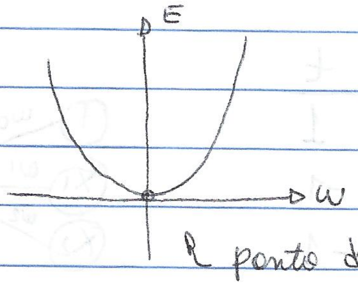
≡ ADALINE ≡ EXERCICIOS

Ex:

$$E(w) = w^2$$



ponto de mínimo

$E(0) = 0$

$$\text{grad } E = \frac{d}{dw}(w^2)$$

$$\text{grad } E = 2w$$

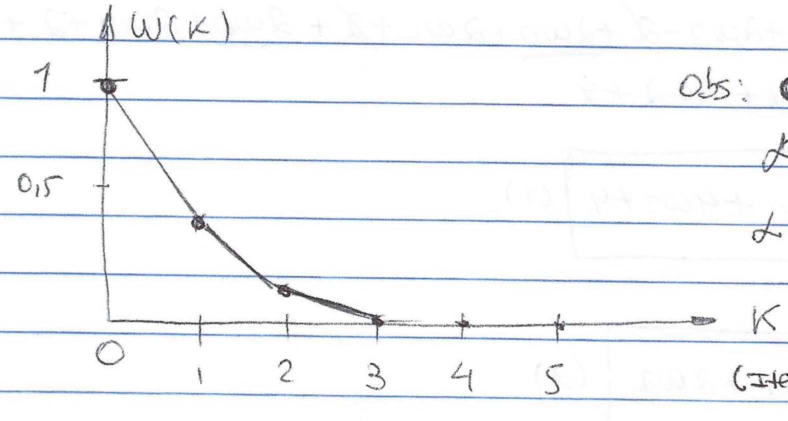
resolução numérica:

$$w(k+1) = w(k) - \overset{\leftarrow \text{passo}}{\alpha} \cdot 2w(k)$$

↑ sentido oposto ao gradiente

$w = 1$ e $\alpha = 0{,}3$

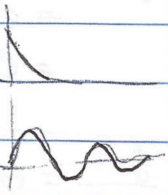
K	w(k)	w(k+1)
0	1	$1 - 0{,}3 \cdot 2 \cdot 1 = 0{,}4$
1	0,4	$0{,}4 - 0{,}3 \cdot 2 \cdot 0{,}4 = 0{,}16$
2	0,16	0,064
3	0,064	0,0256
4	0,0256	0,01024
5	0,01024	$\boxed{0{,}004096}$ ← w=0



Obs: $\alpha = 0{,}45$

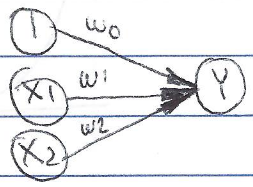
$\alpha = 0{,}90$

$\alpha \geq 1$ ← Diverge !!!



* Resolução Analítica de uma Rede _ Adaline

x_1	x_2	t
1	1	1
1	0	-1
0	1	-1
0	0	-1



$y = y_{in}$

Resolução:

$$E = \sum_{p=1}^{4} [t(p) - y(p)]^2$$

$$E = [1 - (1.w_0 + 1.w_1 + 1.w_2)]^2 + [-1 - (1.w_0 + 1.w_1 + 0.w_2)]^2 + [-1 - (1.w_0 + 0.w_1 + 1.w_2]^2 + [-1 - (1.w_0 + 0.w_1 + 0.w_2)]^2$$

$$E = (1 - w_0 - w_1 - w_2)^2 + (-1 - w_0 - w_1)^2 + (-1 - w_0 - w_2)^2 + (-1 - w_0)^2$$

$$E = (w_0 + w_1 + w_2 - 1)^2 + (w_0 + w_1 + 1)^2 + (w_0 + w_2 + 1)^2 + (w_0 + 1)^2 \quad (0)$$

