Quantum tomography using state-preparation unitaries

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Joran van Apeldoorn[1,2], Arjan Cornelissen[1,4], András Gilyén[3], Giacomo Nannicini[4]

[1]QuSoft, University of Amsterdam, the Netherlands
[2]IViR, University of Amsterdam, the Nethelands
[3]Alfréd Rényi Institute of Mathematics, Budapest, Hungary
[4]IBM Quantum, IBM T.J. Watson Research Center, Yorktown Heights, NY, USA

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Research Center for Quantum Software

Institutuut voor Informatierecht
Institute for Information Law

Quantum state tomography (1/2) – pure states

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Model	Output	
$|\psi\rangle$ j	$[|\alpha_j|]_{j=1}^{d}$	
$|\psi\rangle$	$e^{i\chi}|\psi\rangle$	
$\frac{|0\rangle|0\rangle+|1\rangle|\psi\rangle}{\sqrt{2}}$	$|\psi\rangle$	

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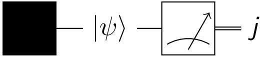
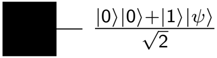
Model	Output	Approximation (ℓ_q-norms)		
		$\lVert\cdot\rVert_\infty \leq \varepsilon$	$\lVert\cdot\rVert_2 \leq \varepsilon$	$\lVert\cdot\rVert_q \leq \varepsilon$, $q \in [2, \infty]$
	$[\lvert\alpha_j\rvert]_{j=1}^{d}$			
	$e^{i\chi}\lvert\psi\rangle$	$\widetilde{\mathcal{O}}\left(\frac{1}{\varepsilon^2}\right)$ [KP20] $\Omega\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\mathcal{O}}\left(\frac{d}{\varepsilon^2}\right)$ [KP20] $\Omega\left(\frac{d}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^{\frac{1}{\frac{1}{2}-\frac{1}{q}}}}, \frac{d^{\frac{2}{q}}}{\varepsilon^2}\right\}\right)$
	$\lvert\psi\rangle$			

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$|\psi\rangle$ — j	$[\lvert\alpha_j\rvert]_{j=1}^{d}$			
$|\psi\rangle$	$e^{i\chi}|\psi\rangle$	$\widetilde{\mathcal{O}}\left(\frac{1}{\varepsilon^2}\right)$ [KP20] $\Omega\left(\frac{1}{\varepsilon^2}\right)$	$\widetilde{\mathcal{O}}\left(\frac{d}{\varepsilon^2}\right)$ [KP20] $\Omega\left(\frac{d}{\varepsilon^2}\right)$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^{\frac{1}{\frac{1}{2}-\frac{1}{q}}}}, \frac{d^{\frac{2}{q}}}{\varepsilon^2}\right\}\right)$
$\frac{|0\rangle|0\rangle+|1\rangle|\psi\rangle}{\sqrt{2}}$	$|\psi\rangle$			
$|0\rangle$ —U— $|\psi\rangle$, $|\psi\rangle$ —$U^\dagger$— $|0\rangle$	$|\psi\rangle$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right)$	$\widetilde{\Theta}\left(\frac{d}{\varepsilon}\right)$	$\widetilde{\Theta}\left(\min\left\{\frac{1}{\varepsilon^{\frac{1}{\frac{1}{2}-\frac{1}{q}}}}, \frac{d^{\frac{1}{2}+\frac{1}{q}}}{\varepsilon}\right\}\right)$

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$\rho = \sum_{j=1}^{r} p_j \left|\psi_j\middle\rangle\!\middle\langle\psi_j\right| \in \mathbb{C}^{d \times d}$

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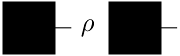
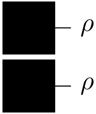
	ρ	–	–	$\mathcal{O}\left(\frac{dr^2}{\varepsilon^2}\right)$ [GLF+10] $\Omega\left(\frac{dr^2}{\varepsilon^2}\right)$ [HHJ+17; CHL+22]	–
	ρ	–	–	$\widetilde{\mathcal{O}}\left(\frac{dr}{\varepsilon^2}\right)$ [OW16; HHJ+17] $\Omega\left(\frac{dr}{\varepsilon^2}\right)$ [HHJ+17; Yue22]	–

