Quantum algorithms for multivariate mean estimation

A.J. Cornelissen[1]
Joint work with S. Jerbi[2] & Y. Hamoudi[3]

[1]QuSoft, University of Amsterdam
[2]Institute for Theoretical Physics, University of Innsbruck
[3]Department of Electrical Engineering and Computer Sciences, University of California, Berkeley

April 20th, 2022

QuSoft
Research Center for Quantum Software

SIMONS INSTITUTE for the Theory of Computing

universität innsbruck

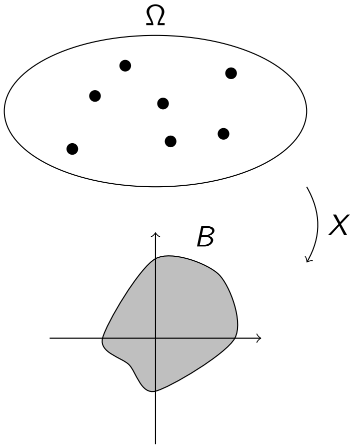
Problem statement

Problem statement

1. Given
   1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$
   2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$

Problem statement

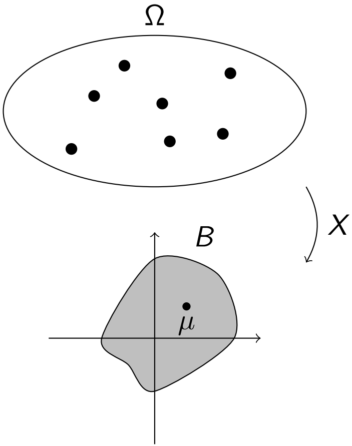
1. Given
   1. A probability space $(\Omega, 2^{\Omega}, \mathbb{P})$
   2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$



Problem statement

1. Given
   1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$
   2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$
2. *Mean:*

$$\mu = \mathbb{E}[X] = \sum_{\omega \in \Omega} \mathbb{P}(\omega) X(\omega) \in \mathbb{R}^d$$



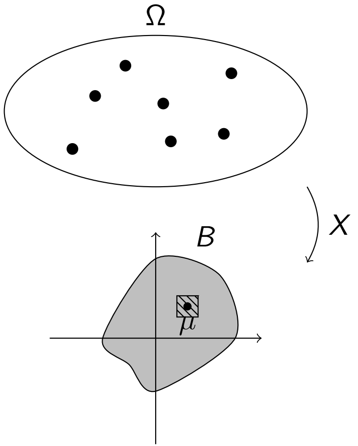
Problem statement

1. Given
   1. A probability space $(\Omega, 2^{\Omega}, \mathbb{P})$
   2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$
2. *Mean:*

$$\mu = \mathbb{E}[X] = \sum_{\omega \in \Omega} \mathbb{P}(\omega) X(\omega) \in \mathbb{R}^d$$

3. *Mean estimation problem:* find $\widetilde{\mu} \in \mathbb{R}^d$ s.t.

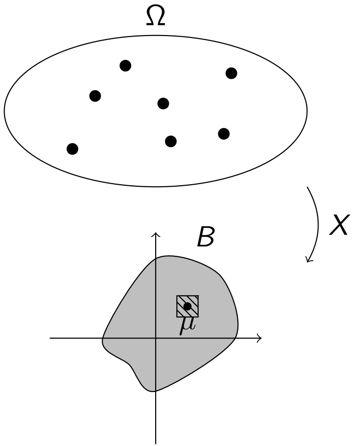
$$\|\widetilde{\mu} - \mu\|_{\infty} < \varepsilon$$



Problem statement

1. Given
   1. A probability space $(\Omega, 2^{\Omega}, \mathbb{P})$
   2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$
2. *Mean:*
$$\mu = \mathbb{E}[X] = \sum_{\omega \in \Omega} \mathbb{P}(\omega) X(\omega) \in \mathbb{R}^d$$
3. *Mean estimation problem:* find $\widetilde{\mu} \in \mathbb{R}^d$ s.t.
$$\|\widetilde{\mu} - \mu\|_{\infty} < \varepsilon$$
4. Variants:

	Classically	Quantumly
Univariate ($d = 1$)	Textbook	Textbook
Multivariate ($d > 1$)	Textbook	*Topic of this talk*



Access models

Access models

Given

1. A probability space $(\Omega, 2^{\Omega}, \mathbb{P})$,
2. A random variable
   $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Access models

Given

1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$,
2. A random variable
   $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Classical access model:

1. Obtain outcome $\omega \sim \mathbb{P}$.
2. Function $\omega \mapsto X(\omega)$.

Access models

Given

1. A probability space $(\Omega, 2^{\Omega}, \mathbb{P})$,
2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Classical access model:

1. Obtain outcome $\omega \sim \mathbb{P}$.
2. Function $\omega \mapsto X(\omega)$.

Quantum access model:

