Draw any figure with a quantum algorithm

Arjan Cornellissen, Farrokh Labib
(QuSoft, CWI, Amsterdam)

Fourier series

Any 1-periodic (nice enough) function $f\colon [0,1] \to \mathbb{C}$ can be written as

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi i k t}.$$

Fourier series

Any 1-periodic (nice enough) function $f\colon [0,1] \to \mathbb{C}$ can be written as

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi ikt}.$$

We can extract the coefficients as follows

$$c_k = \int_0^1 f(t) e^{-2\pi ikt} dt.$$

Fourier series

Any 1-periodic (nice enough) function $f\colon [0,1] \to \mathbb{C}$ can be written as

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi i k t}.$$

We can extract the coefficients as follows

$$c_k = \int_0^1 f(t) e^{-2\pi i k t} dt.$$

Approximations: 1) $f(t) \approx \sum_{k=-K}^{K} c_k e^{2\pi i k t}$ and 2) $c_k \approx \frac{e^{-\pi i k/M}}{M} \hat{f}_k$ here we use the midpoint rule for approximating the integral and

$$\hat{f}_k := \sum_{j=0}^{M-1} f\left(\frac{j}{M} + \frac{1}{2M}\right) e^{-2\pi i j k}.$$

Let $z = (z_0, \ldots, z_{M-1})$ be M points in $\mathbb{C}$. Define

$$|\psi\rangle := \frac{1}{\|z\|} \sum_{j=0}^{M-1} z_j |j\rangle$$

Let $z = (z_0, \ldots, z_{M-1})$ be M points in $\mathbb{C}$. Define

$$|\psi\rangle := \frac{1}{\|z\|} \sum_{j=0}^{M-1} z_j |j\rangle$$

Recall QFT: $QFT|j\rangle = \sum_{k=0}^{M-1} e^{-2\pi ijk}|k\rangle$. Then

$$QFT|\psi\rangle = \frac{1}{\|z\|} \sum_{j=0}^{M-1} \left(\sum_{k=0}^{M-1} z_k e^{-2\pi ijk} \right) |j\rangle$$

$$\frac{1}{\|z\|} \sum_{j=0}^{M-1} \hat{z}_j |j\rangle.$$

Setup

Circuit consists of three parts:

- Preparing the state $|\psi\rangle$ from set of points,

Setup

Circuit consists of three parts:

- Preparing the state $|\psi\rangle$ from set of points,
- apply quantum Fourier transform,

Setup

Circuit consists of three parts:

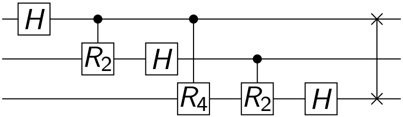
- Preparing the state $|\psi\rangle$ from set of points,
- apply quantum Fourier transform,
- apply measurements to obtain the coefficients of the output state.

Quantum Fourier transform

This is a well-known circuit:

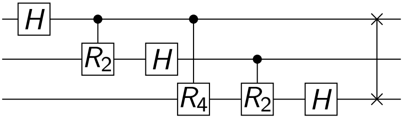
Quantum Fourier transform

This is a well-known circuit:



Quantum Fourier transform

This is a well-known circuit:



Takes $\mathcal{O}(n^2)$ gates, so logarithmic in the number of data points.

Divide and conquer approach:

$$\begin{bmatrix} \alpha_{00\cdots0} \\ \vdots \\ \alpha_{01\cdots1} \\ \hline \alpha_{10\cdots0} \\ \vdots \\ \alpha_{11\cdots1} \end{bmatrix} = \begin{bmatrix} r_{00\cdots00} \\ \vdots \\ r_{01\cdots1} e^{i\phi_{\mathbf{01\cdots1}}} \\ \hline r_{10\cdots0} e^{i\phi_{\mathbf{10\cdots0}}} \\ \vdots \\ \alpha_{11\cdots1} e^{i\phi_{\mathbf{11\cdots1}}} \end{bmatrix} = \begin{bmatrix} a_0 \\ \hline a_1 \end{bmatrix}$$

Divide and conquer approach:

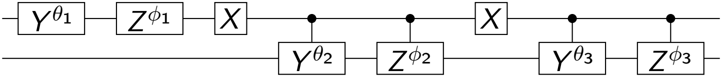
$$\begin{bmatrix} \alpha_{00\cdots0} \\ \vdots \\ \alpha_{01\cdots1} \\ \hline \alpha_{10\cdots0} \\ \vdots \\ \alpha_{11\cdots1} \end{bmatrix} = \begin{bmatrix} r_{00\cdots00} \\ \vdots \\ r_{01\cdots1}e^{i\phi_{\mathbf{01\cdots1}}} \\ \hline r_{10\cdots0}e^{i\phi_{\mathbf{10\cdots0}}} \\ \vdots \\ \alpha_{11\cdots1}e^{i\phi_{\mathbf{11\cdots1}}} \end{bmatrix} = \begin{bmatrix} a_0 \\ \hline a_1 \end{bmatrix}$$

We perform the following operation recursively:

$$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} \|a_0\| \\ 0 \\ \vdots \\ 0 \\ \hline e^{i\phi_{\mathbf{10\cdots0}}}\|a_1\| \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

State preparation

Circuit looks like this:



Takes $\mathcal{O}(2^n)$ gates, so linear in the number of data points.

Measure coefficients

Measure coefficients

- Three step process:
  - Measuring the amplitudes by computational basis measurements.
  - Measure absolute value of relative phases by first applying H to one of the qubits.
  - Measure sign of relative phases by first applying S and H to one the qubits.

Measure coefficients

- Three step process:
  - Measuring the amplitudes by computational basis measurements.
  - Measure absolute value of relative phases by first applying H to one of the qubits.
  - Measure sign of relative phases by first applying S and H to one the qubits.
- Takes $\mathcal{O}(2^n)$ number of measurements, i.e., linear in the number of data points.

Demo

Conclusion

Conclusion

- No real speed-up here.

Conclusion

- No real speed-up here.
- Useful for benchmarking.