In [1]:

```
import pandas as pd
import numpy as np
from numpy import loadtxt
import matplotlib.pyplot as plt
import math
```

Task 1

Reading datasets

In [2]:

```
train_crime=open("../pp2data/train-crime.csv","rb").read().split()
train_house=open("../pp2data/train-housing.csv","rb").read().split()
trainR_crime=open("../pp2data/trainR-crime.csv","rb").read().split()
trainR_house=open("../pp2data/trainR-housing.csv","rb").read().split()
test_crime=open("../pp2data/test-crime.csv","rb").read().split()
test_house=open("../pp2data/test-housing.csv","rb").read().split()
testR_crime=open("../pp2data/testR-crime.csv","rb").read().split()
testR_house=open("../pp2data/testR-housing.csv","rb").read().split()
```

In [3]:

```
train_c=loadtxt(train_crime,delimiter=",")
trainR_c=loadtxt(trainR_crime,delimiter=",")
test_c=loadtxt(test_crime,delimiter=",")
testR_c=loadtxt(testR_crime,delimiter=",")
train_h=loadtxt(train_house,delimiter=",")
trainR_h=loadtxt(trainR_house,delimiter=",")
test_h=loadtxt(test_house,delimiter=",")
testR_h=loadtxt(testR_house,delimiter=",")
```

Calculating MSE

In [4]:

```
def calc_MSE(x,y):
    MSE=np.square(np.subtract(x,y)).mean()
    return MSE
```

Calculating MLE

In [5]:

```
def calc_MLE(x,y,z,l=0):
    pred=z.dot((np.linalg.pinv(l*np.identity(x.shape[1])+np.transpose(x).dot(x))).dot(np.tr
    return pred
```

Bayesian Function

In [6]:

```
def bayes(a,b,x,y,z):
    pred=z.dot(b*np.dot(np.dot(np.linalg.inv(a*np.identity(x.shape[1])+b*np.dot(x.T,x)),x.T
    return pred
```

In [7]:

```
pred_c=calc_MLE(train_c,trainR_c,test_c)
pred_h=calc_MLE(train_h,trainR_h,test_h)
```

In [8]:

```
MSE_c=calc_MSE(testR_c,pred_c)
MSE_c
```

Out[8]:

```
0.3450569330423569
```

In [9]:

```
MSE_h=calc_MSE(testR_h,pred_h)
MSE_h
```

Out[9]:

```
0.29443262001271514
```

Splitting Datasets according to Question

In [10]:

```
new_train_c=[]
l=len(train_c)
for i in range(0,10,1):
    new_train_c.append(train_c[0:int(l*((i+1)/10)),:])
new_trainR_c=[]
l=len(trainR_c)
for i in range(0,10,1):
    new_trainR_c.append(trainR_c[0:int(l*((i+1)/10))])

new_train_h=[]
l=len(train_h)
for i in range(0,10,1):
    new_train_h.append(train_h[0:int(l*((i+1)/10)),:])
new_trainR_h=[]
l=len(trainR_h)
for i in range(0,10,1):
    new_trainR_h.append(trainR_h[0:int(l*((i+1)/10))])
```

Model Selection

In [11]:

```
def ModelSelection(phi,t):
    N=phi.shape[0]
    d=phi.shape[1]
    a0=2
    b0=10
    a_change=1
    b_change=1
    while not (a_change<=0.001 and b_change<=0.001):
        lam=np.linalg.eigvals(b0*np.dot(phi.T,phi))
        gam=0
        for i in range(0,len(lam)):
            gam+=(lam[i]/(a0+lam[i]))
        m=b0*np.dot(np.dot(np.linalg.inv(a0*np.identity(d)+b0*np.dot(phi.T,phi)),phi.T),t)
        a1=gam/np.dot(m,m)
        b1=1/((1/(N-gam))*(np.dot(np.dot(phi,m)-t,np.dot(phi,m)-t)))
        a_change=abs(a1-a0)
        b_change=abs(b1-b0)
        a0=a1
        b0=b1
    return a0,b0,(a0/b0)
```

