Quantum approximate counting

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University of Amsterdam

QuSoft Research Center for Quantum Software

CWI

Quantum computing

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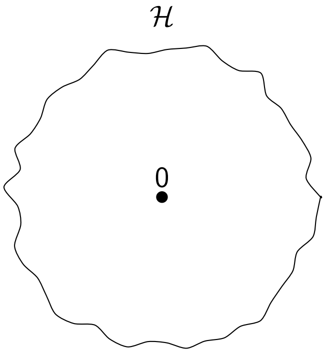

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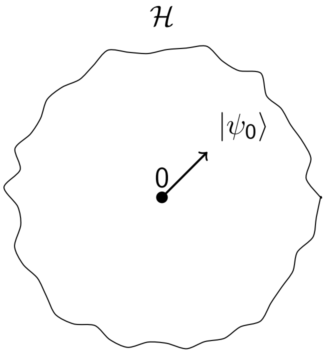

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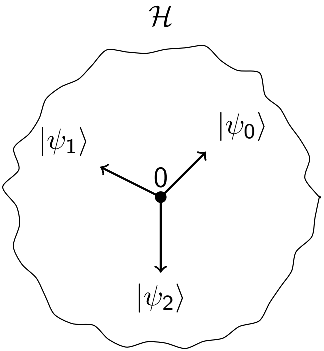

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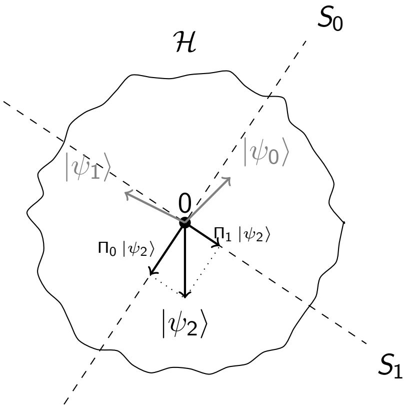

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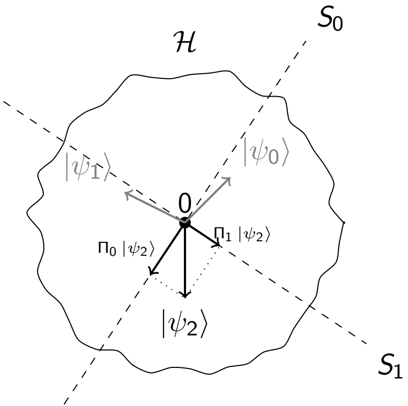

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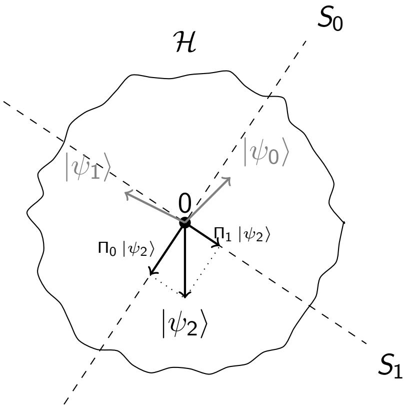

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$$|\psi_0\rangle \overset{U_1}{\mapsto} |\psi_1\rangle \overset{O}{\mapsto} |\psi_2\rangle \overset{U_3}{\mapsto} |\psi_3\rangle \overset{O}{\mapsto} \cdots \overset{U_T}{\mapsto} |\psi_T\rangle.$$

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   2. *Quantumly:*
      1. Hilbert space $\mathcal{H} = \mathbb{C}^n$.
      2. Input-dependent unitary:
         $O_f : |j\rangle \mapsto (-1)^{f(j)} |j\rangle$.
      3. $\Theta(\sqrt{t(n - t + 1)})$ evaluations of O_f.

