Horse shoe prior example

This example reproducts some results of https://ariddell.org/horseshoe-prior-with-stan.html

- **Define the working directory and load CmdStan.m**

In[19]:=
```
(* Linux *)
SetDirectory["~/GitHub/MathematicaStan/Examples/HorseShoePrior"]

(* Windows *)
(* SetDirectory["C:\\Users\\USER_NAME\\Documents\\Mathematica\\STAN\\Examples\\HorseShoePrior"]; *)

Needs["CmdStan`"]
```

Out[19]=
```
/IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior
```

Generate the Horse Shoe Prior Stan code and compile it

In[3]:=
```
stanCode="data {
  int<lower=0> n;
  int<lower=0> p;
  matrix[n,p] X;
  vector[n] y;
}
parameters {
  vector[p] beta;
  vector<lower=0>[p] lambda;
  real<lower=0> tau;
  real<lower=0> sigma;
}
model {
  lambda ~ cauchy(0, 1);
  tau ~ cauchy(0, 1);
  for (i in 1:p)
    beta[i] ~ normal(0, lambda[i] * tau);
  y ~ normal(X * beta, sigma);
}";
StanCodeExport["horseShoePrior",stanCode]

StanCompile["horseShoePrior"]
```

Out[4]= horseShoePrior.stan

Out[5]=

```
--- Translating Stan model to C++ code ---
bin/stanc /IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.stan
  --o=/IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.hpp
Model name=horseShoePrior_model
Input file=/IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.stan
Output file=/IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.hpp

--- Linking C++ model ---
g++ -I src -I stan/src -isystem stan/lib/stan_math/ -isystem stan/lib/stan_math/lib/eigen_3.2.8
  -isystem stan/lib/stan_math/lib/boost_1.60.0 -isystem stan/lib/stan_math/lib/cvodes_2.8.2/include
  -Wall -DEIGEN_NO_DEBUG  -DBOOST_RESULT_OF_USE_TR1 -DBOOST_NO_DECLTYPE -DBOOST_DISABLE_ASSERTS
  -DFUSION_MAX_VECTOR_SIZE=12 -DNO_FPRINTF_OUTPUT -pipe   -lpthread  -O3 -o
  /IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior
  src/cmdstan/main.cpp -include
  /IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.hpp
  stan/lib/stan_math/lib/cvodes_2.8.2/lib/libsundials_nvecserial.a
  stan/lib/stan_math/lib/cvodes_2.8.2/lib/libsundials_cvodes.a
```

■ Load data and save them (RDump file)

In[21]:=
```
yTest = Import["./y-test.dat", "List"];
yTrain = Import["./y-train.dat", "List"];
XTest = Import["./X-test.dat", "Table"];
XTrain = Import["./X-train.dat", "Table"];

(* betaLabel is only used for plot legends *)
betaLabel =
  StringSplit["age sex bmi map tc ldl hdl tch ltg glu age^2 bmi^2 map^2 tc^2 ldl^2 hdl^2 tch^2
     ltg^2 glu^2 age:sex age:bmi age:map age:tc age:ldl age:hdl age:tch age:ltg
     age:glu sex:bmi sex:map sex:tc sex:ldl sex:hdl sex:tch sex:ltg sex:glu
     bmi:map bmi:tc bmi:ldl bmi:hdl bmi:tch bmi:ltg bmi:glu map:tc map:ldl map:hdl
     map:tch map:ltg map:glu tc:ldl tc:hdl tc:tch tc:ltg tc:glu ldl:hdl ldl:tch
     ldl:ltg ldl:glu hdl:tch hdl:ltg hdl:glu tch:ltg tch:glu ltg:glu", " "];

(* Export data *)
RDumpExport["horseShoePrior",
  {{"n", Dimensions[XTrain][[1]]}, {"p", Dimensions[XTrain][[2]]}, {"X", XTrain}, {"y", yTrain}}];

(* Here we just perform an Ordinary Least Squares to get the residue value *)
betaOLS = LeastSquares[XTrain, yTrain];
Norm[XTest.betaOLS - yTest]^2 / Length[yTest]
Norm[Mean[yTrain] - yTest]^2 / Length[yTest]
```

