Power spectrum analysis for quartic potential

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September 26, 2013

1 Power spectrum for quadratic trap

Langevin equation for single colloid of radius, a and mass, m, in a trap of strength, k and a solvent of viscosity, η and Brownian noise, ξ

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = \xi \tag{1}$$

Noting system is overdamped so neglect inertia ($m\ddot{x}(t) \to 0$) and that $\tilde{\dot{x}}(t) = i\omega\tilde{x}(t)$

$$\therefore i\omega\gamma\tilde{x}(t) + k\tilde{x}(t) = \tilde{\xi} \tag{2}$$

$$\therefore \tilde{x}(t) = \frac{\tilde{\xi}}{i\omega\gamma + k} \tag{3}$$

$$\therefore S_x(\omega) = \langle\tilde{x}(t)\tilde{x}(t)^*\rangle = \frac{\tilde{\xi}}{i\omega\gamma + k} \tag{4}$$

$$\therefore S_x(\omega) = \frac{\tilde{\xi}^2}{\gamma^2(\omega_c^2 + \omega^2)} \tag{5}$$

Similarly, if the system is underdamped and inertia becomes significant then,

$$\therefore -\omega^2 m\tilde{x}(t) + i\omega\gamma\tilde{x}(t) + k\tilde{x}(t) = \tilde{\xi} \tag{6}$$

Let $\Omega = \sqrt{k/m}$ and $\Gamma = \gamma/m$

$$\therefore -\omega^2\tilde{x}(t) + i\omega\Gamma\tilde{x}(t) + \Omega^2\tilde{x}(t) = \tilde{\xi} \tag{7}$$

$$\therefore \tilde{x}(t) = \frac{\tilde{\xi}}{\Omega^2 - \omega^2 + i\omega\Gamma} \tag{8}$$

$$\therefore S_x(\omega) = \frac{\tilde{\xi}^2}{(\omega^2 - \Omega^2)^2 + \omega^2\Gamma^2} \tag{9}$$

Noting that $\tilde{\xi}^2 \equiv \langle \tilde{\xi}\tilde{\xi}^* \rangle \equiv S_\xi(\omega) = \frac{\Gamma K_B T}{\pi m}$

$$\therefore S_x(\omega) = \frac{K_B T}{\pi k} \frac{\Omega^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \omega^2\Gamma^2} \tag{10}$$

2 Power spectrum for quartic trap

Underdamped quartic potential, $U(x) = \Delta U \left(\left(\frac{x(t)}{r} \right)^2 - 1 \right)^2$

$$\therefore \gamma \dot{x}(t) + \frac{\partial U(x)}{\partial x} = \xi \tag{11}$$

This is a nonlinear ODE

$$\gamma \dot{x}(t) - \left(\frac{4\Delta U x(t)(d^2 - x(t)^2)}{d^4} \right) = \xi \tag{12}$$

$$\langle \hat{x}^*(\nu)\hat{x}(\nu) \rangle = \frac{k_B T}{\pi k} \frac{f_c}{\nu^2 + f_c^2} + \frac{d^2}{4\pi} \frac{2 f_h}{\nu^2 + (2 f_h)^2}. \tag{13}$$