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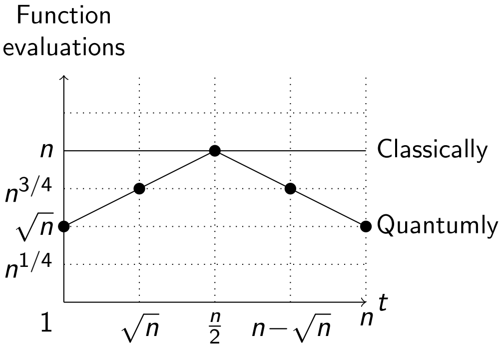

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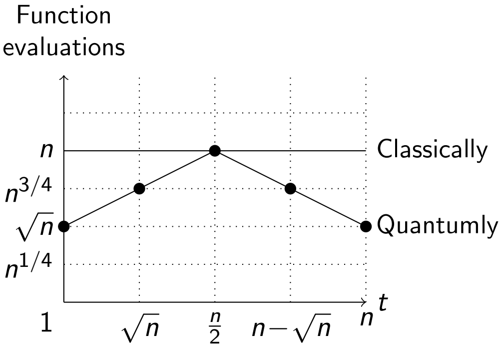
4. *Goal for today:* look at the mathematics behind this phenomenon.



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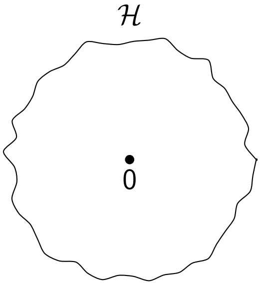

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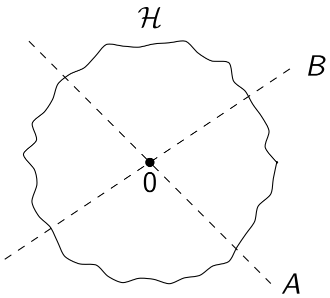

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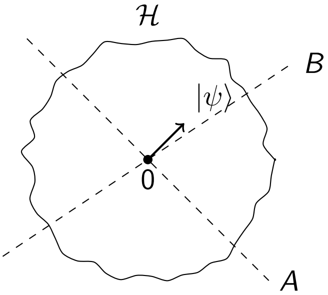

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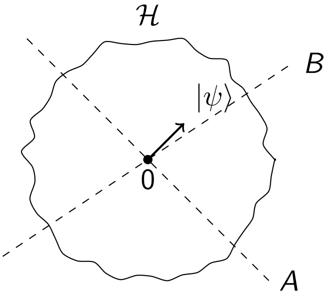

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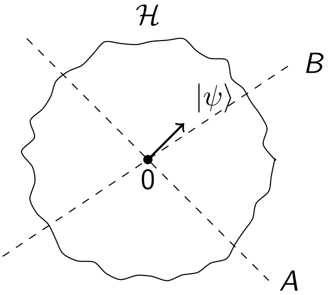

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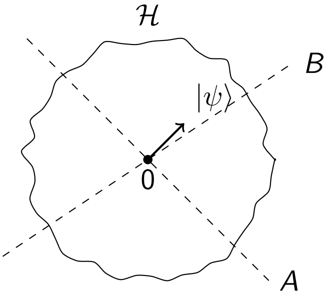
1. U acts as I on $(A \cap B) \oplus (A^\perp \cap B^\perp)$.



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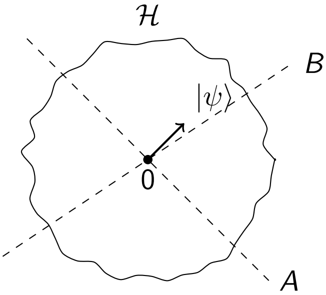

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1. U acts as I on $(A \cap B) \oplus (A^\perp \cap B^\perp)$.
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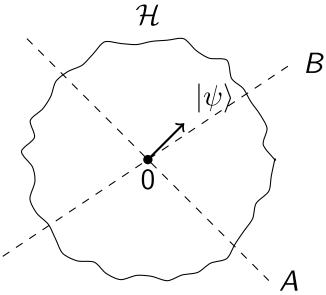
3. The remainder of $\mathcal{H}$ is decomposed into 2-dimensional rotation spaces.