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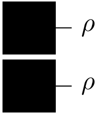
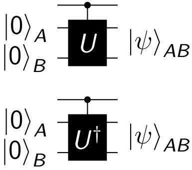
	ρ	–	–	$\widetilde{\mathcal{O}}\left(\frac{dr}{\varepsilon^2}\right)$ [OW16; HHJ+17] $\Omega\left(\frac{dr}{\varepsilon^2}\right)$ [HHJ+17; Yue22]	–
	$\mathrm{Tr}_B[\|\psi\rangle\langle\psi\|]$	$\widetilde{\Theta}(\frac{d}{\varepsilon})$	$\widetilde{\Theta}\left(\min\left\{\frac{d\sqrt{r}}{\varepsilon}, \frac{d}{\varepsilon^2}\right\}\right)$	$\widetilde{\Theta}(\frac{dr}{\varepsilon})$	$\widetilde{\Theta}\left(\min\left\{\frac{dr^{\frac{1}{q}}}{\varepsilon}, \frac{d}{\varepsilon^{\frac{1}{1-\frac{1}{q}}}}\right\}\right)$

Techniques (1/3) – learning observables

$O_1, \dots, O_M$ observables, with $\|O_j\| \leq 1$

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 $\|\psi\rangle$	$[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$	$\mathcal{O}\left(\frac{\log(M)}{\varepsilon^2}\right)$	If the observables commute
		$\mathcal{O}\left(\frac{\log(M)}{\varepsilon^2}\right)$	Shadow tomography [HKP20]

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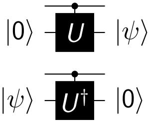	$[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$	$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\sum_{j=1}^M \|O_j\|^2}}{\varepsilon}\right)$	[HWC+22]
		$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\left\|\sum_{j=1}^M O_j^2\right\|}}{\varepsilon}\right)$	$f(\mathbf{x}) = \mathbf{x}^T[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$ Compute $\nabla_{\mathbf{x}} f$

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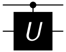
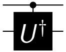
	$[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$	$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\sum_{j=1}^M \|O_j\|^2}}{\varepsilon}\right)$	[HWC+22]
		$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\left\|\sum_{j=1}^M O_j^2\right\|}}{\varepsilon}\right)$	$f(\mathbf{x}) = \mathbf{x}^T[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$ Compute $\nabla_{\mathbf{x}} f$

Density matrix:

$$\rho = \begin{array}{cc} i & \begin{bmatrix} & \vert & \\ - & \rho_{ij} & — \\ & \vert & \end{bmatrix} \\ & j \end{array}$$

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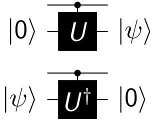	$[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$	$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\sum_{j=1}^M \|O_j\|^2}}{\varepsilon}\right)$	[HWC+22]
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Observables:

$$O_{ij}^+ = \frac{|i\rangle\langle j| + |j\rangle\langle i|}{2}$$
$$O_{ij}^- = \frac{|i\rangle\langle j| - |j\rangle\langle i|}{2i}$$
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$\blacksquare - \lvert\psi\rangle$	$[\langle\psi\rvert O_j \lvert\psi\rangle]_{j=1}^M$	$\mathcal{O}\left(\frac{\log(M)}{\varepsilon^2}\right)$	If the observables commute
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$\lvert 0\rangle - U - \lvert\psi\rangle$; $\lvert\psi\rangle - U^\dagger - \lvert 0\rangle$	$[\langle\psi\rvert O_j \lvert\psi\rangle]_{j=1}^M$	$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\sum_{j=1}^M \|O_j\|^2}}{\varepsilon}\right)$	[HWC+22]
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Norm bound:

$$\sum_{\substack{i \leq j \\ i,j=1}}^d (O_{ij}^+)^2 + \sum_{\substack{i<j \\ i,j=1}}^d (O_{ij}^-)^2 = dI_d$$

