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Problem 1

Problem Statement: How many ways can 11 cards be chosen from a standard 52-card deck if exactly 3 must be Kings and 3 must be Queens?

Given:

- Total cards = 52
- Kings = 4, Queens = 4
- Choose 11 cards: exactly 3 Kings, 3 Queens, and 5 others
- Order does not matter

Solution:

- Choose 3 Kings from 4: $C(4,3) = \frac{4!}{3!(4-3)!} = \frac{4}{1} = 4$
- Choose 3 Queens from 4: $C(4,3) = \frac{4!}{3!(4-3)!} = \frac{4}{1} = 4$
- Remaining cards needed = $11 - (3 + 3) = 5$
- Remaining cards available = $52 - 4 \text{ Kings} - 4 \text{ Queens} = 44$
- Choose 5 cards from 44: $C(44,5) = \frac{44 \cdot 43 \cdot 42 \cdot 41 \cdot 40}{(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = 1086008$
- Total ways = $4 \cdot 4 \cdot 1086008 = 16 \cdot 1086008 = 17376128$

Final Answer: 17,376,128 ways

Problem 2

Problem Statement: A student must select 5 subjects from 7, with 1 compulsory subject. How many ways can they do this?

Given:

- Total subjects = 7
- 1 subject is compulsory
- Choose 5 subjects total
- Order does not matter

Solution:

- Compulsory subject is included, so choose 4 more subjects
- Remaining subjects = $7 - 1 = 6$
- Use combination: $C(6,4) = \frac{6!}{4!(6-4)!} = \frac{(6 \cdot 5)}{(2 \cdot 1)} = \frac{30}{2} = 15$
- Total ways = 15

Final Answer: 15 ways

Problem 3

Problem Statement: In a swim meet with 27 participants and no ties, how many ways can gold, silver, and bronze medals be awarded?

Given:

- Total participants = 27
- Award 3 medals (gold, silver, bronze)
- Order matters

Solution:

- Use permutation: $P(27,3) = 27!/(27-3)! = 27*26*25 = 17550$

Final Answer: 17550 ways

Problem 4

Problem Statement: How many different 3-card hands can be chosen from a standard 52-card deck?

Given:

- Total cards = 52
- Choose 3 cards
- Order does not matter

Solution:

- Use combination: $C(52,3) = 52!/(3!(52-3)!) = (52*51*50)/(3*2*1) = 132600/6 = 22100$

Final Answer: 22100 hands

Problem 5

Problem Statement: How many ways can 8 cards be chosen from a standard 52-card deck if all must be from the same suit?

Given:

- Total cards = 52
- 4 suits, each with 13 cards
- Choose 8 cards from one suit
- Order does not matter

Solution:

- Choose 1 suit: 4 choices
- Choose 8 cards from 13 in the suit: $C(13,8) = 13!/(8!(13-8)!) = 13!/(8!5!) = (13 \cdot 12 \cdot 11 \cdot 10 \cdot 9)/(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 1287$
- Total ways = $4 \cdot 1287 = 5148$

Final Answer: 5148 ways

Problem 6

Problem Statement: Given $P(A) = 0.37$, $P(A \text{ or } B) = 0.15$, and $P(A \text{ and } B) = 0.56$, find $P(B)$.

Given:

- $P(A) = 0.37$
- $P(A \text{ or } B) = 0.15$
- $P(A \text{ and } B) = 0.56$

Solution:

- Use union formula: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Substitute: $0.15 = 0.37 + P(B) - 0.56$
- Simplify: $0.15 = P(B) - 0.19$
- Solve: $P(B) = 0.15 + 0.19 = 0.34$

Final Answer: $P(B) = 0.34$

Problem 7

Problem Statement: A number is chosen from 1 to 10. Find the probability of not selecting a multiple of 2.

Given:

- Total numbers = 10
- Find probability of not selecting a multiple of 2

Solution:

- Multiples of 2: 2, 4, 6, 8, 10 \rightarrow 5 numbers
- Non-multiples of 2: $10 - 5 = 5$
- Probability: $P(\text{not multiple of } 2) = 5/10 = 1/2 = 0.5$

Final Answer: $1/2 = 0.5$ (or 50%)

Problem 8

Problem Statement: A number is chosen from 1 to 25. Find the probability of selecting a composite number.

