Arpon Basu, School: AECS-4 , Mumbai-400094
Solution for problem U401

We apply Lagrange Multipliers.Thus our function is to be optimized is

$$f(x) = x^x y^y z^z$$

and constraint $g(x, y, z)$ is $x + y + z - 3$.

$$\mathcal{L}(x, y, z, \lambda) = f(x) - \lambda g(x)$$

$$\Rightarrow \nabla_{x,y,z,\lambda}\mathcal{L} = 0$$
$$\Rightarrow \lambda = x^x y^y z^z(1 + \ln(x))$$

(Cyclic equations hold for y, z)

$$\therefore x^x y^y z^z(1 + \ln(x)) = x^x y^y z^z(1 + \ln(y)) = x^x y^y z^z(1 + \ln(z))$$

$$\Rightarrow \ln(x) = \ln(y) = \ln(z)$$
$$\Rightarrow x = y = z$$
$$\Rightarrow x = y = z = 1$$

$$\therefore f(1, 1, 1) = 1$$

Putting any other (x, y, z) such that $x + y + z = 3$ make f greater than 1, confirming that 1 is the global minima of f on $x + y + z = 3$.

$$\therefore f \geq 1$$
$$\Rightarrow x^x y^y z^z \geq 1$$

Hence Proved.