

$$a^2 + b^2 + 2 = a^2 + b^2 + 1 + 1 = a^2 + 1 + b^2 + 1 \geq \frac{(a+1)^2}{2} + \frac{(b+1)^2}{2} \geq 2\sqrt{\frac{(a+1)^2(b+1)^2}{4}} = (a+1)(b+1)$$

whereby the inequalities are true by direct application of the AM-GM inequality.

Therefore we get that:

$$(a^2 + b^2 + 2)(b^2 + c^2 + 2)(c^2 + a^2 + 2) \geq (a+1)(b+1)(b+1)(c+1)(c+1)(a+1) = (a+1)^2(b+1)^2(c+1)^2$$

Since $a^2 + 1 \geq \frac{(a+1)^2}{2}$ is true for all $a \geq -1$, we conclude that the inequality holds for all real $a, b, c \geq -1$