Calculo do ponto de minimo:

$$\frac{\partial E}{\partial w_0} = 2(w_0 + w_1 + w_2 - 1).1 + 2(w_0 + w_1 + 1).1 + 2(w_0 + w_2 + 1).1 + 2(w_0 + 1).1$$

$$= \underline{2w_0} + 2w_1 + 2w_2 - \cancel{2} + \underline{2w_0} + 2w_1 + \cancel{2} + \underline{2w_0} + 2w_2 + 2 + \underline{2w_0} + 2$$

$$= 8w_0 + 4w_1 + 4w_2 + 4$$

$$\boxed{\frac{\partial E}{\partial w_0} = 8w_0 + 4w_1 + 4w_2 + 4} \quad (1)$$

resolvendo:

$$\boxed{\frac{\partial E}{\partial w_1} = 4w_0 + 4w_1 + 2w_2} \quad (2)$$

resolvendo:

$$\boxed{\frac{\partial E}{\partial w_2} = 4w_0 + 2w_1 + 4w_2} \quad (3)$$

>>> para ponto de mínimo:

$$\frac{\partial E}{\partial w_0} = \frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial w_2} = 0 \quad e \quad \frac{\partial^2 E}{\partial w_0^2}, \frac{\partial^2 E}{\partial w_1^2}, \frac{\partial^2 E}{\partial w_2^2} \text{ têm que serem } > 0.$$

$$\frac{\partial^2 E}{\partial w_0} = 8 > 0 \checkmark \qquad \frac{\partial^2 E}{\partial w_1} = 4 > 0 \checkmark \qquad \frac{\partial^2 E}{\partial w_2} = 4 > 0 \checkmark$$

$\therefore$ $\exists$ ponto de mínimo.

Calculo do ponto de mínimo do erro quadratico:

$$\begin{array}{l} ① \rightarrow 8w_0 + 4w_1 + 4w_2 + 4 = 0 \\ ② \rightarrow 4w_0 + 4w_1 + 2w_2 = 0 \\ ③ \rightarrow 4w_0 + 2w_1 + 4w_2 = 0 \end{array} \Rightarrow \begin{cases} 2w_0 + w_1 + w_2 = -1 & \text{(I)} \\ 2w_0 + 2w_1 + w_2 = 0 & \text{(II)} \\ 2w_0 + w_1 + 2w_2 = 0 & \text{(III)} \end{cases}$$

resolução do sistema:

(II) e (III)

$$\begin{cases} 2w_0 + 2w_1 + w_2 = 0 \\ -2w_0 - w_1 - 2w_2 = 0 \end{cases}$$

$$w_1 - w_2 = 0 \Rightarrow \boxed{w_1 = w_2} \quad ④$$

Substituindo em (II)

$$2w_0 + 2w_1 + w_1 = 0 \Rightarrow 2w_0 + 3w_1 = 0$$

$$\Downarrow$$

$$\boxed{w_0 = \frac{-3}{2} w_1} \quad ⑤$$

tomando-se (I)

$$2w_0 + w_1 + w_2 = -1$$

$$2\left(\frac{-3}{2} w_1\right) + w_1 + w_1 = -1$$

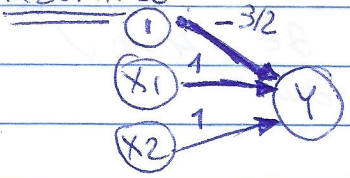
$$-3w_1 + 2w_1 = -1$$

$$-w_1 = -1 \Rightarrow \boxed{w_1 = 1}$$

como $w_1 = w_2 \Rightarrow \boxed{w_2 = 1}$ //

$$w_0 = -\frac{3}{2} w_1 \Rightarrow \boxed{w_0 = -\frac{3}{2}}$$

RESUMINDO:

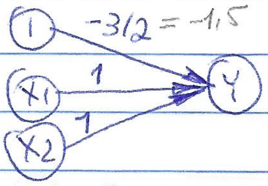


$$\text{Erro Quadratico} = (w_0 + w_1 + w_2 - 1)^2 + (w_0 + w_1 + 1)^2 + (w_0 + w_2 + 1)^2 + (w_0 + 1)^2$$

$$= (-3/2 + 1 + 1 - 1)^2 + (-3/2 + 1 + 1)^2 + (-3/2 + 1 + 1)^2 + (-3/2 + 1)^2$$

$$= 1 \text{ // } (\text{é o } \overset{\text{pt de}}{\text{mínimo}} \text{ !!!!})$$

Recapitulação:



x_1	x_2	y-in	y^*	t
1	1	+0,5	+1	+1
1	0	-0,5	-1	-1
0	1	-0,5	-1	-1
0	0	-1,5	-1	-1

OK

$$* \; y = \begin{cases} 1 \text{ se } y\text{-}in \geq 0 \\ -1 \text{ se } yin < 0 \end{cases}$$