Crime dataset model

In [12]:

```
model_crime=[]
for i in range(10):
    d={}
    alpha,beta,regularization=ModelSelection(new_train_c[i],new_trainR_c[i])
    d["partition"]=(i+1)/10
    d["alpha"]=alpha
    d["beta"]=beta
    d["regularization"]=regularization
    model_crime.append(d)
    print(model_crime[i])
```

```
{'partition': 0.1, 'alpha': 188.13418564070818, 'beta': 3.0590610997375354,
'regularization': 61.50063026098103}
{'partition': 0.2, 'alpha': 284.7219942674846, 'beta': 2.964418873946527, 'r
egularization': 96.04647871116627}
{'partition': 0.3, 'alpha': 266.99009988195087, 'beta': 2.847152772909647,
'regularization': 93.77441998277473}
{'partition': 0.4, 'alpha': 280.4488295991778, 'beta': 2.8496845641386455,
'regularization': 98.41399049159224}
{'partition': 0.5, 'alpha': 284.08760024259936, 'beta': 2.9156222181703813,
'regularization': 97.4363545702676}
{'partition': 0.6, 'alpha': 263.4754615387708, 'beta': 2.962261804049953, 'r
egularization': 88.9440160820869}
{'partition': 0.7, 'alpha': 254.16163967645417, 'beta': 3.0875611127863833,
'regularization': 82.31793003996117}
{'partition': 0.8, 'alpha': 254.20674738385384, 'beta': 3.1241961399624962,
'regularization': 81.36708964338757}
{'partition': 0.9, 'alpha': 247.4375126121574, 'beta': 3.046255677039333, 'r
egularization': 81.22677110696198}
{'partition': 1.0, 'alpha': 239.71632213512905, 'beta': 3.0852982512664147,
'regularization': 77.69632061883588}
```

Housing dataset model

In [13]:

```
model_housing=[]
for i in range(10):
    d={}
    alpha,beta,regularization=ModelSelection(new_train_h[i],new_trainR_h[i])
    d["partition"]=(i+1)/10
    d["alpha"]=alpha
    d["beta"]=beta
    d["regularization"]=regularization
    model_housing.append(d)
    print(model_housing[i])
```

```
{'partition': 0.1, 'alpha': (17.115748346919897-4.8112392413123525e-30j), 'b
eta': (55.43127666781543+4.35640031600882e-29j), 'regularization': (0.308774
204308696-3.2946525412022132e-31j)}
{'partition': 0.2, 'alpha': 16.334287029416586, 'beta': 3.4586561427608227,
'regularization': 4.722726502779191}
{'partition': 0.3, 'alpha': 17.047085057337306, 'beta': 4.1144505963678375,
'regularization': 4.1432226874679605}
{'partition': 0.4, 'alpha': 18.406145860229955, 'beta': 4.974604278779374,
'regularization': 3.700022118090225}
{'partition': 0.5, 'alpha': 17.80860565102691, 'beta': 4.411526273698912, 'r
egularization': 4.0368354501706305}
{'partition': 0.6, 'alpha': 19.53644116765738, 'beta': 4.691276605105255, 'r
egularization': 4.1644189443864725}
{'partition': 0.7, 'alpha': 18.642552046513647, 'beta': 4.393500189872348,
'regularization': 4.243211844963025}
{'partition': 0.8, 'alpha': 19.27321350878941, 'beta': 4.549846965772047, 'r
egularization': 4.23601357447393}
{'partition': 0.9, 'alpha': 16.95077861294148, 'beta': 4.119603837235635, 'r
egularization': 4.114662303139301}
{'partition': 1.0, 'alpha': 20.412460531028803, 'beta': 4.041255751833772,
'regularization': 5.051019233753366}
```