Given:

- Total numbers = 25
- Find probability of selecting a composite number

Solution:

- Composite numbers: greater than 1, not prime
- Primes in 1 to 25: 2, 3, 5, 7, 11, 13, 17, 19, 23 → 9 primes
- 1 is neither prime nor composite
- Non-composite = 9 primes + 1 = 10
- Composite numbers = 25 - 10 = 15
- Probability: $P(\text{composite}) = 15/25 = 3/5 = 0.6$

Final Answer: $3/5 = 0.6$ (or 60%)

Problem 9

Problem Statement: If you roll a pair of dice, find the probability of not rolling a difference of 1.

Given:

- Each die has 6 faces
- Total outcomes = $6 * 6 = 36$

Solution:

- Pairs with $|a - b| = 1$: (2,1), (1,2), (3,2), (2,3), (4,3), (3,4), (5,4), (4,5), (6,5), (5,6) → 10 outcomes
- Outcomes where difference is not 1: $36 - 10 = 26$
- Probability: $P(\text{not difference of 1}) = 26/36 = 13/18 \approx 0.7222$

Final Answer: $13/18 \approx 0.7222$ (or 72.22%)

Problem 10

Problem Statement: A number is chosen from 1 to 50. Find the probability of selecting a number greater than 6 and less than 18.

Given:

- Total numbers = 50
- Find probability of selecting a number from 7 to 17

Solution:

- Numbers from 7 to 17: 7, 8, ..., 17 $\rightarrow 17 - 6 = 11$ numbers
- Probability: $P(7 \text{ to } 17) = 11/50 = 0.22$

Final Answer: $11/50 = 0.22$ (or 22%)

Problem 11

Problem Statement: A card is drawn from a standard 52-card deck. Find the probability of drawing a black face card.

Given:

- Total cards = 52
- Black suits = Spades, Clubs
- Face cards = Jack, Queen, King

Solution:

- Black face cards: 3 (Spades) + 3 (Clubs) = 6
- Probability: $P(\text{black face card}) = 6/52 = 3/26 \approx 0.1154$

Final Answer: $3/26 \approx 0.1154$ (or 11.54%)

Problem 12

Problem Statement: If you roll a pair of dice, find the probability that both show prime numbers.

Given:

- Each die has 6 faces
- Total outcomes = $6 * 6 = 36$
- Primes on a die: 2, 3, 5

Solution:

- Prime outcomes per die = 3
- Favorable outcomes: $3 * 3 = 9$
- Probability: $P(\text{both primes}) = 9/36 = 1/4 = 0.25$

Final Answer: $1/4 = 0.25$ (or 25%)

Problem 13

Problem Statement: Two cards are drawn from a 52-card deck without replacement. Find the probability of drawing a red card then the 4 of spades.

Given:

- Total cards = 52
- Red cards = 26 (13 Hearts + 13 Diamonds)
- 4 of spades = 1
- Without replacement, order matters

Solution:

- Probability of red card first: $P(\text{red}) = 26/52 = 1/2$
- After drawing a red card, 51 cards remain
- Probability of 4 of spades: $P(4 \text{ of spades} | \text{red}) = 1/51$
- Multiply: $P(\text{red then 4 of spades}) = (1/2) * (1/51) = 1/102 \approx 0.0098$

Final Answer: $1/102 \approx 0.0098$ (or 0.98%)

Problem 14

Problem Statement: Two cards are drawn from a 52-card deck with replacement. Find the probability of drawing the 5 of Hearts then a face card.

Given:

- Total cards = 52
- 5 of Hearts = 1
- Face cards = 12 (3 per suit * 4 suits)
- With replacement, order matters

Solution:

- Probability of 5 of Hearts first: $P(5 \text{ of Hearts}) = 1/52$
- With replacement, deck is unchanged
- Probability of face card: $P(\text{face card}) = 12/52 = 3/13$
- Multiply: $P(5 \text{ of Hearts then face card}) = (1/52) * (3/13) = 3/676 \approx 0.0044$

Final Answer: $3/676 \approx 0.0044$ (or 0.44%)

Problem 15

Problem Statement: Given $P(\text{Calculus and Dean's List}) = 0.44$ and $P(\text{Dean's List}) = 0.68$, find $P(\text{Calculus} | \text{Dean's List})$.

Given:

- $P(\text{Calculus and Dean's List}) = 0.44$
- $P(\text{Dean's List}) = 0.68$

Solution:

- Use conditional probability: $P(\text{Calculus} \mid \text{Dean's List}) = P(\text{Calculus and Dean's List})/P(\text{Dean's List})$
- Calculate: $0.44/0.68 \approx 0.6471$

Final Answer: 0.6471 (or 64.71%)