Span programs and quantum time complexity

A. J. Cornelissen[1] S. Jeffery[2] M. Ozols[1] A. Piedrafita[2]

[1]QuSoft – University of Amsterdam
[2]QuSoft – CWI

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Synopsis of our results

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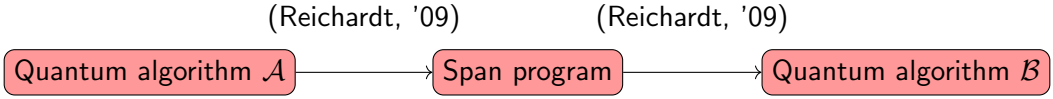

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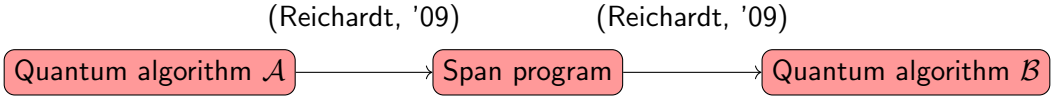

	Quantum algorithm $\mathcal{A}$	Quantum algorithm $\mathcal{B}$
Query complexity	Q	$\mathcal{O}(Q)$

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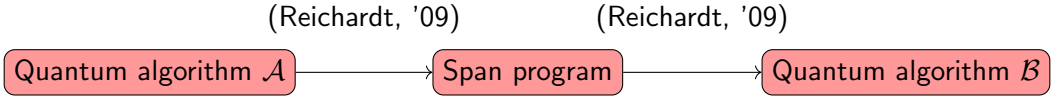

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Question: Can we do the same with time complexity?

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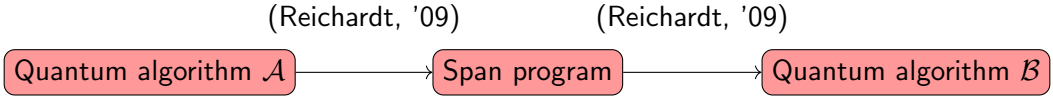

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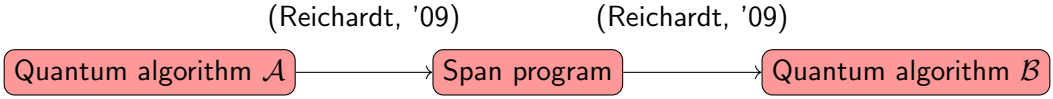

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*if $\mathcal{A}$ allows for *efficient uniform access*.

Model

Quantum algorithm $\mathcal{A}$

Quantum algorithm $\mathcal{B}$

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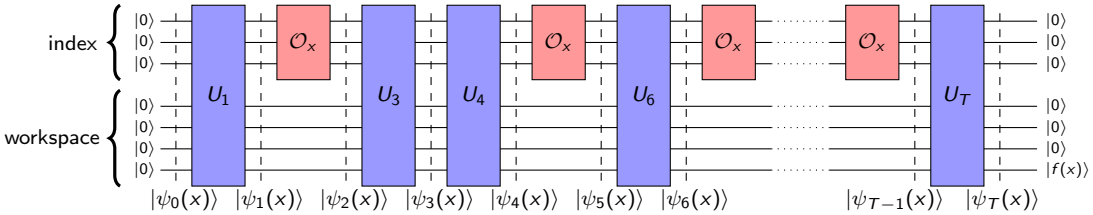
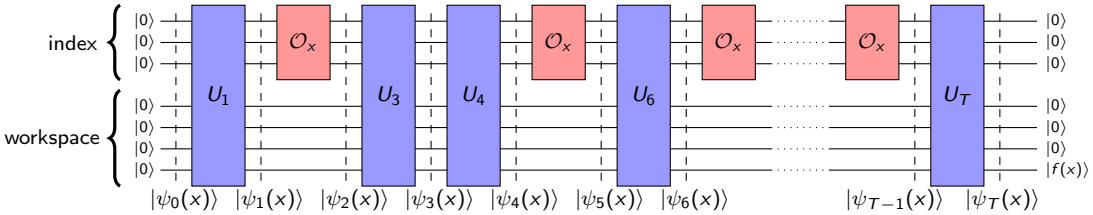
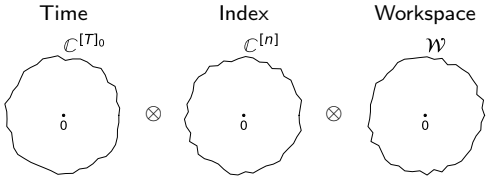