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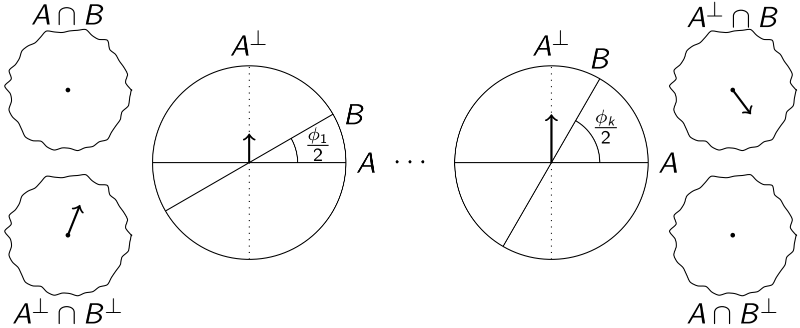
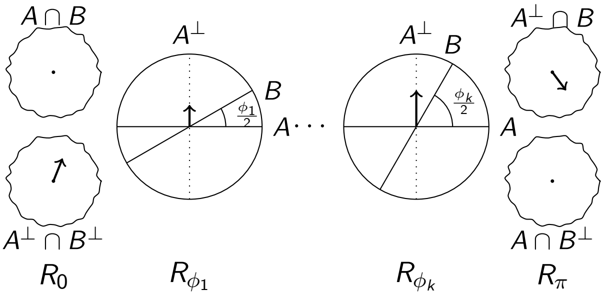


Peak diagrams

1. Given $\mathcal{H}$, A, B, $|\psi\rangle$.



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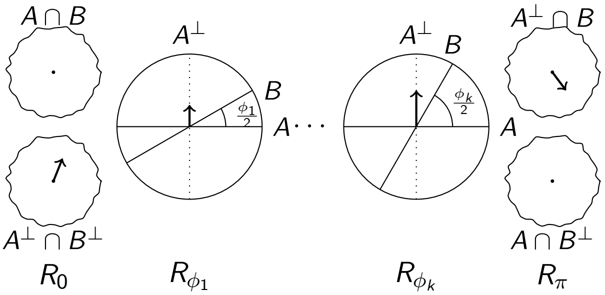

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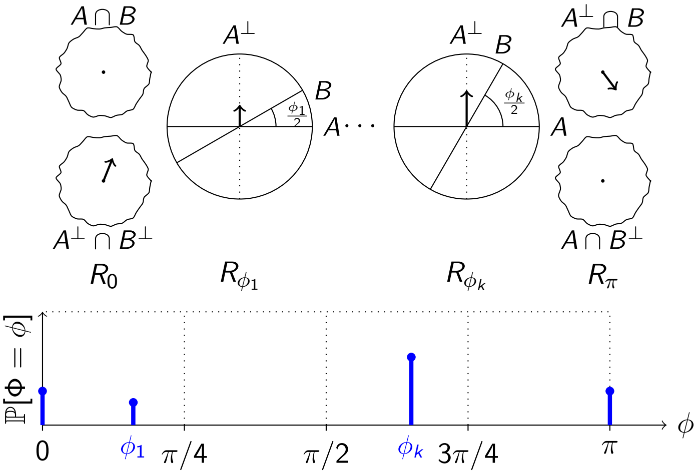

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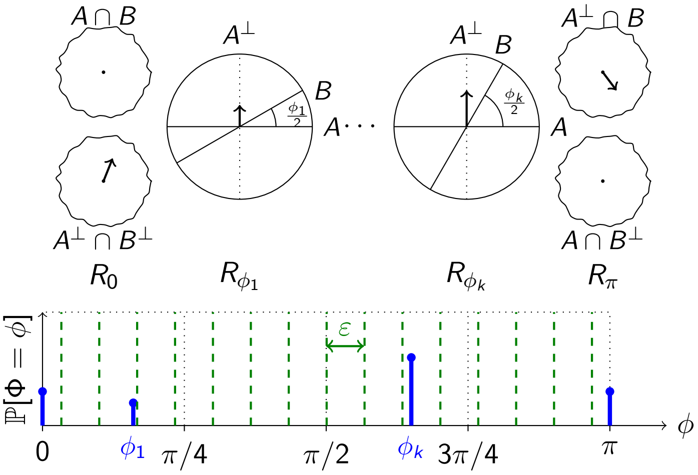
3. *Phase estimation:* One can sample from this binned distribution with $\mathcal{O}(1/\varepsilon)$ calls to $U = (2\Pi_B - I)(2\Pi_A - I)$.



