CS754 Assignment 1 Report

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Introduction

Welcome to our report on CS215 Assignment 3. We have tried to make this report comprehensive and self-contained. We hope reading this would give you a proper flowing description of our work, methods used and the results obtained. Feel free to keep our code scripts alongside to know the exact implementation of our tasks.

We have referred to some sites on the web for finding the MATLAB implementations (generic documentation pages) and the same has been added in the references section.

In many places, to better give context to the place from which the questions could have arisen, some theoretical discussions have been engaged in.

Hope you enjoy reading the report. Here we go!

1 Problem 1

We have been given the following problem-

Let $\boldsymbol{\theta^\star}$: $\min\|\boldsymbol{\theta}\|_1$ such that $\|\boldsymbol{y} - \boldsymbol{\Phi\Psi\theta}\|_2 \leq \varepsilon$, where $\boldsymbol{x} = \boldsymbol{\Psi\theta}$ and $\boldsymbol{y} = \boldsymbol{\Phi x} + \boldsymbol{\eta}$. ε is an upper bound on the magnitude of the noise vector $\boldsymbol{\eta}$.

Also, Theorem 3 states-
If $\boldsymbol{\Phi}$ obeys the restricted isometry property with isometry constant $\delta_{2s} < \sqrt{2} - 1$, then we have $\|\boldsymbol{\theta} - \boldsymbol{\theta^\star}\|_2 \leq C_1 s^{-1/2}\|\boldsymbol{\theta} - \boldsymbol{\theta_s}\|_1 + C_2\varepsilon$ where C_1 and C_2 are functions of only δ_{2s} and where $\forall i \in \mathcal{S}, \boldsymbol{\theta_s}_i = \theta_i; \forall i \notin \mathcal{S}, \boldsymbol{\theta_s}_i = 0$.

1.1 Trend of Error Bound with s

This is not a discrepancy. In reality, the error bound becomes worse as the value of s increases. The point is, we are only focusing on the effect of $s^{-1/2}$ and $||\boldsymbol{\theta} - \boldsymbol{\theta_s}||_1$. We must also see the change in C_1 and C_2. These constants increase as the value of δ_{2s} changes. Thus, as the value of s increases, we observe that the bound on $delta_{2s}$ also increases which leads to an increase in the value of C_1 and C_2. Thus, we cannot claim that the error bound improves as the sparsity measure, s increases in value.

2 Problem 2

3 Problem 3

4 Problem 4

5 Problem 5

6 Problem 6