Solution for Problem J369

After removing square roots from both sides of the original equation we obtain

$$x^6 + 4x^5 + 4x^4 - 2x^3 - 4x^2 + 1 = 0. \tag{1}$$

$$\Rightarrow (x+1)^2(x^4 + 2x^3 - x^2 - 2x + 1) = 0$$

Thus **??** has a double root $= -1$, but -1 is not the root of original equation hence they must be discarded.

Dividing **??** by $(x+1)^2$ we obtain

$$x^4 + 2x^3 - x^2 - 2x + 1 = 0 \tag{2}$$

$$\Rightarrow (x-1)(x)(x+1)(x+2) = -1 \Rightarrow (x^2 + x - 1)^2 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}.$$

Plugging these roots to original equation , we get that $x = \frac{\sqrt{5}-1}{2}$ is the only solution of the given equation.