Out[28]= 0.670749

Out[29]= 0.965737

■ Run Stan and get result

In[30]:=
```
(* use the same seed as the blog post *)
StanSetOptionSample["random seed",5]
```

Out[30]= {{random seed, 5}}

In[31]:=
```
(* Run stan *)
StanRunSample["horseShoePrior"]
```

Out[31]=
```
method = sample (Default)
  sample
    num_samples = 1000 (Default)
    num_warmup = 1000 (Default)
    save_warmup = 0 (Default)
    thin = 1 (Default)
    adapt
      engaged = 1 (Default)
      gamma = 0.050000000000000003 (Default)
      delta = 0.80000000000000004 (Default)
      kappa = 0.75 (Default)
      t0 = 10 (Default)
      init_buffer = 75 (Default)
      term_buffer = 50 (Default)
      window = 25 (Default)
    algorithm = hmc (Default)
      hmc
        engine = nuts (Default)
          nuts
            max_depth = 10 (Default)
```

```
        metric = diag_e (Default)
        stepsize = 1 (Default)
        stepsize_jitter = 0 (Default)
id = 0 (Default)
data
  file = /IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/horseShoePrior.data.R
init = 2 (Default)
random
  seed = 5
output
  file = /IS006139/home/pix/GitHub/MathematicaStan/Examples/HorseShoePrior/output.csv
  diagnostic_file =  (Default)
  refresh = 100 (Default)


Gradient evaluation took 0.000208 seconds
1000 transitions using 10 leapfrog steps per transition would take 2.08 seconds.
Adjust your expectations accordingly!


Iteration:    1 / 2000 [  0%]  (Warmup)
Iteration:  100 / 2000 [  5%]  (Warmup)
Iteration:  200 / 2000 [ 10%]  (Warmup)
Iteration:  300 / 2000 [ 15%]  (Warmup)
Iteration:  400 / 2000 [ 20%]  (Warmup)
Iteration:  500 / 2000 [ 25%]  (Warmup)
Iteration:  600 / 2000 [ 30%]  (Warmup)
Iteration:  700 / 2000 [ 35%]  (Warmup)
Iteration:  800 / 2000 [ 40%]  (Warmup)
Iteration:  900 / 2000 [ 45%]  (Warmup)
Iteration: 1000 / 2000 [ 50%]  (Warmup)
Iteration: 1001 / 2000 [ 50%]  (Sampling)
Iteration: 1100 / 2000 [ 55%]  (Sampling)
Iteration: 1200 / 2000 [ 60%]  (Sampling)
Iteration: 1300 / 2000 [ 65%]  (Sampling)
Iteration: 1400 / 2000 [ 70%]  (Sampling)
Iteration: 1500 / 2000 [ 75%]  (Sampling)
Iteration: 1600 / 2000 [ 80%]  (Sampling)
Iteration: 1700 / 2000 [ 85%]  (Sampling)
Iteration: 1800 / 2000 [ 90%]  (Sampling)
Iteration: 1900 / 2000 [ 95%]  (Sampling)
Iteration: 2000 / 2000 [100%]  (Sampling)

 Elapsed Time: 70.0086 seconds (Warm-up)
               32.0924 seconds (Sampling)
               102.101 seconds (Total)
```

