Short proofs without interference

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Trying to merge proofs…

all unsat, over the same variables

F_1 F_2 F_3

Trying to merge proofs...

$u_1 \vee F_1$ $\qquad$ $u_2 \vee F_2$ $\qquad$ $u_3 \vee F_3$

Trying to merge proofs...

still unsat!

$$u_1 \lor F_1 \qquad u_2 \lor F_2 \qquad u_3 \lor F_3 \qquad \overline{u_1} \lor \overline{u_2} \lor \overline{u_3}$$

Trying to merge proofs...

$u_1 \vee F_1$ $\quad u_2 \vee F_2$ $\quad u_3 \vee F_3$ $\quad \overline{u_1} \vee \overline{u_2} \vee \overline{u_3}$

Very Smartest Distributed Solver

$u_1 \vee F_1$ $\quad u_2 \vee F_2$ $\quad u_3 \vee F_3$

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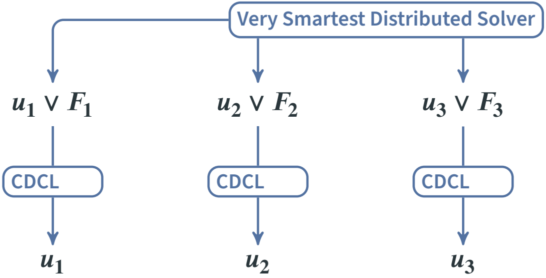

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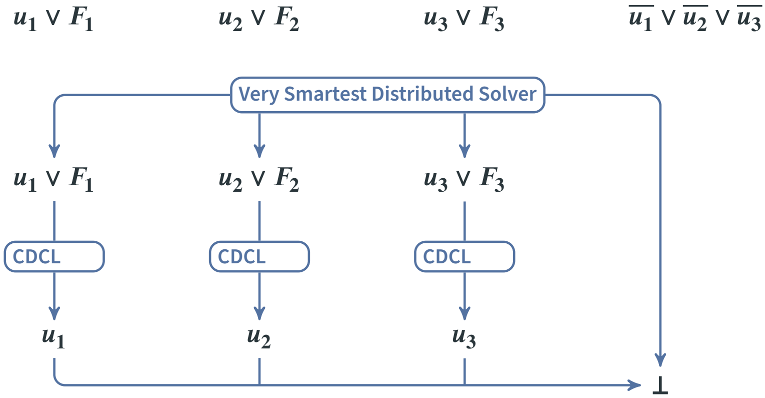

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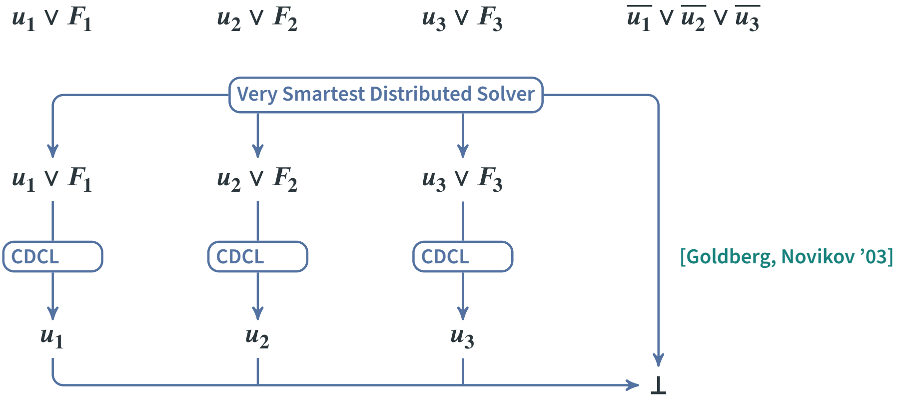

$$\pi_1 : u_1 \lor F_1 \vdash u_1$$
$$\pi_2 : u_2 \lor F_2 \vdash u_2$$
$$\pi_3 : u_3 \lor F_3 \vdash u_3$$