MLE and Bayesian for crime dataset

In [14]:

```
MSE_crime=[]
l=len(train_c)
for i in range(0,10,1):
    x=train_c[0:int(round(l*((i+1)/10),0))]
    y=trainR_c[0:int(round(l*((i+1)/10),0))]
    pred=calc_MLE(x,y,test_c)
    MSE=calc_MSE(testR_c,pred)
    MSE_crime.append(((i+1)/10,MSE))
```

In [15]:

```
MSE_crime
```

Out[15]:

```
[(0.1, 1.3002479408636172),
 (0.2, 0.4967427267380541),
 (0.3, 0.456348556463142 15),
 (0.4, 0.38714667059641883),
 (0.5, 0.3713426121845869),
 (0.6, 0.3644705541248856),
 (0.7, 0.35484728890387435),
 (0.8, 0.34830539217648143),
 (0.9, 0.3463928082504861),
 (1.0, 0.3450569330423569)]
```

In [16]:

```
bayes_crime=[]
for i in range(10):
    x=train_c[0:int(round(l*((i+1)/10),0))]
    y=trainR_c[0:int(round(l*((i+1)/10),0))]
    pred=bayes(model_crime[i]["alpha"],model_crime[i]["beta"],x,y,test_c)
    MSE=calc_MSE(testR_c,pred)
    bayes_crime.append(((i+1)/10,MSE))
```

In [17]:

```
bayes_crime
```

Out[17]:

```
[(0.1, 0.396984001877673 74),
 (0.2, 0.3452471828496966),
 (0.3, 0.34826092450288),
 (0.4, 0.33788161858136473),
 (0.5, 0.33862318460111307),
 (0.6, 0.3328208435831047),
 (0.7, 0.33077656253536186),
 (0.8, 0.33244628962304645),
 (0.9, 0.33214869354426235),
 (1.0, 0.3345435937647672)]
```

MLE and Bayesian for housing dataset

In [18]:

```
MSE_housing=[]
l=len(train_h)
for i in range(0,10,1):
    x=train_h[0:int(round(l*((i+1)/10),0))]
    y=trainR_h[0:int(round(l*((i+1)/10),0))]
    pred=calc_MLE(x,y,test_h)
    MSE=calc_MSE(testR_h,pred)
    MSE_housing.append(((i+1)/10,MSE))
```

In [19]:

```
MSE_housing
```

Out[19]:

```
[(0.1, 0.8678327090953848),
 (0.2, 0.6531445531536013),
 (0.3, 0.3736963444198246),
 (0.4, 0.3414166696345161),
 (0.5, 0.3644021055620389),
 (0.6, 0.32080556108532404),
 (0.7, 0.3141824758819998),
 (0.8, 0.30051685692609026),
 (0.9, 0.31907890073457246),
 (1.0, 0.29443262001271514)]
```

In [20]:

```
bayes_housing=[]
for i in range(10):
    x=train_h[0:int(round(l*((i+1)/10),0))]
    y=trainR_h[0:int(round(l*((i+1)/10),0))]
    pred=bayes(model_housing[i]["alpha"],model_housing[i]["beta"],x,y,test_h)
    MSE=calc_MSE(testR_h,pred)
    bayes_housing.append(((i+1)/10,MSE))
```

