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Solution for Problem U374

Without loss of generality,we can assume that the two equal roots are a and b, thus $a = b$
Therefore the Viete's relations give that:-

$$a + b + c = -3p \tag{1}$$

As $a = b$,

$$2b + c = -3p \tag{2}$$

$$ab + bc + ca = 3q \tag{3}$$

As $a = b$,

$$b^2 + 2bc = 3q \tag{4}$$

$$abc = -3pq \tag{5}$$

As $a = b$,

$$b^2c = -3pq \tag{6}$$

Since $-3pq = \frac{(-3p)(3q)}{3} = \frac{(2b+c)(b^2+2bc)}{3} \Rightarrow 3b^2c = 2b^3 + b^2c + 4b^2c + 2bc^2$ from equation (6), (2) and (4)
$\Rightarrow 0 = 2b^3 + 2b^2c + 2bc^2$

$$b^3 + b^2c + bc^2 = 0 \tag{7}$$

But,$a^2b + b^2c + c^2a = b^3 + b^2c + bc^2$ as $a = b$

Thus, $a^2b + b^2c + c^2a = b^3 + b^2c + bc^2 = 0$.(from equation (7))