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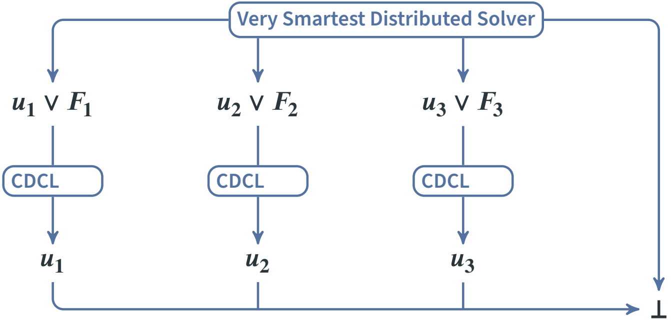

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$\pi_3 : u_3 \lor F_3 \vdash u_3$

$\pi : u_1 \land u_2 \land u_3 \land (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3}) \vdash \bot$

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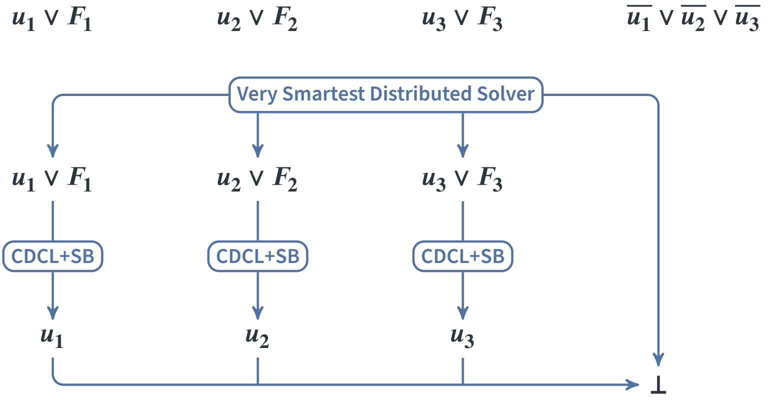

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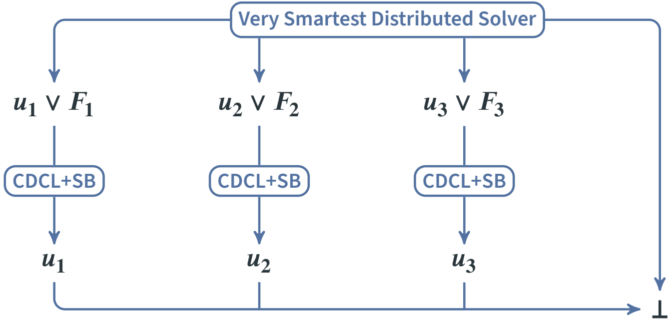
$\sigma_1 = \{x_1 \leftrightarrow y_1\}$ $\sigma_2 = \{x_2 \leftrightarrow y_2\}$ $\sigma_3 = \{x_3 \leftrightarrow y_3\}$

$u_1 \vee F_1$ $u_2 \vee F_2$ $u_3 \vee F_3$ $\overline{u_1} \vee \overline{u_2} \vee \overline{u_3}$



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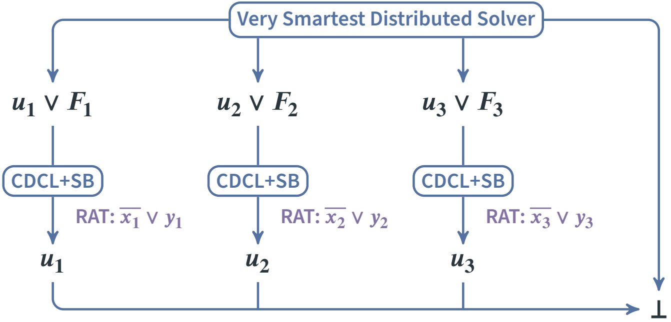

[Heule, Hunt, Wezler ’15]

$\pi_1 : u_1 \vee F_1 \vdash u_1$

$\pi_2 : u_2 \vee F_2 \vdash u_2$ $\qquad$ $\pi : u_1 \wedge u_2 \wedge u_3 \wedge (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3}) \vdash \perp$