In [21]:

```
bayes_housing
```

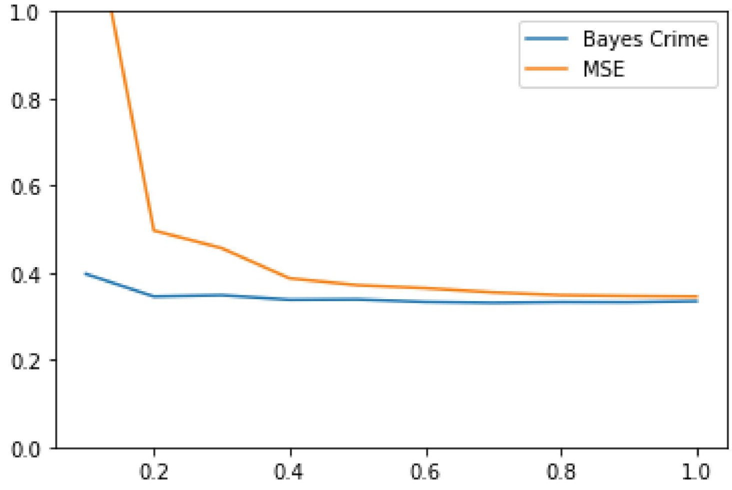
Out[21]:

```
[(0.1, (0.47393428813876126+6.53776744194863e-32j)),
 (0.2, 0.3178536604019471),
 (0.3, 0.3141356507494856),
 (0.4, 0.3098564718648854),
 (0.5, 0.32089215353385886),
 (0.6, 0.29966191365297895),
 (0.7, 0.2942475355630813),
 (0.8, 0.2879615393914638),
 (0.9, 0.3025004284077777),
 (1.0, 0.2884936266054806)]
```

Graph for Crime dataset

In [22]:

```
plt.plot(*zip(*bayes_crime),label="Bayes Crime")
plt.plot(*zip(*MSE_crime),label="MSE")
plt.ylim([0,1])
plt.legend()
plt.show()
```

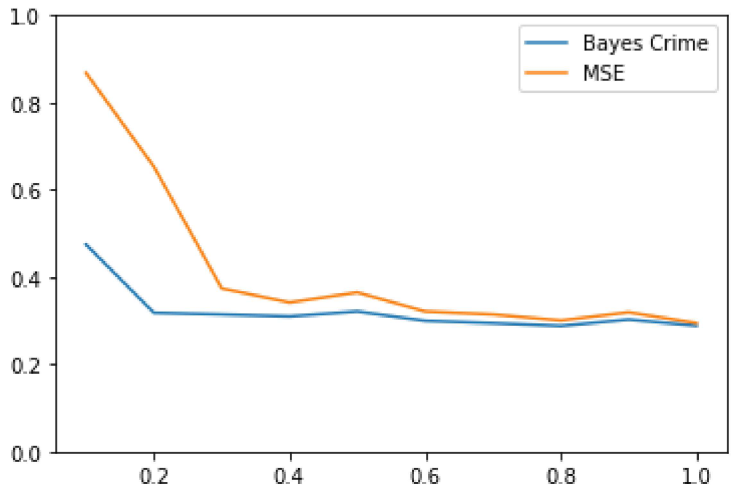


Graph for Housing Dataset

In [23]:

```
plt.plot(*zip(*bayes_housing),label="Bayes Crime")
plt.plot(*zip(*MSE_housing),label="MSE")
plt.ylim([0,1])
plt.legend()
plt.show()
```

```
C:\Users\aarya\anaconda3\lib\site-packages\matplotlib\cbook\__init__.py:129
8: ComplexWarning: Casting complex values to real discards the imaginary par
t
  return np.asarray(x, float)
```



Calculating MSE for different lamda values

In [24]:

```
lam=[1.0,33.0,100.0,1000.0]
mse_c=[]
for la in lam:
    MSE_crime=[]
    l=len(train_c)
    for i in range(0,10,1):
        x=train_c[0:int(round(l*((i+1)/10),0))]
        y=trainR_c[0:int(round(l*((i+1)/10),0))]
        pred=calc_MLE(x,y,test_c,la)
        MSE=calc_MSE(testR_c,pred)
        MSE_crime.append(((i+1)/10,MSE))
    mse_c.append(MSE_crime)
```