1. Distribution oracle:

$$|0\rangle \mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} \, |\omega\rangle .$$

Access models

Given

1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$,
2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Classical access model:

1. Obtain outcome $\omega \sim \mathbb{P}$.
2. Function $\omega \mapsto X(\omega)$.

Quantum access model:

1. Distribution oracle:
$$|0\rangle \mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} \, |\omega\rangle.$$
2. Random variable oracle:
$$|\omega\rangle |0\rangle \mapsto |\omega\rangle |X(\omega)\rangle.$$

Access models

Given

1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$,
2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Classical access model:

1. Obtain outcome $\omega \sim \mathbb{P}$.
2. Function $\omega \mapsto X(\omega)$.

Quantum access model:

1. Distribution oracle:
$$|0\rangle \mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle.$$
2. Random variable oracle:
$$|\omega\rangle|0\rangle \mapsto |\omega\rangle|X(\omega)\rangle.$$

Think of
$$|X(\omega)\rangle = |X(\omega)_1\rangle \otimes |X(\omega)_2\rangle \otimes \cdots \otimes |X(\omega)_d\rangle$$

Access models

Given

1. A probability space $(\Omega, 2^\Omega, \mathbb{P})$,
2. A random variable $X : \Omega \to B \subseteq \mathbb{R}^d$.

We want to approximate μ.

Classical access model:

1. Obtain outcome $\omega \sim \mathbb{P}$.
2. Function $\omega \mapsto X(\omega)$.

Quantum access model:

1. Distribution oracle:
$$|0\rangle \mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle.$$
2. Random variable oracle:
$$|\omega\rangle |0\rangle \mapsto |\omega\rangle |X(\omega)\rangle.$$

Think of
$$|X(\omega)\rangle = |X(\omega)_1\rangle \otimes |X(\omega)_2\rangle \otimes \cdots \otimes |X(\omega)_d\rangle$$

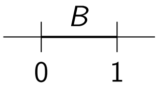
Calls to these routines are *samples*.

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.

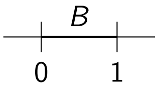
Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.

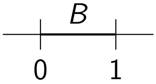


Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N} \sum_{j=1}^{N} X(\omega_j)$.

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



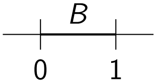
Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



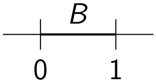
Classical algorithm:

1. Take samples $\omega_1, \ldots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



Classical algorithm:

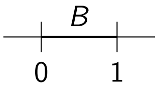
1. Take samples $\omega_1, \ldots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N} \sum_{j=1}^{N} X(\omega_j)$.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0,1]$.



Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

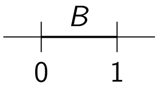
Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

$$U : |0\rangle |0\rangle \mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle$$

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



Classical algorithm:

1. Take samples $\omega_1, \ldots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N} \sum_{j=1}^{N} X(\omega_j)$.

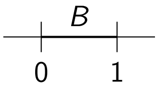
Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

$$U : |0\rangle |0\rangle \mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle$$
$$\mapsto \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle \left(\sqrt{X(\omega)} |1\rangle + \sqrt{1 - X(\omega)} |0\rangle \right)$$

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0,1]$.



Classical algorithm:

1. Take samples $\omega_1, \ldots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

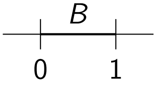
Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

$$
\begin{aligned}
U : |0\rangle\,|0\rangle &\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle\,|0\rangle \\
&\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle \left(\sqrt{X(\omega)}\,|1\rangle + \sqrt{1 - X(\omega)}\,|0\rangle\right) \\
&= \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)X(\omega)}\,|\omega\rangle\,|1\rangle + (\cdots)\,|0\rangle
\end{aligned}
$$

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0,1]$.



Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

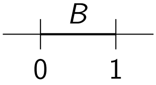
Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

$$\begin{aligned}
U : |0\rangle|0\rangle &\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle|0\rangle \\
&\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle \left(\sqrt{X(\omega)}\,|1\rangle + \sqrt{1-X(\omega)}\,|0\rangle\right) \\
&= \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)X(\omega)}\,|\omega\rangle|1\rangle + (\cdots)|0\rangle \\
&= \sqrt{\mathbb{E}[X]}\,|\psi_1\rangle|1\rangle + \sqrt{1-\mathbb{E}[X]}\,|\psi_0\rangle|0\rangle.
\end{aligned}$$

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0,1]$.



Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^N X(\omega_j)$.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

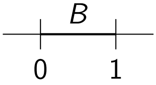
Quantum algorithm:

$$
\begin{aligned}
U : |0\rangle |0\rangle &\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle \\
&\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle \left(\sqrt{X(\omega)} |1\rangle + \sqrt{1 - X(\omega)} |0\rangle \right) \\
&= \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega) X(\omega)} |\omega\rangle |1\rangle + (\cdots) |0\rangle \\
&= \sqrt{\mathbb{E}[X]} |\psi_1\rangle |1\rangle + \sqrt{1 - \mathbb{E}[X]} |\psi_0\rangle |0\rangle.
\end{aligned}
$$

Amplitude estimation finds $\mathbb{E}[X] \pm \varepsilon$
with $N = \mathcal{O}(1/\varepsilon)$ calls to U.

Univariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$.
Let $X : \Omega \to [0, 1]$.



Classical algorithm:

1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j)$.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$

Quantum algorithm:

$$
\begin{aligned}
U : |0\rangle\,|0\rangle &\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle\,|0\rangle \\
&\mapsto \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}\,|\omega\rangle \left(\sqrt{X(\omega)}\,|1\rangle + \sqrt{1 - X(\omega)}\,|0\rangle \right) \\
&= \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)X(\omega)}\,|\omega\rangle\,|1\rangle + (\cdots)\,|0\rangle \\
&= \sqrt{\mathbb{E}[X]}\,|\psi_1\rangle\,|1\rangle + \sqrt{1 - \mathbb{E}[X]}\,|\psi_0\rangle\,|0\rangle.
\end{aligned}
$$

Amplitude estimation finds $\mathbb{E}[X] \pm \varepsilon$
with $N = \mathcal{O}(1/\varepsilon)$ calls to U.

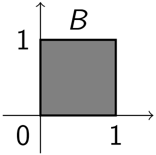
Quadratic quantum speed-up!

Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.

Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.
Let $X : \Omega \to [0, 1]^d$.

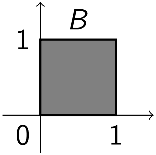


Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.
Let $X : \Omega \to [0,1]^d$.

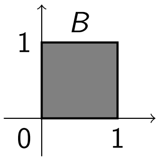
Classical algorithm:

1. Run 1D-algorithm with N samples:
   1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
   2. Compute $\frac{1}{N} \sum_{j=1}^{N} X(\omega_j) \in [0,1]^d$.
2. *Median trick:* Repeat $\mathcal{O}(\log(d))$ times and take the coordinate-wise median.



Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.
Let $X : \Omega \to [0,1]^d$.



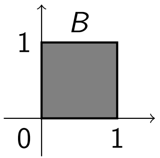
Classical algorithm:

1. Run 1D-algorithm with N samples:
   1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
   2. Compute $\frac{1}{N}\sum_{j=1}^N X(\omega_j) \in [0,1]^d$.
2. *Median trick:* Repeat $\mathcal{O}(\log(d))$ times and take the coordinate-wise median.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ in ℓ_∞-norm w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$.
Total number of samples: $\mathcal{O}(\log(d)/\varepsilon^2)$.

Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.
Let $X : \Omega \to [0,1]^d$.



Classical algorithm:

1. Run 1D-algorithm with N samples:
   1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
   2. Compute $\frac{1}{N}\sum_{j=1}^{N} X(\omega_j) \in [0,1]^d$.
2. *Median trick:* Repeat $\mathcal{O}(\log(d))$ times and take the coordinate-wise median.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ in ℓ_∞-norm w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$.
Total number of samples: $\mathcal{O}(\log(d)/\varepsilon^2)$.

Quantum algorithm:

$$U : |0\rangle|0\rangle \mapsto \sqrt{\mathbb{E}[X]}\,|\psi_1\rangle|1\rangle + \sqrt{1-\mathbb{E}[X]}\,|\psi_0\rangle|0\rangle.$$

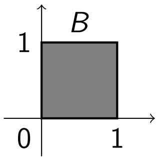
Multivariate mean estimation

Problem: Compute $\mathbb{E}[X] \pm \varepsilon$ in ℓ_∞-norm.
Let $X : \Omega \to [0,1]^d$.

Classical algorithm:

1. Run 1D-algorithm with N samples:
   1. Take samples $\omega_1, \dots, \omega_N \in \Omega$.
   2. Compute $\frac{1}{N}\sum_{j=1}^N X(\omega_j) \in [0,1]^d$.
2. *Median trick:* Repeat $\mathcal{O}(\log(d))$ times and take the coordinate-wise median.

Computes $\mathbb{E}[X] \pm 1/\sqrt{N}$ in ℓ_∞-norm w.h.p.
$\Rightarrow N = \mathcal{O}(1/\varepsilon^2)$.
Total number of samples: $\mathcal{O}(\log(d)/\varepsilon^2)$.



Quantum algorithm:

$U : |0\rangle|0\rangle \mapsto$
$\sqrt{\mathbb{E}[X]}\,|\psi_1\rangle|1\rangle + \sqrt{1 - \mathbb{E}[X]}\,|\psi_0\rangle|0\rangle$.

Roadblock: It is not clear how to represent $\mathbb{E}[X] \in \mathbb{R}^d$ as an amplitude.

