Avogadro's Number from Diff. Coeff.

Avogadro's number (N_A) can be calculated from the translational diffusion coefficient (D_T) using the **Stokes-Einstein equation** in combination with the **Ideal Gas Law**.

Stokes-Einstein equation

$D_T = \frac{k_B T}{6\pi\eta r}$

Where:

- D_T = diffusion coefficient (m^2/s),
- k_B = Boltzmann constant (J/K),
- T = temperature (K),
- η = dynamic viscosity of the medium ($Pa\mathring{u}s$),
- r = radius of the particle (m).

Ideal Gas Law

$PV = nRT$

- P = Pressure,
- V = Volume,
- n = Number of moles,
- R = Ideal gas constant,

Microscopic Ideal Gas Law

On a molecular scale, the ideal gas law is,

$PV = Nk_BT$.

- N = Number of particles,

Relating n and N

The number of particles N is related to the number of moles n by Avogadro's number,

$N = nN_A$.

Substituting into $PV = Nk_BT$, we get,

$PV = nN_Ak_BT$.

Comparing this with the macroscopic ideal gas law $PV = nRT$,

$R = N_A k_B$.

$\therefore k_B = \frac{R}{N_A}$

Substitute into Stokes-Einstein and solve for N_A

$$N_A = \frac{RT}{6\pi\eta r D_T}$$

Example

Imagine a spherical particle in water at $T = 298K$

- $D = 5.0 \times 10^{-13}\ m^2/s$,
- $\eta = 0.001\ Pa\mathring{u}s$,
- $r = 1.0 \times 10^{-6}\ m$.

The ideal gas constant $R = 8.314\ J/\text{mol}\mathring{u}K$.

$N_A = \frac{(8.314)(298)}{6\pi(0.001)(10^{-6})(5.0\times10^{-13})}$

First, calculate the denominator:

$6\pi(0.001)(10^{-6})(5.0 \times 10^{-13}) = 9.4248 \times 10^{-18}$

Now calculate:

$N_A = \frac{2480.572}{9.4248\times10^{-18}} \approx 6.6 \times 10^{23}\,\text{mol}^{-1}$.