



United International University

Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics

Mid-term Examination : Spring 2022

Total Marks: 30 Time: 1 hour and 45 minutes

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

Answer all the 5 questions. Numbers to the right of the questions denote their marks.

1. (a) My favorite football team qualifies for final whenever I buy a ticket. [1.5]
- Find inverse and contrapositive of the above sentence.
 - Suppose, p = My favorite football team qualifies for final, q = I buy a ticket
Translate the given sentence into a proposition.
- (b) Find the truth table of the following compound proposition and comment on whether the proposition is a tautology, contradiction, or neither of them. [2.5]

$$\neg((x \rightarrow \neg y) \vee \neg(z \longleftrightarrow \neg z))$$

- (c) Using propositional laws prove that, [2]

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

2. (a) Consider the following predicates- [3]

$D(x)$: x is a student of discrete mathematics course.

$T(x)$: x is an instructor of discrete mathematics course.

$A(x)$: x got an A in their final exam.

$C(x, y)$: x receives counseling from y regarding discrete mathematics course.

Represent the following statements using the above predicates, quantifiers and logical connectives. The domain of all variables consists of all people associated with UIU.

- All students of discrete mathematics course get A on their final exam.
 - Some students that receive counseling from an instructor, get A in their final exam.
 - All students of this course receive counseling from some course instructor.
- (b) Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers. [3]
- $\forall x \exists y (xy = 1)$
 - $\forall x \forall y (((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$
 - $\exists x \exists y ((x^2 > y) \wedge (x < y))$

3. (a) Consider the Venn diagram in Figure 1: [3]

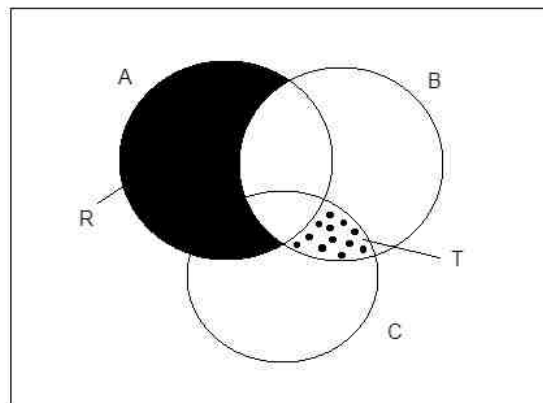


Figure 1: Venn Diagram for Question 3(a)

Here, the three circles represent the sets A, B, and C respectively. The shaded portion on the left represents set R. And the dotted portion on the right represents the set T. Here, $A = \{1, 2, 3, 4, 5\}$, $B = \{5, 2, 1, 0, 9\}$, and $C = \{0, 1, 7, 5, 8\}$.

Find $R \cup T$. You must use the values given in the Set definitions above.

(b) Given that, $A = \{x, y\}$, $B = \{a, b, c\}$, $C = \{1, 2, 3\}$, and $D = \{x, y, z\}$. [1.5 × 2 = 3]

i. Find $A \times B \times C$; ii. What is the value of $|P(A \times B \times C \times D)|$?

4. (a) Find $f \circ g$, $g \circ f$ and $f \circ f$, where $f(x) = x^2 + 1$ and $g(x) = \sqrt{x} - 1$ [3]

(b) Provide a counterexample for each of the following functions to show that they are **not** one-to-one: [3]

i. $f : R \rightarrow R, f(x) = 2x^2 - 5x + 1$

ii. $f : Z \rightarrow Z, f(x) = x^2 + 1$

iii. $f : Z \rightarrow Z, f(x) = \left\lfloor \frac{x}{2} \right\rfloor + x^4$

5. (a) Prove the following statement: [3]

“If mn is even, then m is even or n is even.”

(b) Prove the following statement using a direct proof: [3]

“The sum of two rational numbers is rational.”