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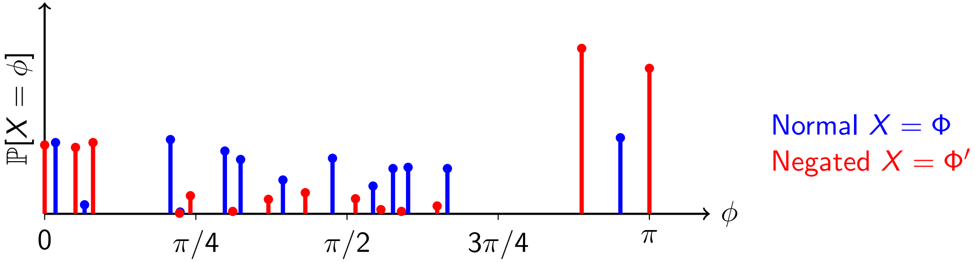

Characteristic function

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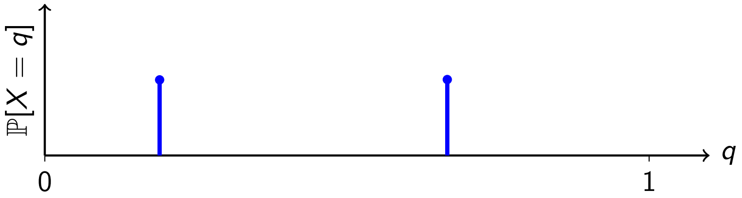

Normal $X = Q$

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1. Let $Q = \sin^2(\Phi/2)$.
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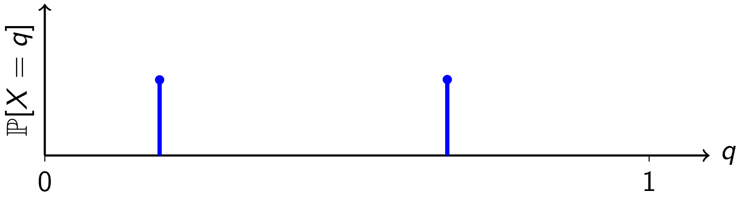
   Let $\chi : [0,1] \to \mathbb{R}$,
   $\chi(q) = \sum_j \frac{p_j}{q - q_j}$



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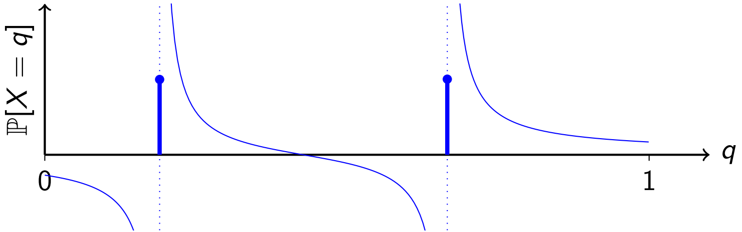

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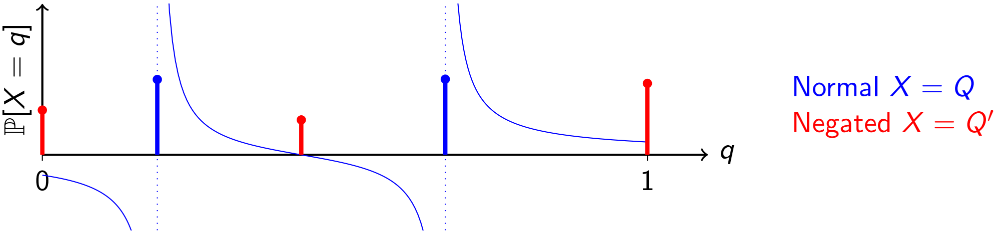

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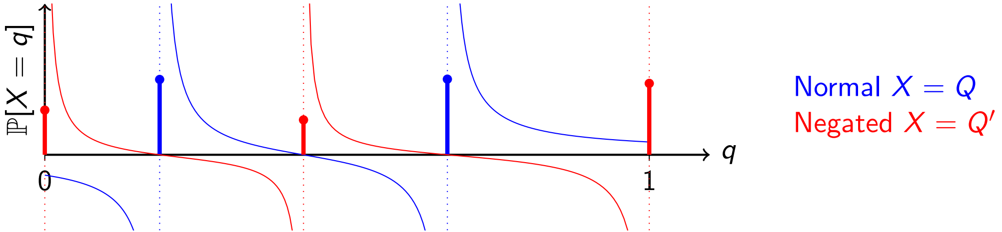