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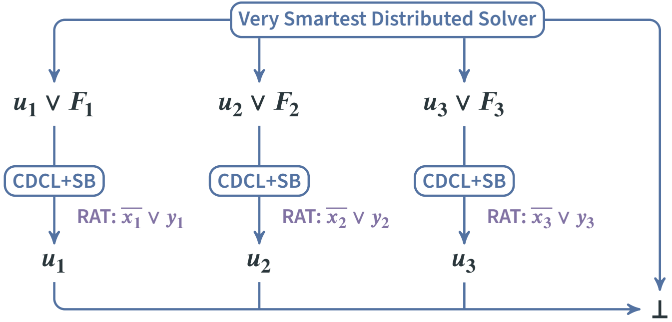

[Järvisalo, Heule, Biere ’12]

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$\pi : u_1 \wedge u$ … NOPE … $\vee \overline{u_3}) \vdash \perp$

… and failing because of interference

What are DRAT proofs really doing?

$\pi : F \vdash G$ proves that for each $I \models F$ we have $\mathrm{mut}(I) \models G$

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$$\pi_2 : u_2 \lor F_2 \vdash (u_2 \lor F_2) \land (\overline{x_2} \lor y_2) \vdash u_2 \qquad I \models u_2 \lor F_2 \Rightarrow \mathrm{mut}_2(I) \models u_2$$
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what we need is this!

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Interference-based proof systems force a single concurrent mut prefix…

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Interference-based proof systems force a single concurrent mut prefix...

... because of the accumulated formula

Kripke semantics for redundance rules

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Dynamic constraints a static constraint holds executing a program

$$I \vDash \varepsilon.C \quad \text{iff} \quad J \vDash C \text{ for all } J \text{ such that } I \otimes J \vDash \varepsilon$$

What’s in the box? Programs!

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Composition $I \otimes J \vDash \varepsilon_1\, \varepsilon_2 \quad \text{iff} \quad \exists K,\ I \otimes K \vDash \varepsilon_1 \text{ and } K \otimes J \vDash \varepsilon_2$

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Choice $I \otimes J \vDash \varepsilon_1 \sqcup \varepsilon_2 \quad \text{iff} \quad I \otimes J \vDash \varepsilon_1 \text{ or } I \otimes J \vDash \varepsilon_2$

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Test $I \otimes J \vDash T? \quad \text{iff} \quad I = J \text{ and } I \vDash T$

What's in the box? Programs!

Noop $I \otimes J \models 1$ iff $I = J$

Composition $I \otimes J \models \varepsilon_1\, \varepsilon_2$ iff $\exists K,\ I \otimes K \models \varepsilon_1$ and $K \otimes J \models \varepsilon_2$

Assignment $I \otimes J \models \langle \sigma \rangle$ iff $J = I \circ \sigma$

Choice $I \otimes J \models \varepsilon_1 \sqcup \varepsilon_2$ iff $I \otimes J \models \varepsilon_1$ or $I \otimes J \models \varepsilon_2$

Test $I \otimes J \models T?$ iff $I = J$ and $I \models T$

Branch $I \otimes J \models \nabla(T : \varepsilon_1 \parallel \varepsilon_0)$ iff $I \otimes J \models (T?\, \varepsilon_1) \sqcup (\overline{T}?\, \varepsilon_0)$

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Test $I \otimes J \vDash T? \quad \text{iff} \quad I = J \text{ and } I \vDash T$

Branch $I \otimes J \vDash \nabla(T : \varepsilon_1 \parallel \varepsilon_0) \quad \text{iff} \quad I \otimes J \vDash (T?\, \varepsilon_1) \sqcup (\overline{T}?\, \varepsilon_0)$

Not even a new thing! [Fischer, Ladner '79] [Balbiani, Herzig, Troquard '13]

Propositional dynamic logic (PDL) defines modalities for each program

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Choice $I \otimes J \vDash \varepsilon_1 \sqcup \varepsilon_2$ iff $I \otimes J \vDash \varepsilon_1$ or $I \otimes J \vDash \varepsilon_2$

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Propositional dynamic logic (PDL) defines modalities for each program

Necessitation law (a.k.a. I can apply programs to a proof)

If $F \vDash G$ holds, then $\varepsilon.F \vDash \varepsilon.G$ holds too

RUP-like inferences in dynamic logic

Unit propagation (BCP) a *blazingly* fast sound but incomplete algorithm for inconsistency in CNF formulas [Zhang, Madigan, Moskewicz, Malik '01]

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$$F \wedge \overline{C} \models \bot \qquad \text{iff} \qquad F \models C$$

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I don’t have diamonds!

