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Solution for Problem J371

After manipulating the expression $\binom{n+3}{2}+6\binom{n+4}{4}+90\binom{n+5}{6}$ we obtain

$$\binom{n+3}{2}+6\binom{n+4}{4}+90\binom{n+5}{6}=\frac{n^2+5n+6}{2}(1+\frac{(n^2+5n+4)(n^2+5n+2)}{4})$$

let $a=(n^2+5n+2)$.

$$\binom{n+3}{2}+6\binom{n+4}{4}+90\binom{n+5}{6}=\frac{a+4}{2}(4+\frac{(a)(a+2)}{4})$$

After manipulation we obtain
$$\binom{n+3}{2}+6\binom{n+4}{4}+90\binom{n+5}{6}=\frac{a+4}{2}(4+\frac{(a)(a+2)}{4})=(\frac{a}{2}+1)^3+(1)^3$$

For even n, $a=n(n+5)+2$ is even. For odd n, $(n+5)$ is even making a even.Thus for all integer n,$\frac{a}{2}$ is an integer.
So $\binom{n+3}{2}+6\binom{n+4}{4}+90\binom{n+5}{6}=(\frac{a}{2}+1)^3+(1)^3$ is sum of two perfect cubes.