Model

Quantum algorithm $\mathcal{A}$



Quantum algorithm $\mathcal{B}$



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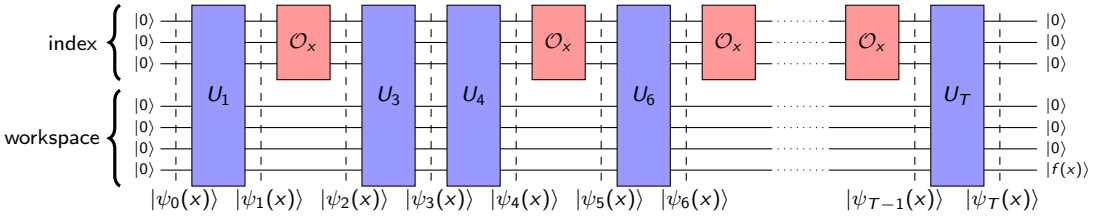
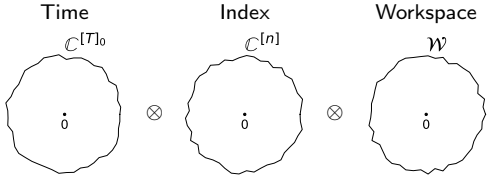
Quantum algorithm $\mathcal{A}$



Quantum algorithm $\mathcal{B}$

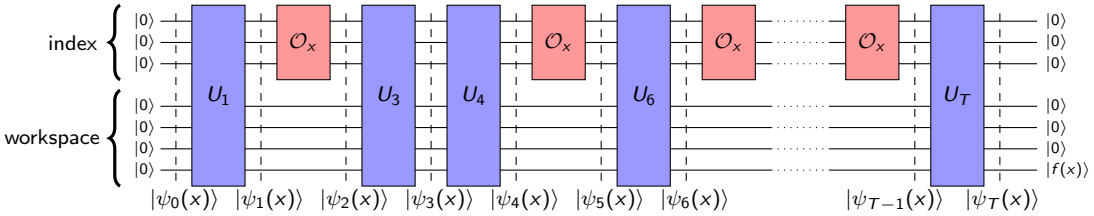


Oracle	# calls
$\mathcal{O}_x : \lvert i\rangle \mapsto (-1)^{x_i} \lvert i\rangle$	$\mathcal{O}(Q)$
$\mathcal{O}_{\mathcal{A}} : \lvert t\rangle \lvert\psi\rangle \mapsto \lvert t\rangle U_t \lvert\psi\rangle$	$\mathcal{O}(T)$
$\mathcal{O}_{\mathcal{Q}} : \lvert t\rangle \mapsto (-1)^{t\in\mathcal{Q}} \lvert t\rangle$	$\mathcal{O}(T)$

No. extra gates: $\mathcal{O}(T \operatorname{polylog}(T))$

Model

Quantum algorithm $\mathcal{A}$



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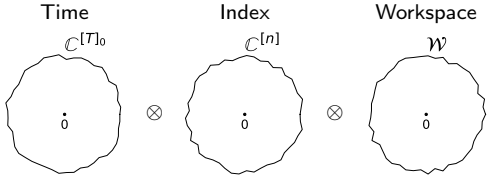

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No. extra gates: $\mathcal{O}(T \operatorname{polylog}(T))$

$\mathcal{A}$ allows for efficient uniform access if $\mathcal{O}_{\mathcal{A}}$ and $\mathcal{O}_{\mathcal{Q}}$ can be implemented with $\mathcal{O}(\operatorname{polylog}(T))$ gates.

