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Solution for problem J439

We claim that $(\mathbf{x},\mathbf{y},\mathbf{z}) = (\mathbf{1},\mathbf{1},\mathbf{1}),(-\mathbf{1},-\mathbf{1},-\mathbf{1})$ are the only solutions.

We subtract the third equation from sum of first two equations.

$$\begin{aligned}(2x^2-3xy+2y^2)+(y^2-3xy+4y^2)-(z^2+3zx-x^2) &= 1+2-3\\ &= 0\\ \Rightarrow 3(x^2+y^2+z^2-xy-yz-zx) &= 0\\ \Rightarrow (x-y)^2+(y-z)^2+(z-x)^2 &= 0\\ \therefore (x-y)=(y-z)=(z-x) &= 0\\ \Rightarrow x=y=z\end{aligned}$$

Putting $x=y=z$ in all the three given equation we obtain $x^2=y^2=z^2=1$
$\therefore (\mathbf{x},\mathbf{y},\mathbf{z}) = (\mathbf{1},\mathbf{1},\mathbf{1}),(-\mathbf{1},-\mathbf{1},-\mathbf{1})$ **are the only solutions.**
Plugging these solutions in the three given equations we verify that they work.