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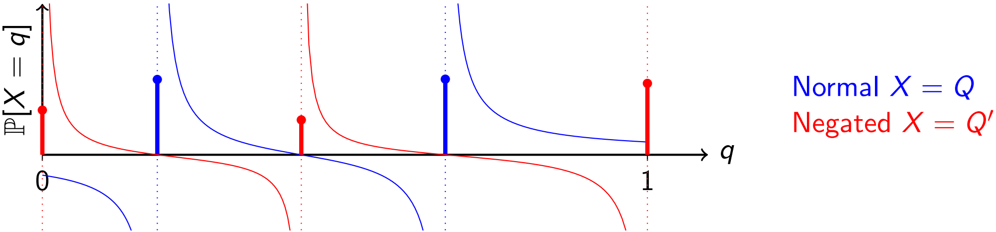

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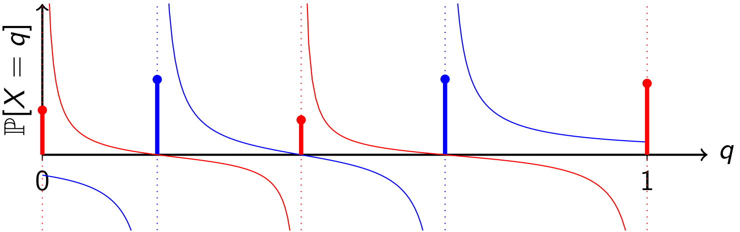
Main result:

$$\chi'(q) = \frac{1}{q(q-1)\chi(q)}.$$



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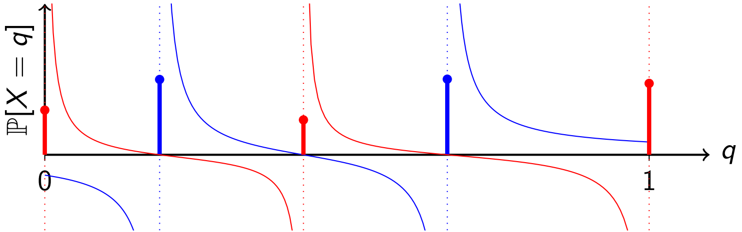

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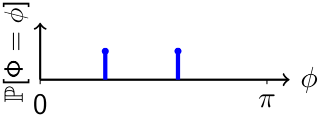

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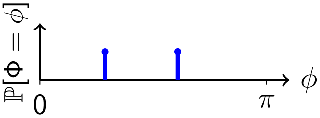

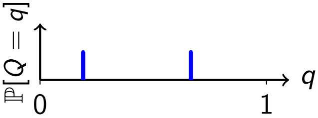

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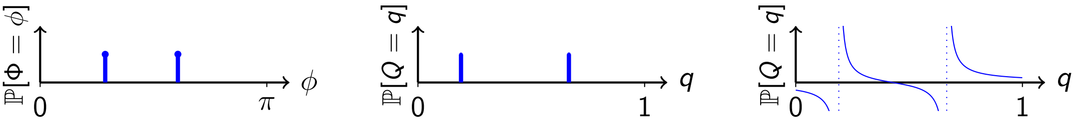
$$Q = \sin^2(\Phi/2) \qquad \chi(q) = \sum_{j=1}^{k} \frac{p_j}{q - q_j}$$



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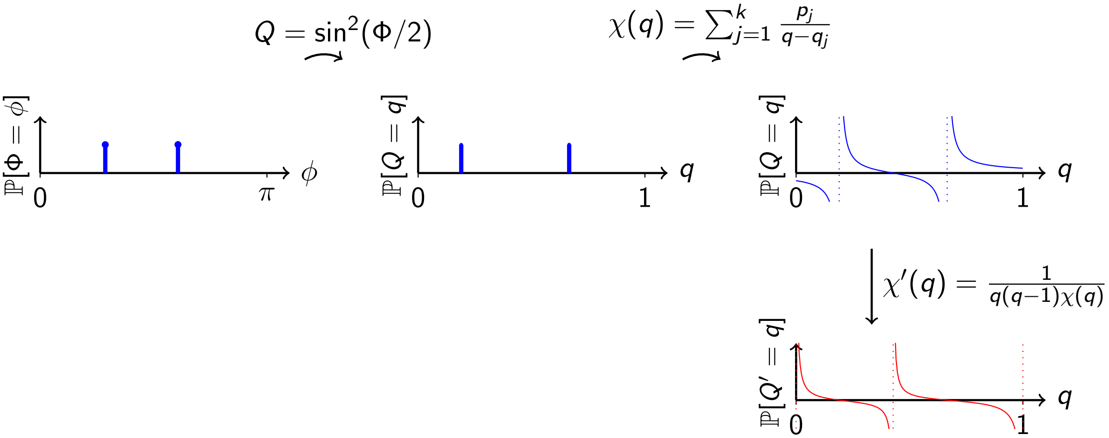