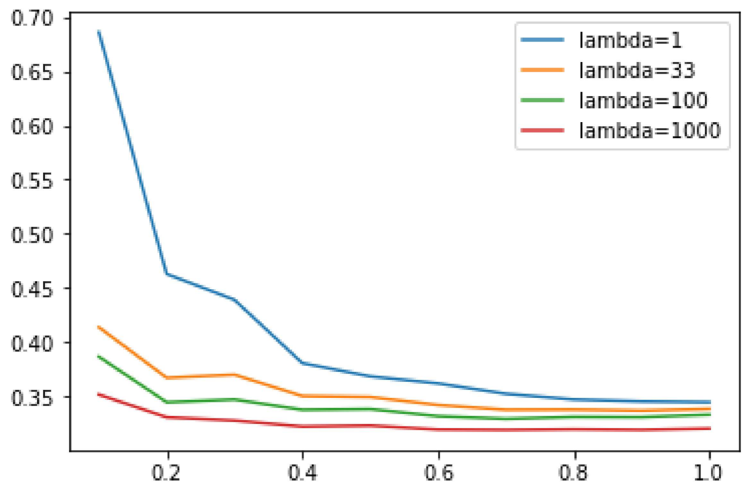
In [25]:

```
lam=[1.0,33.0,100.0,1000.0]
mse_h=[]
for la in lam:
    MSE_housing=[]
    l=len(train_h)
    for i in range(0,10,1):
        x=train_h[0:int(round(l*((i+1)/10),0))]
        y=trainR_h[0:int(round(l*((i+1)/10),0))]
        pred=calc_MLE(x,y,test_h,la)
        MSE=calc_MSE(testR_h,pred)
        MSE_housing.append(((i+1)/10,MSE))
    mse_h.append(MSE_housing)
```

Graph for Crime datset

In [26]:

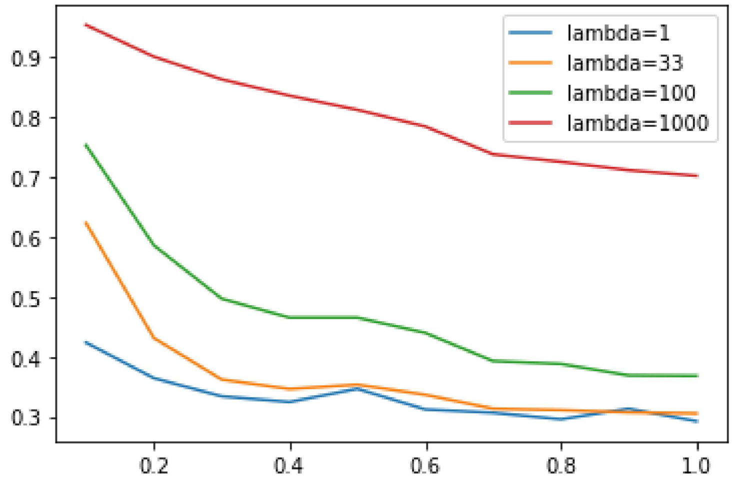
```
plt.plot(*zip(*mse_c[0]),label="lambda=1")
plt.plot(*zip(*mse_c[1]),label="lambda=33")
plt.plot(*zip(*mse_c[2]),label="lambda=100")
plt.plot(*zip(*mse_c[3]),label="lambda=1000")
plt.legend()
plt.show()
```



Graph for Housing dataset

In [27]:

```
plt.plot(*zip(*mse_h[0]),label="lambda=1")
plt.plot(*zip(*mse_h[1]),label="lambda=33")
plt.plot(*zip(*mse_h[2]),label="lambda=100")
plt.plot(*zip(*mse_h[3]),label="lambda=1000")
plt.legend()
plt.show()
```



Task 2

Creating datsets

In [28]:

```
train_f3=open("../pp2data/train-f3.csv","rb").read().split()
train_f5=open("../pp2data/train-f5.csv","rb").read().split()
trainR_f3=open("../pp2data/trainR-f3.csv","rb").read().split()
trainR_f5=open("../pp2data/trainR-f5.csv","rb").read().split()
test_f3=open("../pp2data/test-f3.csv","rb").read().split()
test_f5=open("../pp2data/test-f5.csv","rb").read().split()
testR_f3=open("../pp2data/testR-f3.csv","rb").read().split()
testR_f5=open("../pp2data/testR-f5.csv","rb").read().split()
```

