## The circle constant

The Tau Manifesto is dedicated to one of the most important numbers in mathematics, perhaps the most important: the *circle constant* relating the circumference of a circle to its linear dimension. For millennia, the circle has been considered the most perfect of shapes, and the circle constant captures the geometry of the circle in a single number. Of course, the traditional choice for the circle constant is  $\pi$ -but, as mathematician Bob Palais notes in his delightful article " $\pi$  Is Wrong!",  $\pi$  is wrong. It's time to set things right.

(*Note*: Mathematically sophisticated readers, including those already familiar with *The Tau Manifesto*, can skip directly to Section 5, which is part of a revision released on Tau Day, 2012. This new section builds an irrefutable case against  $\pi$ .)

## An immodest proposal

We begin repairing the damage wrought by  $\pi$  by first understanding the notorious number itself. The traditional definition for the circle constant sets  $\pi$  (pi) equal to the ratio of a circle's circumference to its diameter:<sup>2</sup>

$$\pi \equiv \frac{C}{D} = 3.14159265\dots$$
 (1)

The number  $\pi$  has many remarkable properties—among other things, it is irrational and indeed transcendental—and its presence in mathematical formulas is widespread.

It should be obvious that  $\pi$  is not "wrong" in the sense of being factually incorrect; the number  $\pi$  is perfectly well-defined, and it has all the properties normally ascribed to it by mathematicians. When we say that " $\pi$  is wrong", we mean that  $\pi$  is a confusing and unnatural choice for the circle constant. In particular, since a circle is defined as the set of points a fixed distance—the *radius*—from a given point, a more natural definition for the circle constant uses r in place of D:

circle constant 
$$\equiv \frac{C}{r}$$
. (2)

Because the diameter of a circle is twice its radius, this number is numerically equal to  $2\pi$ . Like  $\pi$ , it is transcendental and hence irrational, and (as we'll see in Section 2) its use in mathematics is similarly widespread.

In " $\pi$  Is Wrong!", Bob Palais argues persuasively in favor of the second of these two definitions for the circle constant, and in my view he deserves principal credit for identifying this issue and bringing it to a broad audience. He calls the true circle constant "one turn", and he also introduces a new symbol to represent it (Figure 1). As we'll see, the description is prescient, but unfortunately the symbol is rather strange, and (as discussed in Section 4) it seems unlikely to gain wide adoption.

Figure 1: The strange symbol for the circle constant from " $\pi$  Is Wrong!".

The Tau Manifesto is dedicated to the proposition that the proper response to " $\pi$  is wrong" is "No, really." And the true circle constant deserves a proper name. As you may have guessed by now, The Tau Manifesto proposes that this name should be the Greek letter  $\tau$  (tau):

$$\tau \equiv \frac{C}{r} = 6.283185307179586\dots$$
 (3)

<sup>&</sup>lt;sup>1</sup>Palais, Robert. " $\pi$  Is Wrong!", *The Mathematical Intelligencer*, Volume 23, Number 3, 2001, pp. 7–8. Many of the arguments in *The Tau Manifesto* are based on or are inspired by " $\pi$  Is Wrong!". It is available online at http://bit.ly/pi-is-wrong.

<sup>&</sup>lt;sup>2</sup>The symbol  $\equiv$  means "is defined as".

Throughout the rest of this manifesto, we will see that the *number*  $\tau$  is the correct choice, and we will show through usage (Section 2 and Section 3) and by direct argumentation (Section 4) that the *letter*  $\tau$  is a natural choice as well.

## A powerful enemy

Before proceeding with the demonstration that  $\tau$  is the natural choice for the circle constant, let us first acknowledge what we are up against—for there is a powerful conspiracy, centuries old, determined to propagate pro- $\pi$  propaganda. Entire books are written extolling the virtues of  $\pi$ . (I mean, books!) And irrational devotion to  $\pi$  has spread even to the highest levels of geekdom; for example, on "Pi Day" 2010 Google changed its logo to honor  $\pi$  (Figure 2).

Figure 2: The Google logo on March 14 (3/14), 2010 ("Pi Day").

Meanwhile, some people memorize dozens, hundreds, even thousands of digits of this mystical number. What kind of sad sack memorizes even 40 digits of  $\pi$  (Figure 3)?<sup>3</sup>

Figure 3: Michael Hartl proves Matt Groening wrong by reciting  $\pi$  to 40 decimal places.

Truly, proponents of  $\tau$  face a mighty opponent. And yet, we have a powerful ally—for the truth is on our side.

<sup>&</sup>lt;sup>3</sup>The video in Figure 3 (available at http://vimeo.com/12914981) is an excerpt from a lecture given by Dr. Sarah Greenwald, a professor of mathematics at Appalachian State University. Dr. Greenwald uses math references from *The Simpsons* and *Futurama* to engage her students' interest and to help them get over their math anxiety. She is also the maintainer of the Futurama Math Page.