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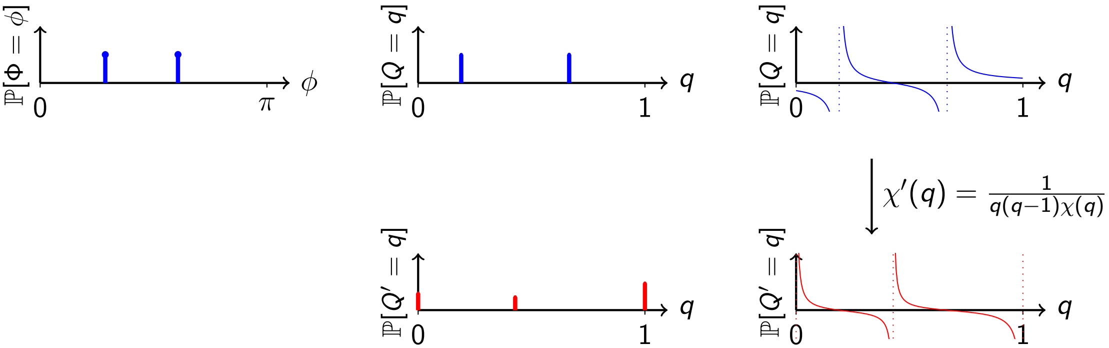

$$\chi'(q) = \frac{1}{q(q-1)\chi(q)}$$

$$\mathbb{P}[Q' = q^*] = \lim_{q \to q^*} \chi'(q)(q - q^*)$$

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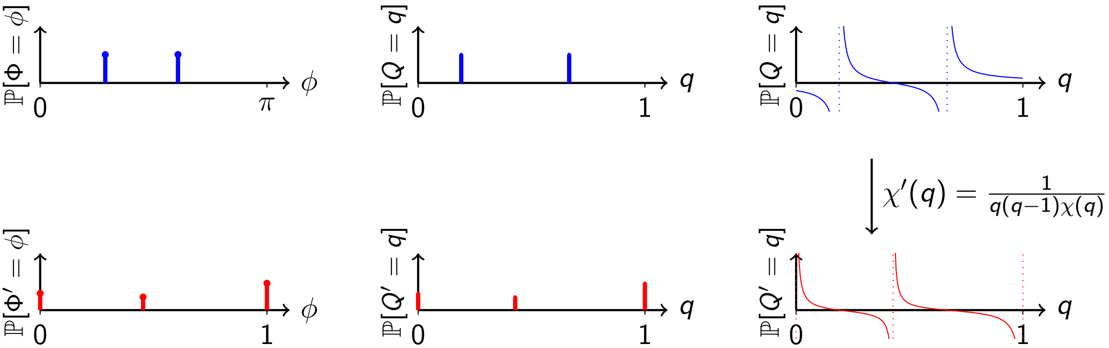

$$\chi'(q) = \frac{1}{q(q-1)\chi(q)}$$

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Application to quantum counting

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1. $f : \{1, 2, \dots, n\} \to \{0, 1\}$.
2. $O_f : |j\rangle \mapsto (-1)^{f(j)} |j\rangle$.
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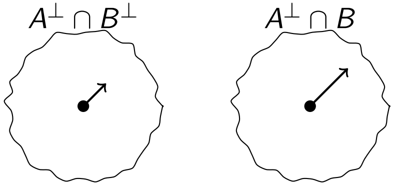

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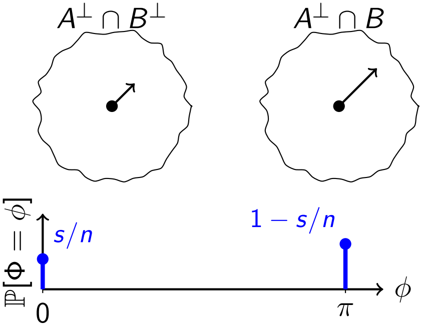

