Arpon Basu,School: AECS-4,Mumbai-400094
Solution for problem O433

Given

$$3qr = s^2 - s + 1 \tag{1}$$

Multiplying both side of equation 1 by s we obtain

$$\begin{aligned}(3qr)s &= (s^2 - s + 1)s \\ &= s^3 - s^2 + s\end{aligned} \tag{2}$$

$$\begin{aligned}\therefore q^3 + r^3 - s^3 + 3qrs &= q^3 + r^3 - s^3 + s^3 - s^2 + s \\ &= q^3 + r^3 - (s^2 - s)\end{aligned} \tag{3}$$

From equation 1 we obtain

$$(s^2 - s) = 3qr - 1$$

Replacing value of $(s^2 - s)$ in quation 3 we obtain

$$\begin{aligned}q^3 + r^3 - s^3 + 3qrs &= q^3 + r^3 - (s^2 - s) \\ &= q^3 + r^3 + 1 - 3qr \\ &= q^3 + r^3 + 1^3 - 3qr(1) \\ &= (q + r + 1)(q^2 + r^2 + 1 - qr - q - r)\end{aligned} \tag{4}$$

$$\therefore (q + r + 1) \mid q^3 + r^3 - s^3 + 3qrs$$

Hence Proved