Intermezzo: Quantum oracle conversions [GAW18]

Let D a finite set, $f : D \to [0, 1]$, and $x \in D$.

Intermezzo: Quantum oracle conversions [GAW18]

Let D a finite set, $f : D \to [0,1]$, and $x \in D$.

1. *Binary oracle:* $B_{f,\varepsilon} : |x\rangle|0\rangle \mapsto |x\rangle\left|\widetilde{f}(x)\right\rangle$, where $|f(x) - \widetilde{f}(x)| < \varepsilon$.

Intermezzo: Quantum oracle conversions [GAW18]

Let D a finite set, $f : D \to [0,1]$, and $x \in D$.

1. *Binary oracle:* $B_{f,\varepsilon} : |x\rangle |0\rangle \mapsto |x\rangle \left|\widetilde{f}(x)\right\rangle$, where $|f(x) - \widetilde{f}(x)| < \varepsilon$.
2. *Probability oracle:* $U_f : |x\rangle |0\rangle |0\rangle \mapsto |x\rangle \left(\sqrt{f(x)} |\psi_1\rangle |1\rangle + \sqrt{1 - f(x)} |\psi_0\rangle |0\rangle\right)$.

Intermezzo: Quantum oracle conversions [GAW18]

Let D a finite set, $f : D \to [0,1]$, and $x \in D$.

1. *Binary oracle:* $B_{f,\varepsilon} : |x\rangle|0\rangle \mapsto |x\rangle\left|\widetilde{f}(x)\right\rangle$, where $|f(x) - \widetilde{f}(x)| < \varepsilon$.
2. *Probability oracle:* $U_f : |x\rangle|0\rangle|0\rangle \mapsto |x\rangle\left(\sqrt{f(x)}|\psi_1\rangle|1\rangle + \sqrt{1-f(x)}|\psi_0\rangle|0\rangle\right)$.
3. *Phase oracle:* $O_f : |x\rangle \mapsto e^{if(x)}|x\rangle$.

Intermezzo: Quantum oracle conversions [GAW18]

Let D a finite set, $f : D \to [0,1]$, and $x \in D$.

1. *Binary oracle:* $B_{f,\varepsilon} : |x\rangle|0\rangle \mapsto |x\rangle\left|\widetilde{f}(x)\right\rangle$, where $|f(x) - \widetilde{f}(x)| < \varepsilon$.
2. *Probability oracle:* $U_f : |x\rangle|0\rangle|0\rangle \mapsto |x\rangle\left(\sqrt{f(x)}|\psi_1\rangle|1\rangle + \sqrt{1-f(x)}|\psi_0\rangle|0\rangle\right)$.
3. *Phase oracle:* $O_f : |x\rangle \mapsto e^{if(x)}|x\rangle$.

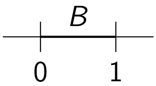
Oracle conversion graph:

$$\begin{tikzcd}
 & B_{f,\varepsilon} & \\
O_f \arrow[ur, "\text{Phase estimation} : \mathcal{O}(\frac{1}{\varepsilon})"] & & U_f \arrow[ul, "\mathcal{O}(\frac{1}{\varepsilon}) : \text{Amplitude estimation}"'] \arrow[ll, "\widetilde{\mathcal{O}}(1)"', "\text{[GAW18]}"]
\end{tikzcd}$$

New univariate mean estimation I

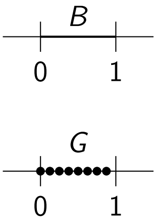
New univariate mean estimation I

1. Let $X : \Omega \to [0, 1]$.



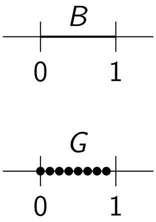
New univariate mean estimation I

1. Let $X : \Omega \to [0, 1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.



New univariate mean estimation I

1. Let $X : \Omega \to [0, 1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.
3. Let $f : G \to [0, 1]$, $f(g) = g\mathbb{E}[X]$.



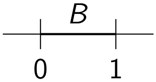
New univariate mean estimation I

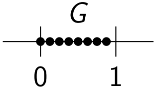
1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. Probability oracle to f:

$$U_f : |g\rangle |0\rangle |0\rangle \mapsto |g\rangle \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle$$

$$\mapsto |g\rangle \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle \left(\sqrt{gX(\omega)} |1\rangle + \sqrt{1 - gX(\omega)} |0\rangle\right)$$

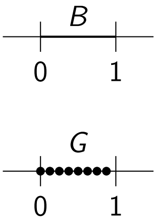
$$= |g\rangle \left(\sqrt{g\mathbb{E}[X]} |\psi_1\rangle |1\rangle + \sqrt{1 - g\mathbb{E}[X]} |\psi_0\rangle |0\rangle\right).$$





