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Solution for problem E-48

$$\cot 36^0 \cdot \cot 72^0 = \frac{1}{\tan 36^0 \cdot \tan 72^0} = \frac{1}{\tan(2 \cdot 18^0) \cdot \tan(90-18)^0} = \frac{1}{\frac{2\tan 18^0}{(1-\tan^2 18^0)} \cot 18^0} = \frac{(1-\tan^2 18^0)}{2}$$

Let $A = 18^0 \therefore 5A = 90^0 \Rightarrow 3A = 90^0 - 2A \Rightarrow \sin(3A) = \sin(90^0 - 2A) = \cos(2A)$

$$\sin(3A) = \cos(2A)$$

$$\Rightarrow 3\sin A - 4\sin^3 A = 1 - 2\sin^2 A$$

$$\Rightarrow 4\sin^3 A - 2\sin^2 A - 3\sin A + 1 = 0$$

$$\Rightarrow (\sin A - 1)(4\sin^2 A + 2\sin A - 1) = 0$$

Thus three solutions of the above equation are $\sin A = 1, \sin A = \frac{\sqrt{5}-1}{4}, \sin A = -\frac{\sqrt{5}-1}{4}$. As $A = 18^0$ so only solution is $\sin A = \frac{\sqrt{5}-1}{4}$.

$$\tan^2 A = \frac{\sin^2 A}{\cos^2 A} = \frac{\sin^2 A}{1-\sin^2 A} = \frac{(\frac{\sqrt{5}-1}{4})^2}{(1-(\frac{\sqrt{5}-1}{4})^2)} = \frac{(3-\sqrt{5})}{(5+\sqrt{5})}$$

$$\therefore (1-\tan^2 A) = (1-\tan^2 18^0) = (1-\frac{(3-\sqrt{5})}{(5+\sqrt{5})}) = \frac{(2+2\sqrt{5})}{(5+\sqrt{5})} = \frac{2(1+\sqrt{5})}{(5+\sqrt{5})} = \frac{2}{\sqrt{5}}$$

$$\therefore \cot 36^0 \cdot \cot 72^0 = \frac{(1-\tan^2 18^0)}{2} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$