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$\|\psi\rangle - U^\dagger - \|0\rangle$		$\widetilde{\mathcal{O}}\left(\frac{\sqrt{\left\|\sum_{j=1}^M O_j^2\right\|}}{\varepsilon}\right)$	$f(\mathbf{x}) = \mathbf{x}^T[\langle\psi\| O_j \|\psi\rangle]_{j=1}^M$ Compute $\nabla_{\mathbf{x}} f$

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Result:

$\|\widetilde{\rho} - \rho\|_{\max} \leq \varepsilon$ costs $\widetilde{\mathcal{O}}(\frac{\sqrt{d}}{\varepsilon})$ queries.

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- Given a copy of $|\psi\rangle$, and U s.t.
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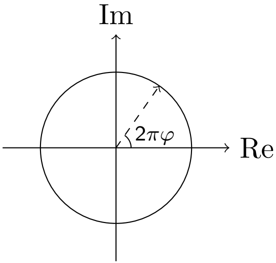

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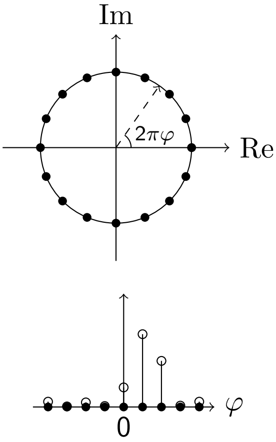

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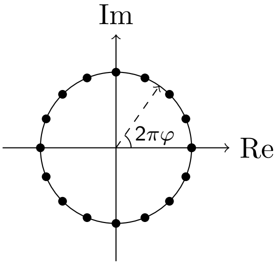
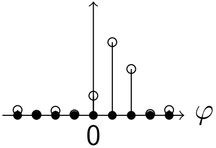
  - Correct for choice of θ.





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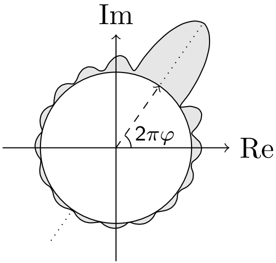

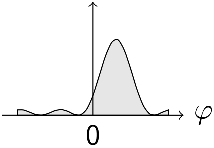

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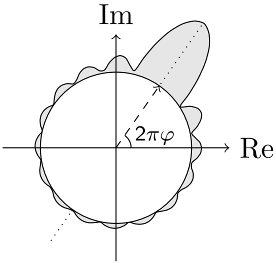
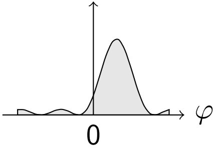
- Boosting techniques directly on the circle.





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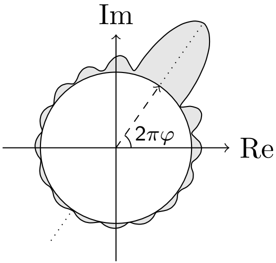

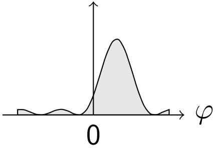

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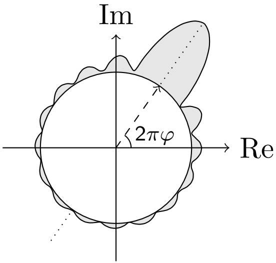

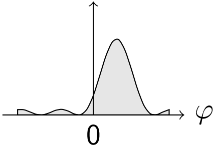

Result:

$$\|\widetilde{\rho} - \rho\|_{\max} \leq \varepsilon \quad \overset{\text{[RV10]}}{\Rightarrow} \quad \|\widetilde{\rho} - \rho\|_{\infty} \leq \sqrt{d}\varepsilon$$

Techniques (3/3) – norm conversion

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Setting:

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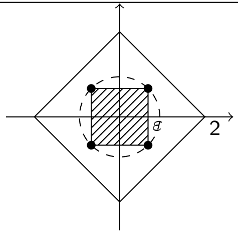
Low-precision regime (ε big)	*High-precision regime* (ε small)
	
	$\|\lambda\|_2 \leq \sqrt{2r} \cdot \varepsilon$

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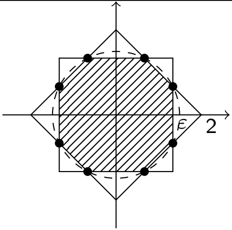
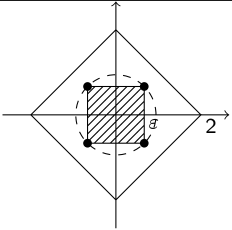
Low-precision regime (ε big)	*High-precision regime* (ε small)
	
$\|\lambda\|_2 \leq \sqrt{2\varepsilon}$	$\|\lambda\|_2 \leq \sqrt{2r} \cdot \varepsilon$

Techniques (3/3) – norm conversion

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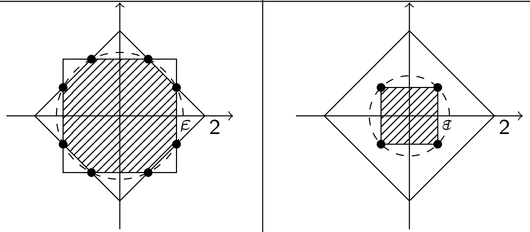

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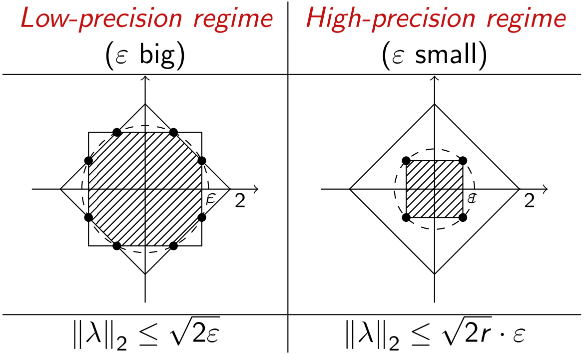

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- Source of all regimes.
- Also relevant for lower bound proofs.

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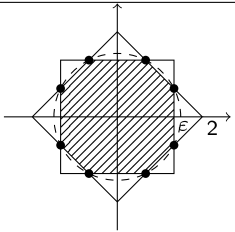	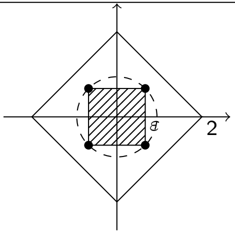
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Quantum state tomography – mixed states – algorithm analysis

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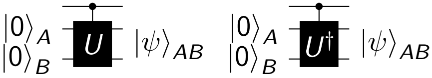
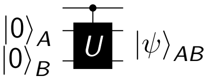
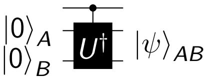


Want to learn: $\rho = \mathrm{Tr}_B[|\psi\rangle\langle\psi|]$

Quantum state tomography – mixed states – algorithm analysis





$\Downarrow$ *Learning observables*

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$\widetilde{\mathcal{O}}(\frac{\sqrt{d}}{\varepsilon'})$ queries

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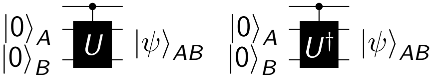

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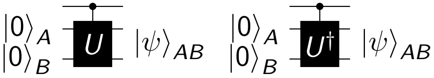

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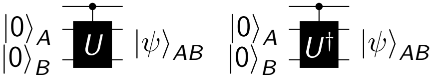

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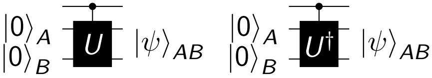

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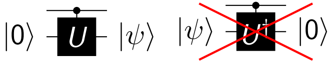

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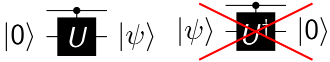

Thanks for your attention!

`arjan@cwi.nl`