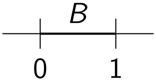
New univariate mean estimation I

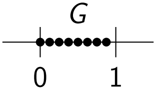
1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. Probability oracle to f:
$$U_f : |g\rangle|0\rangle|0\rangle \mapsto |g\rangle \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}|\omega\rangle|0\rangle$$
$$\mapsto |g\rangle \sum_{\omega\in\Omega} \sqrt{\mathbb{P}(\omega)}|\omega\rangle \left(\sqrt{gX(\omega)}|1\rangle + \sqrt{1 - gX(\omega)}|0\rangle\right)$$
$$= |g\rangle \left(\sqrt{g\mathbb{E}[X]}|\psi_1\rangle|1\rangle + \sqrt{1 - g\mathbb{E}[X]}|\psi_0\rangle|0\rangle\right).$$
5. Can be turned into phase oracle:
$$O_f : |g\rangle \mapsto e^{ig\mathbb{E}[X]}|g\rangle$$
with $\widetilde{O}(1)$ calls to U_f.



New univariate mean estimation II

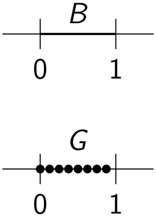
1. Let $X : \Omega \to [0, 1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}$.
3. Let $f : G \to [0, 1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.





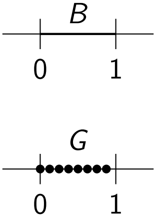
New univariate mean estimation II

1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0, \ldots, 2^n - 1\}$:
$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$



New univariate mean estimation II

1. Let $X : \Omega \to [0, 1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.
3. Let $f : G \to [0, 1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0, \ldots, 2^n - 1\}$:
$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$

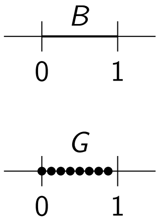


Algorithm:

New univariate mean estimation II

1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0,\dots,2^n-1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0,\dots,2^n-1\}$:

$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$



Algorithm:

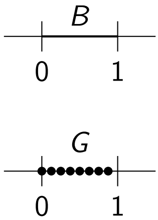
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$$

New univariate mean estimation II

1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0,\dots,2^n-1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0,\dots,2^n-1\}$:
$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$

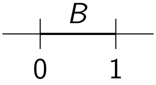
Algorithm:

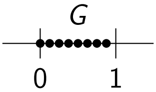
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg\mathbb{E}[X]} |g\rangle$$



New univariate mean estimation II

1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0, \ldots, 2^n - 1\}$:
$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$



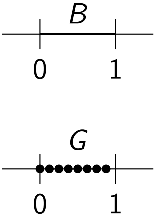


Algorithm:

$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg\mathbb{E}[X]} |g\rangle$$
$$\overset{\mathrm{QFT}^\dagger}{\rightsquigarrow} \left|\mathrm{round}\left(\frac{\mathbb{E}[X]N}{2\pi}\right)\right\rangle.$$

New univariate mean estimation II

1. Let $X : \Omega \to [0,1]$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \ldots, 2^n - 1\}\}$.
3. Let $f : G \to [0,1]$, $f(g) = g\mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $k \in \{0, \ldots, 2^n - 1\}$:
$$|k\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i g k} |g\rangle.$$



Algorithm:

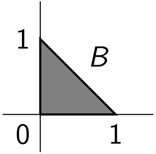
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg\mathbb{E}[X]} |g\rangle$$
$$\overset{\mathrm{QFT}^\dagger}{\rightsquigarrow} \left|\mathrm{round}\left(\frac{\mathbb{E}[X]N}{2\pi}\right)\right\rangle.$$

$\Rightarrow N = \mathcal{O}(1/\varepsilon)$.

Multivariate mean estimation I

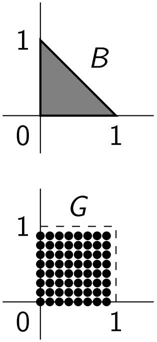
Multivariate mean estimation I

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.



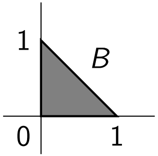
Multivariate mean estimation I

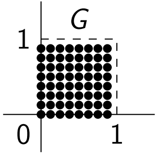
1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.



Multivariate mean estimation I

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0, 1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.





