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Solution for Problem O387
We check the modulo 7 of the expression $3^{(6n-3)}+3^{(3n-1)}+1$.
$3^{(6n-3)}=3^{3(2n-1)}=27^{(2n-1)}\equiv(-1)^{(2n-1)}\equiv-1(\text{mod }7)$
$3^{(3n-1)}=(9)(3^{3n-3)})=(9)27^{(n-1)}\equiv(9)(-1)^{(n-1)}\equiv(2)(-1)^{(n-1)}(\text{mod }7)$
$3^{(6n-3)}+3^{(3n-1)}+1\equiv(-1+(2)(-1)^{(n-1)}+1)(\text{mod }7)\equiv(2)(-1)^{(n-1)}(\text{mod }7)\equiv2,5(\text{mod }7)$
But cubic residude modulo 7 is $0,1,6$. Hence given expression can't be a perfect cube.