In [29]:

```
train_3=loadtxt(train_f3,delimiter=",")
trainR_3=loadtxt(trainR_f3,delimiter=",")
test_3=loadtxt(test_f3,delimiter=",")
testR_3=loadtxt(testR_f3,delimiter=",")
train_5=loadtxt(train_f5,delimiter=",")
trainR_5=loadtxt(trainR_f5,delimiter=",")
test_5=loadtxt(test_f5,delimiter=",")
testR_5=loadtxt(testR_f5,delimiter=",")
```

In [30]:

```
def bayes2(a,b,x,y,z):
    pred=z.dot(b*np.dot(np.dot(np.linalg.inv(a*np.identity(x.shape[1])+b*np.dot(x.T,x)),x.T
    mn=(b*np.dot(np.dot(np.linalg.inv(a*np.identity(x.shape[1])+b*np.dot(x.T,x)),x.T),y))
    sn=(np.linalg.inv(a*np.identity(x.shape[1])+b*np.dot(x.T,x)),x.T)
    return pred,mn,sn
```

Calculating log Evidence and MSE for f3

In [31]:

```
logev_3=[]
MSE_3=[]
bayes_3=[]
for i in range(10):

    new_train_3=np.hstack((np.ones((train_3.shape[0],1)),train_3.reshape(train_3.shape[0],1
    new_test_3=np.hstack((np.ones((test_3.shape[0],1)),test_3.reshape(test_3.shape[0],1)))
    for j in range(2,i+2):
        new_train_3=np.hstack((new_train_3,(new_train_3[:,1]**(j)).reshape(new_train_3.shap
        new_test_3=np.hstack((new_test_3,(new_test_3[:,1]**(j)).reshape(new_test_3.shape[0]

    a,b,l=ModelSelection(new_train_3,trainR_3)

    bayes,mn,sn=bayes2(a,b,new_train_3,trainR_3,new_test_3)

    e=np.dot((trainR_3-np.dot(new_train_3,mn)).T,(trainR_3-np.dot(new_train_3,mn)))*b/2
    e1=(a*np.dot(mn.T,mn))/2
    e2=np.log(np.linalg.det(np.linalg.inv(sn[0])))
    log=(((new_train_3.shape[1]/2*np.log(alpha))+((300/2)*np.log(b))-e-e1-(e2/2)-(300*np.lo
    logev_3.append((i+1,log))


    MSE=calc_MSE(testR_3,bayes)
    bayes_3.append((i+1,MSE))

    pred=calc_MLE(new_train_3,trainR_3,new_test_3)
    MSE=calc_MSE(testR_3,pred)
    MSE_3.append((i+1,MSE))
```

In [32]:

```
logev_3
```

Out[32]:

```
[(1, -3031.2969121330398),
 (2, -3028.564275831095),
 (3, -2695.224252770911),
 (4, -2701.4329184615654),
 (5, -2706.667939500311),
 (6, -2707.3946126834317),
 (7, -2721.462621288671),
 (8, -2732.579184186423),
 (9, -2746.881958887436),
 (10, (-2761.319226132034+0j))]
```

In [33]:

```
MSE_3
```

Out[33]:

```
[(1, 39389142.58553826),
 (2, 39495762.45900575),
 (3, 148429.38836568058),
 (4, 179627.46131843395),
 (5, 186263.6807275151),
 (6, 211370.60715117436),
 (7, 184992.29642495257),
 (8, 196641.43358936673),
 (9, 546257.7759903334),
 (10, 579316.5958650279)]
```