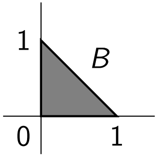
Multivariate mean estimation I

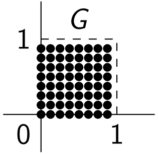
1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. Probability oracle to f:

$$U_f : |\vec{g}\rangle |0\rangle |0\rangle \mapsto |\vec{g}\rangle \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle$$

$$\mapsto |\vec{g}\rangle \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle \left(\sqrt{\vec{g}^T X(\omega)} |1\rangle + \sqrt{1 - \vec{g}^T X(\omega)} |0\rangle \right)$$

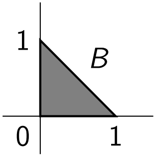
$$= |\vec{g}\rangle \left(\sqrt{\vec{g}^T \mathbb{E}[X]} |\psi_1\rangle |1\rangle + \sqrt{1 - \vec{g}^T \mathbb{E}[X]} |\psi_0\rangle |0\rangle \right).$$

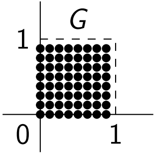




Multivariate mean estimation I

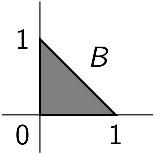
1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. Probability oracle to f:
$$U_f : |\vec{g}\rangle |0\rangle |0\rangle \mapsto |\vec{g}\rangle \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle |0\rangle$$
$$\mapsto |\vec{g}\rangle \sum_{\omega \in \Omega} \sqrt{\mathbb{P}(\omega)} |\omega\rangle \left(\sqrt{\vec{g}^T X(\omega)} |1\rangle + \sqrt{1 - \vec{g}^T X(\omega)} |0\rangle \right)$$
$$= |\vec{g}\rangle \left(\sqrt{\vec{g}^T \mathbb{E}[X]} |\psi_1\rangle |1\rangle + \sqrt{1 - \vec{g}^T \mathbb{E}[X]} |\psi_0\rangle |0\rangle \right).$$
5. Can be turned into phase oracle:
$$O_f : |\vec{g}\rangle \mapsto e^{i\vec{g}^T \mathbb{E}[X]} |\vec{g}\rangle$$
with $\widetilde{O}(1)$ calls to U_f.

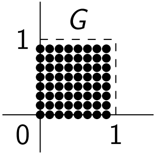




Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.

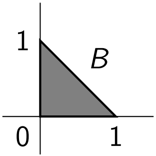


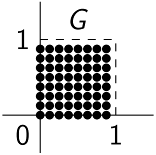


Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0, 1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:

$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} \left|\vec{g}\right\rangle.$$

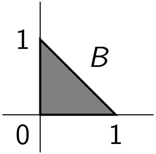


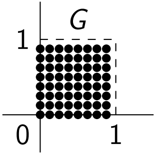


Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0, 1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:
$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} \left|\vec{g}\right\rangle.$$

Algorithm:

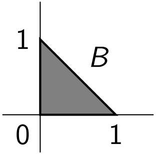


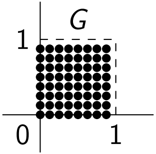


Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:

$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} \left|\vec{g}\right\rangle.$$



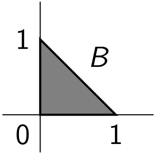


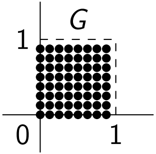
Algorithm:

$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$$

Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0, 1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:
$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} |\vec{g}\rangle.$$





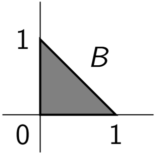
Algorithm:

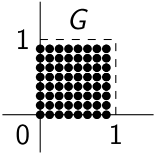
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg^T \mathbb{E}[X]} |g\rangle$$

Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:

$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} \left|\vec{g}\right\rangle.$$





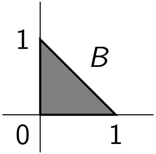
Algorithm:

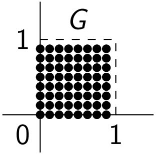
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg^T \mathbb{E}[X]} |g\rangle$$
$$\overset{\mathrm{QFT}^\dagger}{\rightsquigarrow} \left|\mathrm{round}\left(\frac{N\mathbb{E}[X]}{2\pi}\right)\right\rangle.$$

Multivariate mean estimation II

1. Let $X : \Omega \to \{\vec{x} \in \mathbb{R}^d : \vec{x} \geq 0, \|\vec{x}\|_1 \leq 1\} \subseteq \mathbb{R}^d$.
2. Let $G = \{\frac{j}{2^n} : j \in \{0, \dots, 2^n - 1\}\}^d$.
3. Let $f : G \to [0,1]$, $f(\vec{g}) = \vec{g}^T \mathbb{E}[X]$.
4. We can implement O_f with $\widetilde{O}(1)$ overhead.
5. Recall quantum Fourier transform, for $\vec{k} \in \{0, \dots, 2^n - 1\}^d$:

$$\left|\vec{k}\right\rangle \mapsto \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{2\pi i \vec{g}^T \vec{k}} \left|\vec{g}\right\rangle.$$





Algorithm:

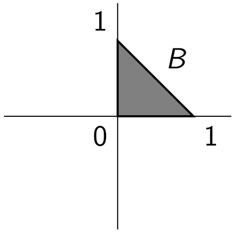
$$\frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle \overset{O_f^N}{\mapsto} \frac{1}{\sqrt{|G|}} \sum_{g \in G} e^{iNg^T \mathbb{E}[X]} |g\rangle$$

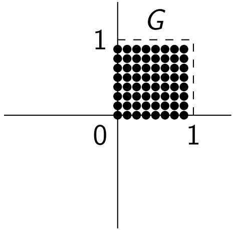
$$\overset{\mathrm{QFT}^\dagger}{\rightsquigarrow} \left|\mathrm{round}\left(\frac{N\mathbb{E}[X]}{2\pi}\right)\right\rangle.$$

$\Rightarrow N = \mathcal{O}(1/\varepsilon)$.

