CSC 8830 Module 3.

Image Blurring and the Convolution Theorem

Convolution theorem states that convolution in spatial domain is equivalent to pointwise multiplication of frequency domain

$$f * g = \mathcal{F}^{-1}\{\mathcal{F}\{f\} \cdot \mathcal{F}\{g\}\}$$

Where

f is the input mage

g is the convolution kernel

$\mathcal{F}$ denotes the fourier transformation

$\mathcal{F}^{-1}$ denotes the inverse Fourier transformation

$*$ denotes convolution

$\cdot$ denotes element-wise multiplication

For a 2D image $I(x, y)$ and kernel $K(x, y)$

$$(I * K)(x, y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} I(x-i, y-j) \cdot K(i, j)$$

The equivalent operation in the frequency domain:

$$I_{blurred} = \mathcal{F}^{-1}\{\mathcal{F}\{I\} \cdot \mathcal{F}\{K\}\}$$

Using a normalized Gaussian kernel:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

For a 15×15 kernel with $\sigma = 2.5$, the kernel is normalized so $\sum G(x, y) = 1$