Calculating Log evidence and MSE for f5

In [34]:

```
logev_5=[]
MSE_5=[]
bayes_5=[]
for i in range(10):

    new_train_5=np.hstack((np.ones((train_5.shape[0],1)),train_5.reshape(train_5.shape[0],1
    new_test_5=np.hstack((np.ones((test_5.shape[0],1)),test_5.reshape(test_5.shape[0],1)))
    j=2
    while j <=i+2:
        new_train_5=np.hstack((new_train_5,(new_train_5[:,1]**(j)).reshape(new_train_5.shap
        new_test_5=np.hstack((new_test_5,(new_test_5[:,1]**(j)).reshape(new_test_5.shape[0]
        j+=1
    a,b,l=ModelSelection(new_train_5,trainR_5)

    bayes,mn,sn=bayes2(a,b,new_train_5,trainR_5,new_test_5)

    e=np.dot((trainR_5-np.dot(new_train_5,mn)).T,(trainR_5-np.dot(new_train_5,mn)))*b/2
    e1=(a*np.dot(mn.T,mn))/2
    e2=np.log(np.linalg.det(np.linalg.inv(sn[0])))
    log=(((new_train_5.shape[1]/2*np.log(alpha))+((300/2)*np.log(b))-e-e1-(e2/2)-(300*np.lo
    logev_5.append((i+1,log))


    MSE=calc_MSE(testR_5,bayes)
    bayes_5.append((i+1,MSE))

    pred=calc_MLE(new_train_5,trainR_5,new_test_5)
    MSE=calc_MSE(testR_5,pred)
    MSE_5.append((i+1,MSE))
```

In [35]:

```
logev_5
```

Out[35]:

```
[(1, -4360.422864141759),
 (2, -3912.9560540194047),
 (3, -3908.1614614004034),
 (4, -2714.974896657237),
 (5, -2721.7392525595783),
 (6, -2729.895548191721),
 (7, -2741.309541933108),
 (8, -2754.408590279996),
 (9, (-2769.3179042194065+0j)),
 (10, -2788.119185549046)]
```

In [36]:

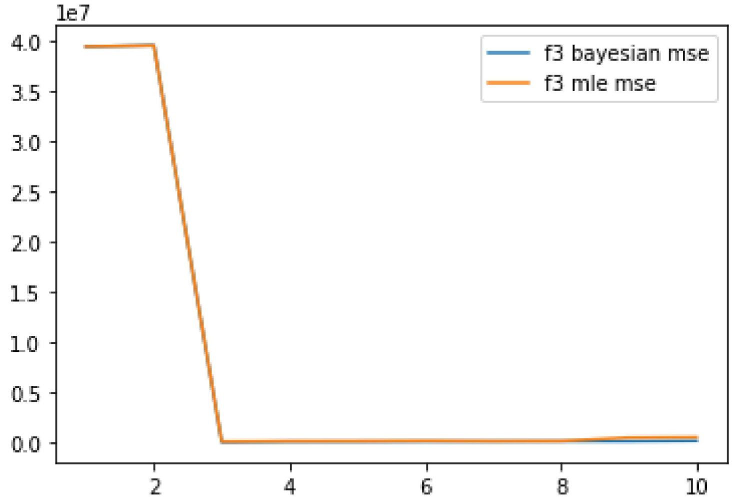
```
MSE_5
```

Out[36]:

```
[(1, 341195638304.2565),
 (2, 17465602121.94851),
 (3, 17435655143.626442),
 (4, 61375.3499006928),
 (5, 79043.03733122443),
 (6, 92512.84710342463),
 (7, 90189.90562969688),
 (8, 126835.276487774),
 (9, 7703653.185180388),
 (10, 238757655.70912683)]
```