Use the results

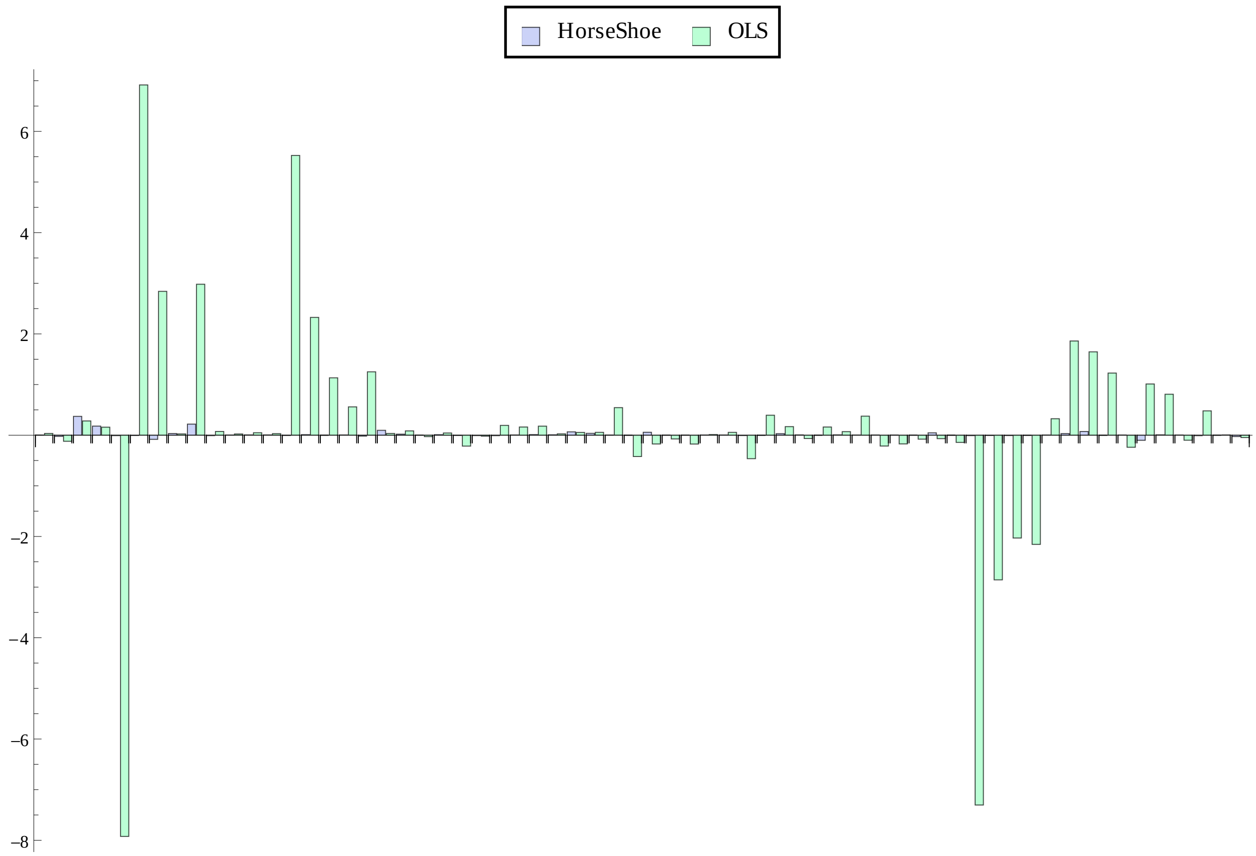
In[32]:=
```
output=StanImport["output.csv"];
```

Compute beta mean and compare it to OLS solution

In[33]:=
```
beta = Mean[StanVariableColumn["beta", output]];
```

In[34]:=
```
BarChart[Transpose[{beta,betaOLS}],ChartLegends→Placed[{"HorseShoe","OLS"},Top]]
```

Out[34]=



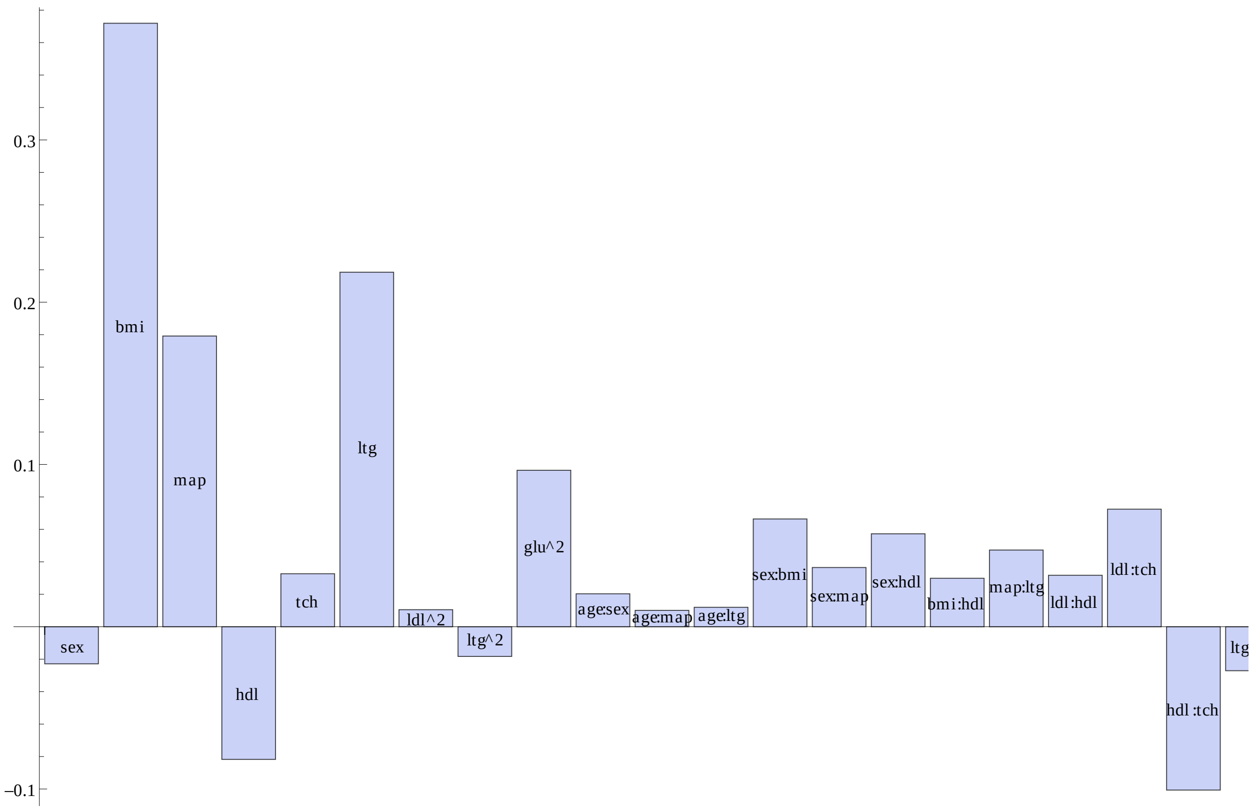
- **Now select all beta such that |beta|>0.01**

In[35]:=
```
selectedBeta=Map[(Abs[#]>0.01)&,beta];
Print["The ",Count[selectedBeta,True]," variables are:\n",
 (betaLabel[[selectedBeta=Flatten[Position[selectedBeta,True]]]])];

selectedBetaLabel=betaLabel[[selectedBeta]];
selectedBeta=beta[[selectedBeta]];

BarChart[selectedBeta,ChartLabels→Placed[selectedBetaLabel,Center]]
```

```
The 21 variables are:
{sex, bmi, map, hdl, tch, ltg, ldl^2, ltg^2, glu^2, age:sex, age:map, age:ltg,
  sex:bmi, sex:map, sex:hdl, bmi:hdl, map:ltg, ldl:hdl, ldl:tch, hdl:tch, ltg:glu}
```

Out[39]=


- **It is interesting to notice that the pruned beta has a better generalization than the OLS solution:**

In[40]:=
```
prunedBeta=Map[If[Abs[#]>0.01,#,0]&,beta];
Norm[XTest.betaOLS-yTest]^2/Length[yTest]
Norm[XTest.prunedBeta-yTest]^2/Length[yTest]
```

Out[41]= 0.670749

Out[42]= 0.50374