Improvements

Improvements on the algorithm [CJ21,CHJ21]:

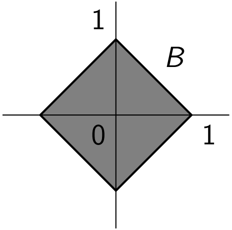


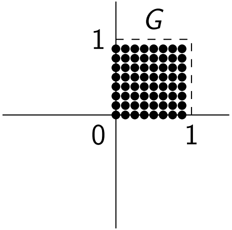


Improvements

Improvements on the algorithm [CJ21,CHJ21]:

1. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_1 \leq 1\}$.
   Core idea: $O_f^\dagger : |x\rangle \mapsto e^{-if(x)} |x\rangle$.

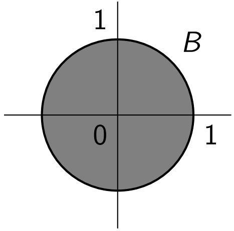


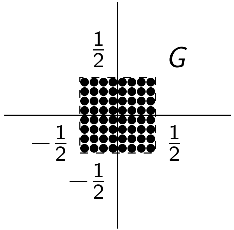


Improvements

Improvements on the algorithm [CJ21,CHJ21]:

1. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_1 \leq 1\}$.
   Core idea: $O_f^\dagger : |x\rangle \mapsto e^{-if(x)} |x\rangle$.
2. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_2 \leq 1\}$.
   Core idea: $\mathop{\mathbb{E}}_{\vec{g}\sim G}\left[\left|\vec{g}^T\vec{v}\right|\right] = \mathcal{O}(\|\vec{v}\|_2)$ for a centered grid.

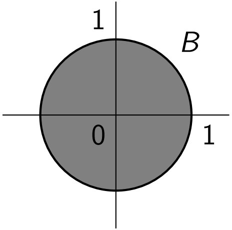


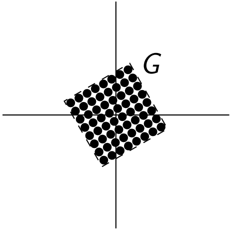


Improvements

Improvements on the algorithm [CJ21,CHJ21]:

1. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_1 \leq 1\}$.
   Core idea: $O_f^\dagger : |x\rangle \mapsto e^{-if(x)} |x\rangle$.
2. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_2 \leq 1\}$.
   Core idea: $\mathop{\mathbb{E}}_{\vec{g}\sim G}\left[\left|\vec{g}^T\vec{v}\right|\right] = \mathcal{O}(\|\vec{v}\|_2)$ for a centered grid.
3. We can compute $O\mathbb{E}[X]$, for some $O^TO = OO^T = I$.
   Core idea: Rotate grid by O^T.

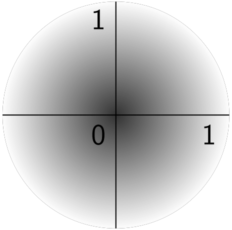


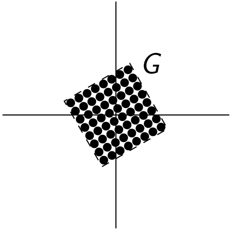


Improvements

Improvements on the algorithm [CJ21,CHJ21]:

1. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_1 \leq 1\}$.
   Core idea: $O_f^\dagger : |x\rangle \mapsto e^{-if(x)} |x\rangle$.
2. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_2 \leq 1\}$.
   Core idea: $\mathop{\mathbb{E}}_{\vec{g}\sim G}\left[\left|\vec{g}^T\vec{v}\right|\right] = \mathcal{O}(\|\vec{v}\|_2)$ for a centered grid.
3. We can compute $O\mathbb{E}[X]$, for some $O^TO = OO^T = I$.
   Core idea: Rotate grid by O^T.
4. We can relax to $\mathrm{Tr}[\mathrm{Cov}(X)] \leq 1$. *[QIP talk]*
   Core idea: Find quantiles and reduce to bounded case.



