TECHNISCHE UNIVERSITÄT WIEN

logics Logical Methods in Computer Science

Microsoft

# Frying the Egg, Roasting the Chicken
# Unit Deletions in DRAT Proofs


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TU Wien



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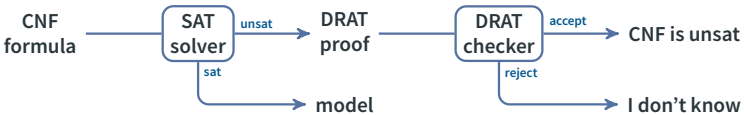
# SAT solving and DRAT proofs



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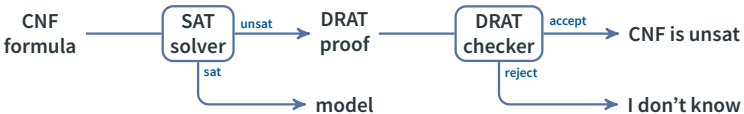


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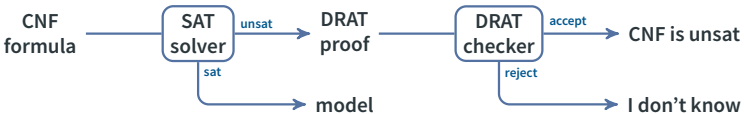


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- RAT introduction is non-monotonic
  *Clause deletion may enable new RAT inferences*

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**Discrepancies exist**

- Short correct proofs in one flavor, incorrect in the other [Rebola, Biere '18]
- About 60% of proofs generated in SAT competitions [Rebola, Cruz '18]

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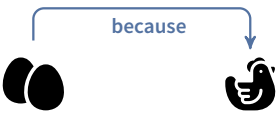


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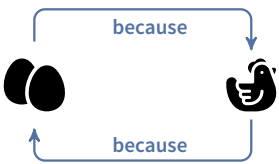


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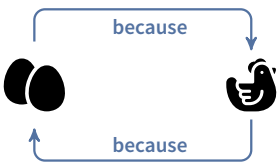


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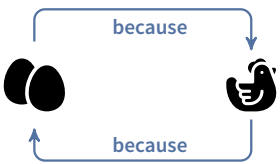


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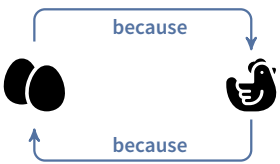


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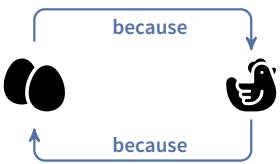


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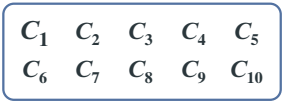


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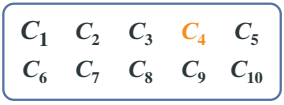


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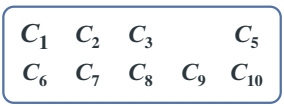


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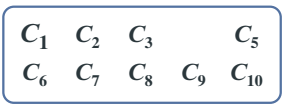


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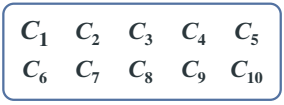


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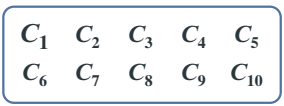



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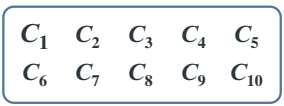



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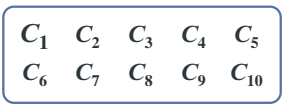



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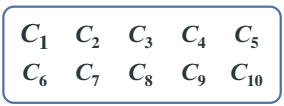



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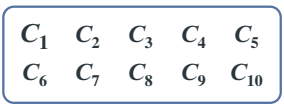



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*inprocessing techniques may still need to delete unit clauses* [Rebola, Biere '18]

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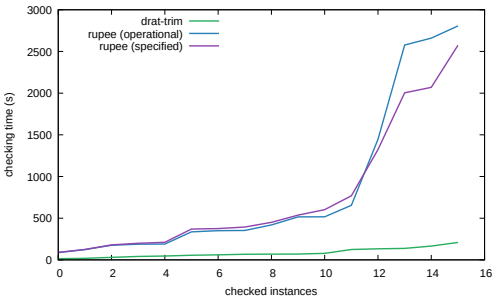

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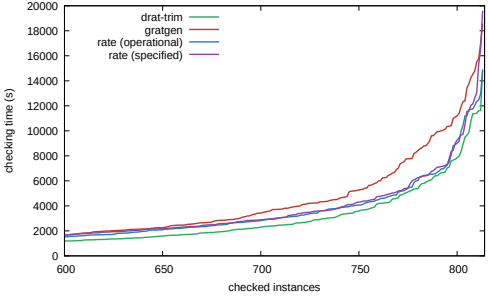

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**SICK certificates** give an UP-model for the failed inference

*All rejected proofs are correctly so, in a certified way*

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*certification through three-valued semantics*