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Can we have this for dynamic formulas? yes, but not built on consistency.

Instead we reduce a dynamic implication check to (several) RUP checks

$F \wedge \overline{\varepsilon.C} \models \bot$ iff $F \models \varepsilon.C$ I don’t have diamonds!

Look Ma! No interference!

$$F = (u_1 \vee F_1) \wedge (u_2 \vee F_2) \wedge (u_3 \vee F_3) \wedge (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \lor y_1$ in $u_1 \lor F_1$

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symmetry: $\sigma_1 = \{x_1 \leftrightarrow y_1\}$

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \vee y_1$ in $u_1 \vee F_1$

symmetry: $\sigma_1 = \{x_1 \leftrightarrow y_1\}$ program: $\nabla(B_1 : 1 \parallel \langle\sigma_1\rangle)$

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$$F \wedge B_1 \vdash 1.\, B_1 \qquad\qquad F \wedge \overline{B_1} \vdash \langle \sigma_1 \rangle.\, B_1$$

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$$F \land B_1 \vdash B_1$$

trivial by inclusion

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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$$F \wedge B_1 \vdash B_1 \qquad\qquad F \wedge \overline{B_1} \vdash B_1\big|_{\sigma_1}$$

trivial by inclusion holds by RUP

Look Ma! No interference!

$$F = (u_1 \vee F_1) \wedge (u_2 \vee F_2) \wedge (u_3 \vee F_3) \wedge (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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$$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, B_1$$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, C$ for $C \in u_1 \vee F_1$

Look Ma! No interference!

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Look Ma! No interference!

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$$F \vdash \nabla(B_1 : 1 \parallel \langle\sigma_1\rangle).\, C \quad \text{for } C \in u_1 \vee F_1$$

$$F \wedge B_1 \vdash 1.\, C \qquad\qquad F \wedge \overline{B_1} \vdash \langle\sigma_1\rangle.\, C$$

$$F \wedge B_1 \vdash C$$

trivial by inclusion

Look Ma! No interference!

$$F \quad = \quad (u_1 \lor F_1) \quad \land \quad (u_2 \lor F_2) \quad \land \quad (u_3 \lor F_3) \quad \land \quad (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3})$$

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$$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, B_1$$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, C$ for $C \in u_1 \lor F_1$

$$F \land B_1 \vdash 1.\, C \qquad F \land \overline{B_1} \vdash \langle \sigma_1 \rangle.\, C$$

$$F \land B_1 \vdash C \qquad F \land \overline{B_1} \vdash C|_{\sigma_1}$$

trivial by inclusion holds by RUP (because of symmetry)

Look Ma! No interference!

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Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \lor y_1$ in $u_1 \lor F_1$

$F \vdash \nabla(B_1 : 1 \parallel \langle\sigma_1\rangle).(u_1 \lor F_1) \land B_1$

symmetry: $\sigma_1 = \{x_1 \leftrightarrow y_1\}$ program: $\nabla(B_1 : 1 \parallel \langle\sigma_1\rangle)$

$$F \vdash \nabla(B_1 : 1 \parallel \langle\sigma_1\rangle).B_1$$

$F \vdash \nabla(B_1 : 1 \parallel \langle\sigma_1\rangle).C$ for $C \in u_1 \lor F_1$

$F \land B_1 \vdash 1.C$ $F \land \overline{B_1} \vdash \langle\sigma_1\rangle.C$

$F \land B_1 \vdash C$ $F \land \overline{B_1} \vdash C|_{\sigma_1}$

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Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \vee y_1$ in $u_1 \vee F_1$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1$

Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

Look Ma! No interference!

$$F \quad = \quad (u_1 \lor F_1) \quad \land \quad (u_2 \lor F_2) \quad \land \quad (u_3 \lor F_3) \quad \land \quad (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3})$$

Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \lor y_1$ in $u_1 \lor F_1$

$F \vdash \nabla(B_1 \,:\, 1 \,\|\, \langle \sigma_1 \rangle).\, (u_1 \lor F_1) \land B_1$

Step 2 derive u_1 from $(u_1 \lor F_1) \land B_1$

$(u_1 \lor F_1) \land B_1 \vdash u_1$ by a bunch of RUP steps

Look Ma! No interference!

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Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

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we haven't derived $(u_1 \vee F_1) \wedge B_1$ though...

Look Ma! No interference!

$$F \quad = \quad (u_1 \lor F_1) \quad \land \quad (u_2 \lor F_2) \quad \land \quad (u_3 \lor F_3) \quad \land \quad (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3})$$

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$F \vdash \nabla(B_1 \,:\, 1 \parallel \langle \sigma_1 \rangle).\, (u_1 \lor F_1) \land B_1$

Step 2 derive u_1 from $(u_1 \lor F_1) \land B_1$

$(u_1 \lor F_1) \land B_1 \vdash u_1$ by a bunch of RUP steps

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Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \vee y_1$ in $u_1 \vee F_1$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\,(u_1 \vee F_1) \wedge B_1$

Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

$(u_1 \vee F_1) \wedge B_1 \vdash u_1$ by a bunch of RUP steps

we haven't derived $(u_1 \vee F_1) \wedge B_1$ though...

but we have necessitation!

$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\,(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$$

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

$\nabla(B_1 \,:\, 1 \parallel \langle \sigma_1 \rangle).\,(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 \,:\, 1 \parallel \langle \sigma_1 \rangle).u_1$

Step 3 derive u_1 from $u_1 \vee F_1$

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

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$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

Look Ma! No interference!

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$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\,(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

Step 3 derive u_1 from $u_1 \vee F_1$

$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

$$B_1 \wedge (1.\,u_1) \vdash u_1 \qquad\qquad \overline{B_1} \wedge (\langle \sigma_1 \rangle.\,u_1) \vdash u_1$$

Look Ma! No interference!

$$F \quad = \quad (u_1 \lor F_1) \quad \land \quad (u_2 \lor F_2) \quad \land \quad (u_3 \lor F_3) \quad \land \quad (\overline{u_1} \lor \overline{u_2} \lor \overline{u_3})$$

Step 1 introduce the symmetry breaker $B_1 = \overline{x_1} \lor y_1$ in $u_1 \lor F_1$

$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, (u_1 \lor F_1) \land B_1$

Step 2 derive u_1 from $(u_1 \lor F_1) \land B_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).\, (u_1 \lor F_1) \land B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

Step 3 derive u_1 from $u_1 \lor F_1$

$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

$B_1 \land (1.\, u_1) \vdash u_1$

$\overline{B_1} \land (\langle \sigma_1 \rangle.\, u_1) \vdash u_1$

$B_1 \land u_1 \vdash u_1$

trivial by inclusion

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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$F \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1$

Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

Step 3 derive u_1 from $u_1 \vee F_1$

$$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$$

$B_1 \wedge (1.u_1) \vdash u_1$	$\overline{B_1} \wedge (\langle \sigma_1 \rangle.u_1) \vdash u_1$
$B_1 \wedge u_1 \vdash u_1$	$B_1 \wedge u_1 \vdash u_1\big\vert_{\sigma_1}$
trivial by inclusion	holds by RUP (because of cleanliness)

[Fazekas, Biere, Scholl ’19]

Look Ma! No interference!

$$F \quad = \quad (u_1 \vee F_1) \quad \wedge \quad (u_2 \vee F_2) \quad \wedge \quad (u_3 \vee F_3) \quad \wedge \quad (\overline{u_1} \vee \overline{u_2} \vee \overline{u_3})$$

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Step 2 derive u_1 from $(u_1 \vee F_1) \wedge B_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).(u_1 \vee F_1) \wedge B_1 \vdash \nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1$

Step 3 derive u_1 from $u_1 \vee F_1$

$\nabla(B_1 : 1 \parallel \langle \sigma_1 \rangle).u_1 \vdash u_1$

Step 4 repeat for u_2 and u_3, then resolve

Conclusion

You don’t need interference for RAT (or PR, or SR, or WSR)

You don’t even need an accumulated formula

You can get new reasoning tools if you stop thinking in terms of redundancy notions

Proof logging and checking is not necessarily more complex