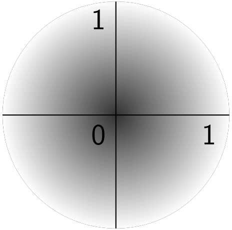


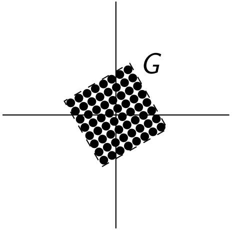
Improvements

Improvements on the algorithm [CJ21,CHJ21]:

1. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_1 \leq 1\}$.
   Core idea: $O_f^\dagger : |x\rangle \mapsto e^{-if(x)}|x\rangle$.
2. We can set $B = \{\vec{x} \in \mathbb{R}^d : \|\vec{x}\|_2 \leq 1\}$.
   Core idea: $\mathbb{E}_{\vec{g}\sim G}\left[\left|\vec{g}^T\vec{v}\right|\right] = \mathcal{O}(\|\vec{v}\|_2)$ for a centered grid.
3. We can compute $O\mathbb{E}[X]$, for some $O^TO = OO^T = I$.
   Core idea: Rotate grid by O^T.
4. We can relax to $\mathrm{Tr}[\mathrm{Cov}(X)] \leq 1$. *[QIP talk]*
   Core idea: Find quantiles and reduce to bounded case.

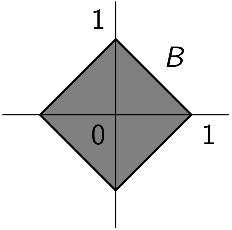
All improvements retain $\widetilde{O}(1/\varepsilon)$.





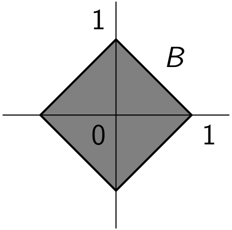
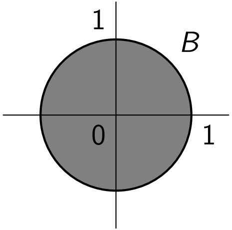
Limits

Limits

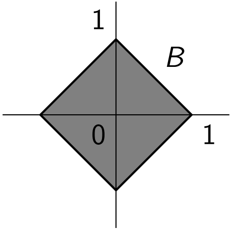
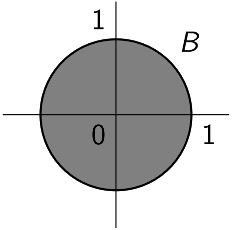
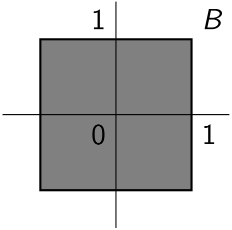


Classically	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	
Quantumly	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	

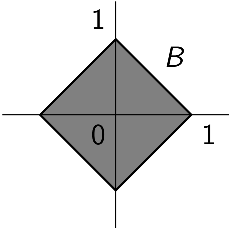
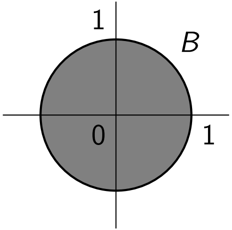
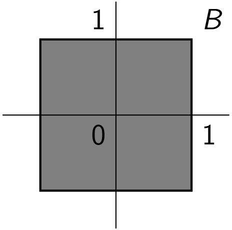
Limits

		
Classically	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$
Quantumly	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$

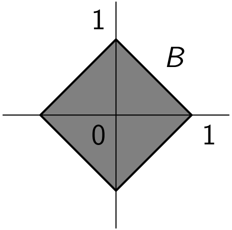
Limits

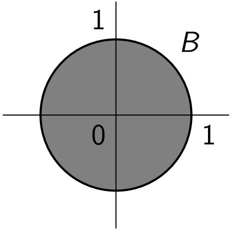
			
Classically	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$
Quantumly	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	

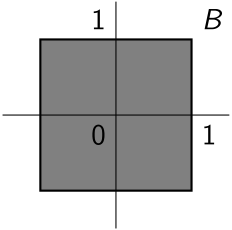
Limits

			
Classically	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$
Quantumly	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right)$

Limits







Classically	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon^2}\right)$
Quantumly	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\frac{1}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right)$

For ℓ_p-approximations: multiply by $d^{\frac{1}{p}}$.

Summary & Outlook

Summary:

Summary & Outlook

Summary:

1. Oracle conversion techniques.

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

1. Log factors?

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

1. Log factors?
2. Other ℓ_p-norms in the bounded-variance case?

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

1. Log factors?
2. Other ℓ_p-norms in the bounded-variance case?
3. (Coordinate-wise) multiplicative error case?

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

1. Log factors?
2. Other ℓ_p-norms in the bounded-variance case?
3. (Coordinate-wise) multiplicative error case?
4. Unbiased estimators?

Summary & Outlook

Summary:

1. Oracle conversion techniques.
2. Multivariate mean estimation algorithm.
3. Overview of improvements.
4. Limits.

Possible further research directions:

1. Log factors?
2. Other ℓ_p-norms in the bounded-variance case?
3. (Coordinate-wise) multiplicative error case?
4. Unbiased estimators?

Thanks for your attention!

`arjan@cwi.nl`