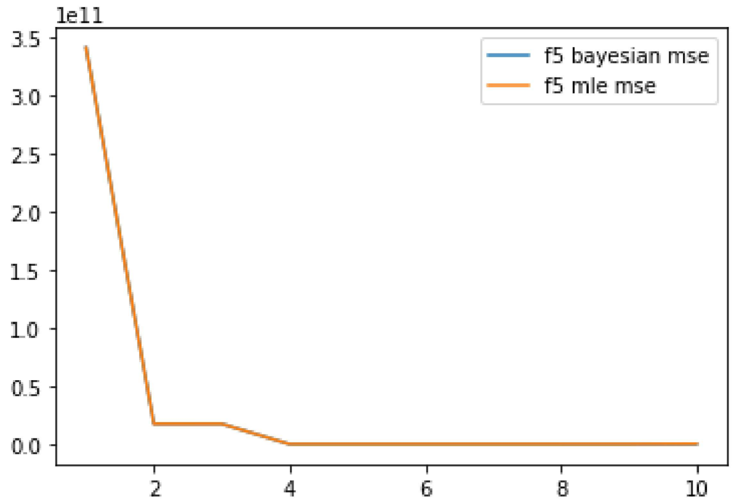
In [37]:

```
plt.plot(*zip(*bayes_3),label='f3 bayesian mse')
plt.plot(*zip(*MSE_3),label='f3 mle mse')
plt.legend()
plt.show()
```



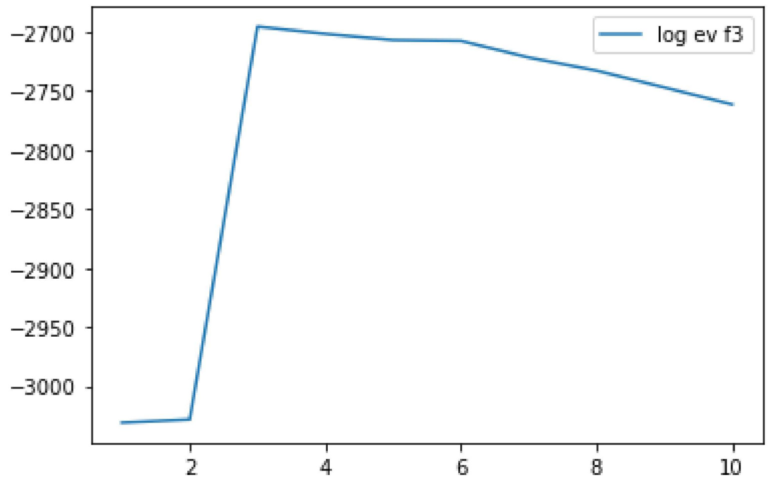
In [38]:

```
plt.plot(*zip(*bayes_5),label='f5 bayesian mse')
plt.plot(*zip(*MSE_5),label='f5 mle mse')
plt.legend()
plt.show()
```



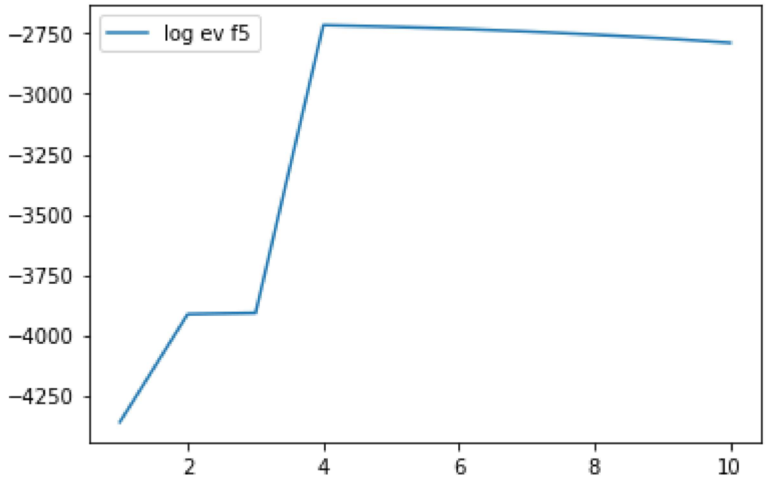
In [39]:

```
plt.plot(*zip(*logev_3),label='log ev f3')
plt.legend()
plt.show()
```



In [40]:

```
plt.plot(*zip(*logev_5),label='log ev f5')
plt.legend()
plt.show()
```



In [ ]: