Problem Set 4

1. Call $L \subseteq \Sigma^+$ non counting if

$$\exists n_0 \forall n \geq n_0 \forall u, v, w \in \Sigma^* (uv^n w \in L \Leftrightarrow uv^{n+1} w \in L)$$

That is for all $n \geq n_0$, either all $uv^n w$ are in L, or none is.
A language L is counting iff it is not non counting.
   - Formulate the condition for a counting language
   - Is $L = (aa)^+$ counting or not?
   - Is $L = (ab)^+$ counting or not?

2. Write second order logic formulae to capture the following:
   (a) There is a path from node s to node t in the graph. The signature is $\tau = \{E\}$.
   (b) Every bounded non empty set has a least upper bound. The signature is $\tau = \{\leq\}$

3. Let Σ be a finite alphabet. The atomic formulae in MSO defined over Σ^* are $x = y, x < y, S(x, y), X(x)$ and $Q_a(x)$, $a \in \Sigma$. Consider the following logic called MSO_0 having atomic formulae of the following forms:

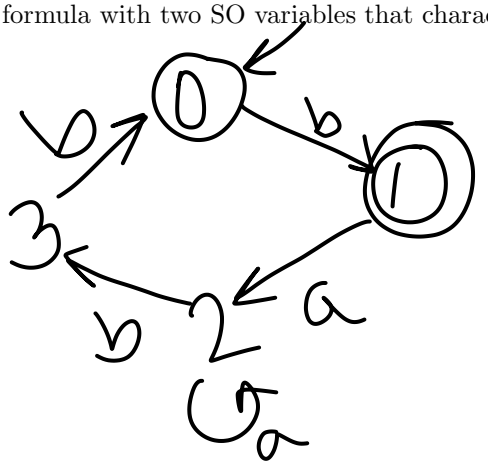
$$Sing(X), X \subseteq Y, X < Y, S(X, Y), Q_a(X)$$

where
   - $Sing(X)$ means that X is a SO variable of cardinality 1;
   - $X \subseteq Y$ means that every element of the SO variable X is contained in the SO variable Y;
   - $X < Y$ means that SO variables X, Y have cardinality 1, and that the element in Y is greater than the element in X;
   - $S(X, Y)$ means that SO variables X, Y have cardinality 1, and Y contains the successor of the element in X; and,
   - $Q_a(X)$ means that all positions in X are decorated by $a \in \Sigma$.

If φ is an atomic formula in MSO, then $\varphi \wedge \varphi, \neg\varphi, \varphi \vee \varphi$, $\forall x\ \varphi$ and $\forall X\ \varphi$ are formulae in MSO. Similarly, if φ is an atomic formula in MSO_0, then, $\varphi \wedge \varphi, \neg\varphi, \varphi \vee \varphi$ and $\forall X\ \varphi$ are formulae in MSO_0.
Compare the expressiveness of MSO and MSO_0.

4. For the formula $\exists x \forall y (x < y \rightarrow Q_a(y))$ give an equivalent MSO_0 formula.

5. Consider the following NFA $N = (\{0, 1, 2, 3\}, \{a, b\}, \Delta, \{0\}, \{1\})$ with $\Delta(0, b) = \{1\}$, $\Delta(1, a) = \{2\}$, $\Delta(2, a) = \{2\}$, $\Delta(2, b) = \{3\}$ and $\Delta(3, b) = \{0\}$. Write an MSO formula with two SO variables that characterizes $L(N)$.



$b(a^+ b^3)^*$