

## Problem Set 5

1. Let  $AP = \{p, q, r\}$ . Formulate the following as  $LT$  properties:
  - (a) Eventually false
  - (b)  $p$  occurs exactly twice, and  $q$  never occurs between two occurrences of  $p$
  - (c) If  $r$  occurs only finitely often, then  $p$  continuously occurs from some point
  - (d)  $r$  is true continuously upto somepoint, and at the next point, both  $p, q$  hold, and then  $q$  and  $r$  alternate infinitely often
  - (e) Infinitely often there are two consecutive occurrences of  $p$
  - (f) Between every consecutive occurrences of  $p$ , there is a  $q$ , and there is a prefix of  $r$ 's of even length
2. Let  $TS$  and  $TS'$  be two transition systems without terminal states on the same set of atomic propositions  $AP$ . Then show that  $Traces(TS) = Traces(TS')$  iff  $TS$  and  $TS'$  satisfy the same set of LT properties.
3. Consider a set of atomic propositions  $AP$ . Consider the following logic  $\mathcal{X}$  defined as follows:

$$\varphi ::= (a \in AP) \mid \varphi \wedge \varphi \mid \neg \varphi \mid \varphi \Delta \varphi$$

with semantics as follows:

Given a word  $w = A_0 A_1 \dots$  over  $2^{AP}$  and a position  $i \in \mathbb{N}$ , we define

- (a)  $w, i \models a$  iff  $a \in A_i$  for  $a \in AP$
- (b)  $w, i \models \varphi_1 \wedge \varphi_2$  iff  $w, i \models \varphi_1$  and  $w, i \models \varphi_2$
- (c)  $w, i \models \neg \varphi$  iff  $w, i \not\models \varphi$
- (d)  $w, i \models \varphi \Delta \psi$  iff  $\exists j > i, w, j \models \psi$  and  $\forall i < k < j, w, k \models \varphi$ .

Comment on the equivalence of LTL and  $\mathcal{X}$ .

4. Exercises 5.1, 5.2, 5.5, 5.6 and 5.7 from Baier-Katoen.