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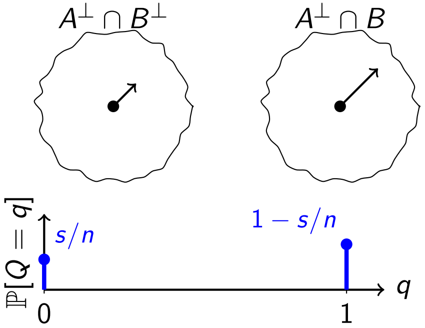

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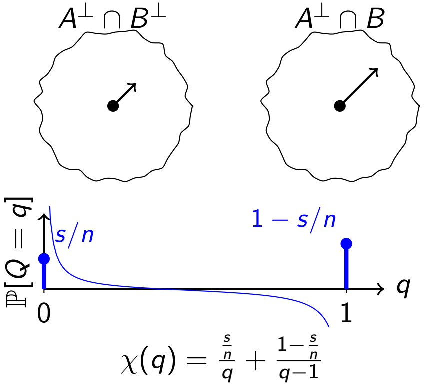

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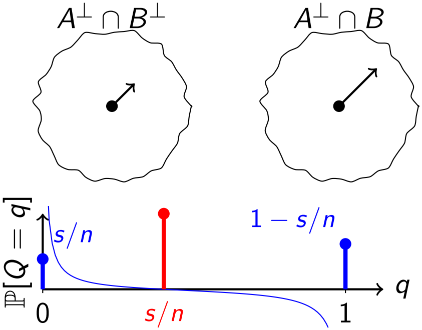

$$\chi(q) = \frac{\frac{s}{n}}{q} + \frac{1-\frac{s}{n}}{q-1}$$

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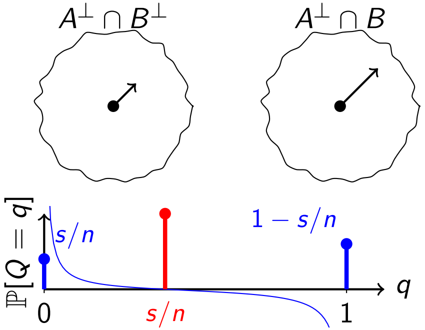

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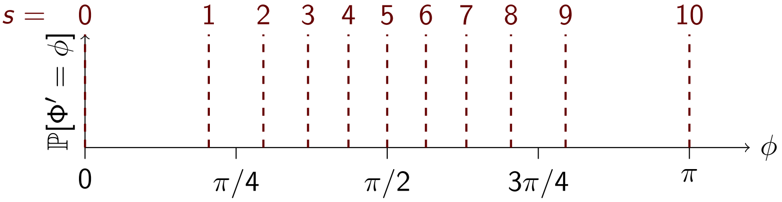

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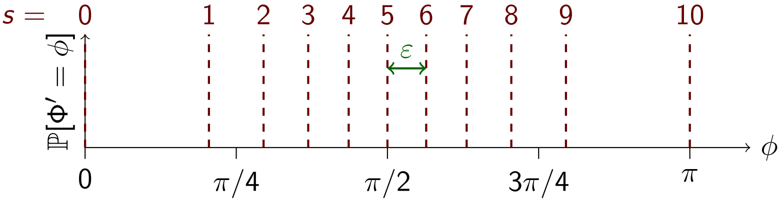

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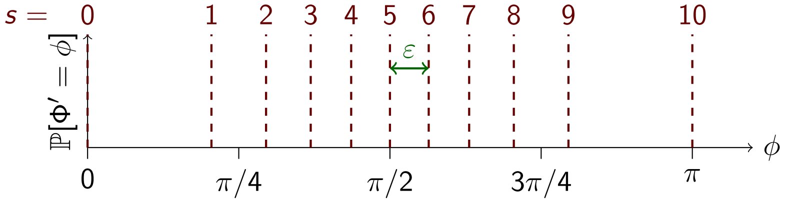

Thus,

$$\varepsilon = \arcsin\left(\sqrt{\frac{t}{n}}\right) - \arcsin\left(\sqrt{\frac{t-1}{n}}\right) \quad \Leftrightarrow \quad \frac{1}{\varepsilon} = \mathcal{O}\left(\sqrt{t(n-t+1)}\right).$$

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Thanks for your attention!

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