Application: variable-time search problem

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Quantum algorithm $\mathcal{A}_1$

Quantum algorithm $\mathcal{A}_2$

$\vdots$

Quantum algorithm $\mathcal{A}_n$

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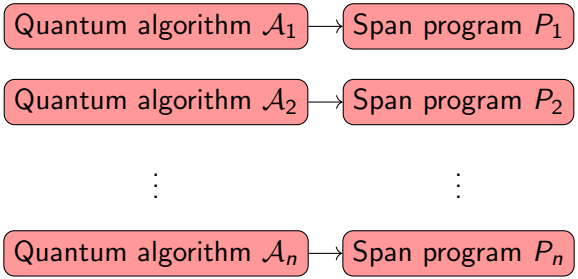
Quantum algorithm $\mathcal{A}_n$

Is there at least one algorithm that outputs 1?

Application: variable-time search problem



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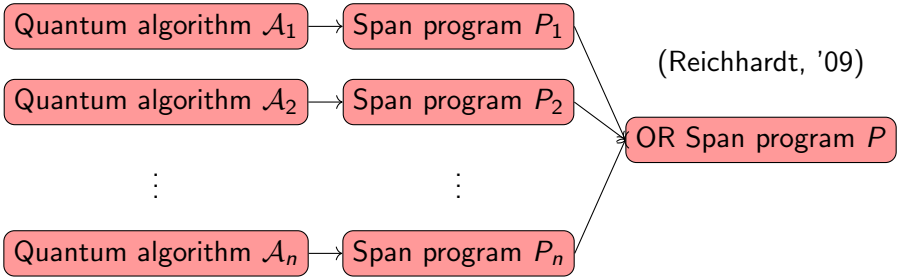

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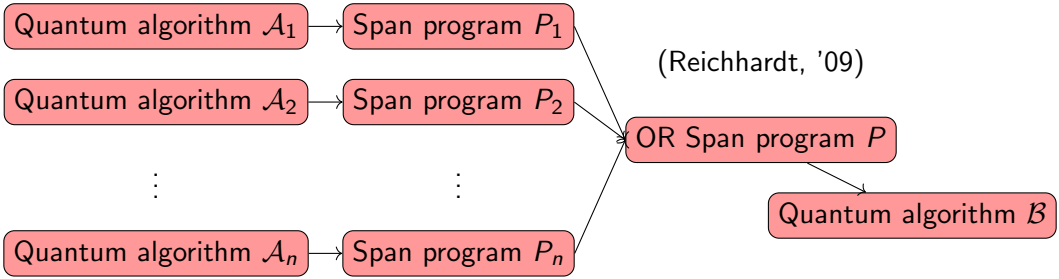

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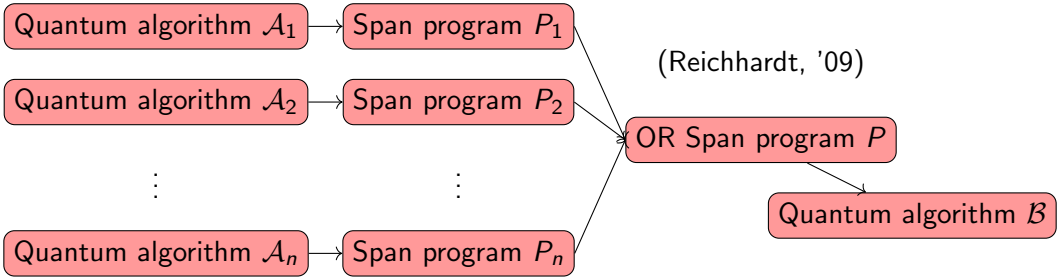

Is there at least one algorithm that outputs 1?

	No. queries to $\mathcal{O}_x$	No. queries to $\mathcal{O}_\mathcal{A}$ & $\mathcal{O}_\mathcal{Q}$	No. extra gates
Total	$\widetilde{\mathcal{O}}\left(\sqrt{\sum_{j=1}^n Q_j^2}\right)$	$\widetilde{\mathcal{O}}\left(\sqrt{\sum_{j=1}^n T_j^2}\right)$	$\widetilde{\mathcal{O}}\left(\sqrt{\sum_{j=1}^n T_j^2}\right)$

The end

Thanks for your attention!
Contact: arjan@cwi.nl