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Solution for Problem U373

We define $a_k = 1 + \frac{1}{1+2+3+...+k+(k+1)} = 1 + \frac{2}{k+1} - \frac{2}{k+2}$ ,where $1 \leq k \leq n-1$.

Applying the AM-GM inequality to the numbers $a_1$,$a_2$,$a_3$,...,$a_{n-1}$,we get that:-
$(\frac{a_1+a_2+a_3+...+a_{n-1}}{n-1})^{n-1} > a_1a_2a_3...a_{n-1}$ (Note that a strict inequality holds because $a_1 \neq a_2 \neq a_3 \neq ... \neq a_{n-1}$ for $n \geq 2$).
Now, $(\frac{a_1+a_2+a_3+...+a_{n-1}}{n-1})^{n-1} = (\frac{(n-1)+2(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{n}-\frac{1}{n+1})}{n-1})^{n-1} = (1+\frac{1}{2n+2})^{n-1} < (1+\frac{1}{n-1})^{n-1}$.
But,the upper bound of $(1+\frac{1}{n-1})^{n-1}$ as $n \to \infty$ is $e$,where $e$ is the base of natural logarithms.And since $e < 3$,we get that:-
$a_1a_2a_3...a_{n-1} < (\frac{a_1+a_2+a_3+...+a_{n-1}}{n-1})^{n-1} < e < 3$.