**Can we have this for dynamic formulas?** yes, but not built on consistency.

Instead we reduce a dynamic implication check to (several) RUP checks

$$F \wedge \overline{\varepsilon.C} \vDash \bot \quad \text{iff} \quad F \vDash \varepsilon.C$$

I don't have diamonds!

# Look Ma! No interference!

$$F = (u_1 \vee F_1) \wedge (u_2 \vee F_2) \wedge (u_3 \vee F_3) \wedge (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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trivial by inclusion

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$$F \wedge B_1 \vdash B_1 \qquad\qquad F \wedge \overline{B_1} \vdash B_1|_{\sigma_1}$$

trivial by inclusion holds by RUP

# Look Ma! No interference!

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$F \land B_1 \vdash 1.\, C$ $F \land \overline{B_1} \vdash \langle \sigma_1 \rangle.\, C$

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trivial by inclusion holds by RUP (because of symmetry)

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symmetry: $\sigma_1 = \{x_1 \leftrightarrow y_1\}$

program: $\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle)$

$$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, B_1$$

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**Step 3** derive $u_1$ from $u_1 \vee F_1$

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$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

$$B_1 \wedge (1.u_1) \vdash u_1 \qquad\qquad \overline{B_1} \wedge (\langle \sigma_1 \rangle.u_1) \vdash u_1$$

# Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

**Step 1** introduce the symmetry breaker $B_1 = \overline{x_1} \vee y_1$ in $u_1 \vee F_1$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1$

**Step 2** derive $u_1$ from $(u_1 \vee F_1) \wedge B_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

**Step 3** derive $u_1$ from $u_1 \vee F_1$

$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

$B_1 \wedge (1.u_1) \vdash u_1$ $\qquad\qquad$ $\overline{B_1} \wedge (\langle \sigma_1 \rangle.u_1) \vdash u_1$

$B_1 \wedge u_1 \vdash u_1$

trivial by inclusion

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| | |
|---|---|
| $B_1 \wedge (1. u_1) \vdash u_1$ | $\overline{B_1} \wedge (\langle \sigma_1 \rangle. u_1) \vdash u_1$ |
| $B_1 \wedge u_1 \vdash u_1$ | $B_1 \wedge u_1 \vdash u_1\big\vert_{\sigma_1}$ |
| trivial by inclusion | holds by RUP (because of cleanliness) |

[Fazekas, Biere, Scholl '19]

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**Step 4** repeat for $u_2$ and $u_3$, then resolve

# Conclusion

You don’t need interference for RAT (or PR, or SR, or WSR)

You don’t even need an accumulated formula

You can get new reasoning tools if you stop thinking in terms of redundancy notions

Proof logging and checking is not necessarily more complex