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Optical performance of vertical single-axis tracked solar panels

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ABSTRACT

To investigate the optical performance of the vertical single-axis (*v*-axis, in short) tracked solar panels as compared with fixed and full 2-axis tracked solar panels, a mathematical procedure to estimate the annual collectible radiation on fixed and tracked panels is suggested based on the monthly horizontal radiation. Calculation results showed that the yearly optimal tilt-angle of a *v*-axis tracked solar panel for maximizing the annual energy collection was almost linearly proportional to the site latitude, and the corresponding maximum annual collectible radiation on such tracked panel was about 96% of solar radiation annually collected by a dual-axis tracked panel. Compared with a traditional fixed south-facing solar panel inclined at the optimal tilt-angle, the annual collectible radiation due to the use of the *v*-axis sun-tracking was increased by 28% in the areas with abundant solar resources and increased by16% in the areas with poor solar resources. An empirical correlation for a quick estimation of yearly optimal tilt-angles of *v*-axis tracked solar panels was also proposed based on climatic data of 31 sites in China.

1. Introduction

To maximize energy collection, solar panels, such as collectors and photovoltaic modules, are usually oriented toward the equator with an optimal tilt-angle from the horizon which depends on climatic conditions and site latitude [1–4]. However, tracking the sun is required to further increase the solar gain of solar panels. There are basically two kinds of tracking system: single-axis and dual-axis, and they usually operate using either an electric or thermal power mechanism.

Kacira et al. [5] experimentally investigated the effect of dual-axis sun-tracking on the energy gain as compared with a fixed PV panel in Sanliurfa, Turkey, and found that the gross gain was 29.3% in radiation and 34.6% in power for a particular day in July. Theoretical analysis by Rabl [6] indicated that the annual collectible radiation on a solar panel tracking the sun about the polar axis was above 96% of that on a panel with full 2-axis sun-tracking according to the extraterrestrial radiation. Chang [7] theoretically analyzed solar gain of inclined south-north single-axis tracked panels based on the extraterrestrial radiation for different time periods and latitudes, and found that the ratio of the annual collectible radiation on the tracked panel to that of a fixed panel was close to 1.5 for areas with the site latitude below 65°, gradually increasing for latitudes above this with a maximum value of 1.8 in the Arctic zone. Another theoretical study by Chang [8] indicated that the annual collectible radiation on the horizontal east-west axis tracked panel was much less than that on the inclined south-north single-axis tracked panels. A study performed by Morcos [9] showed that changing collectors' azimuth and tilt-angles daily to their optimum values in Egypt resulted in an increase of 29.2% in total solar radiation compared with a fixed collector with the tilt-angle equal to its geographic latitude. Huang and Sun [10] designed a single-axis three-positions tracking panel, which was south-north oriented and rotated eastward or westward about an inclined south-north axis, and daily adjusted the panel's position at three fixed angles, i.e. in the morning, noon, and afternoon. Theoretical results based on radiation predicted under clear sky conditions showed that the optimal stopping angle of the panel during the morning or afternoon was about 50° from the noon position, independent of the site latitude, and there was a gain of 24.5% in power generation as compared to a fixed PV module for geographic latitude below 50°. Experimental investigations performed by Al-Mohamad [11] indicated that the daily output power of a vertical axis tracked PV module was above 20% higher at least than that of a fixed module. Abdallah [12] conducted an experimental study to investigate the effect of different sun-tracking systems on the electrical generation of a PV module, and found that there were electrical gains up to 43.87%, 37.5%, 34.43% and 15.69% for the two axes, horizontal east–west, vertical and horizontal north–south tracking, respectively, as compared with the fixed south-facing PV module tilted at 32° from the horizon in Amman of Jordan.

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All these studies showed that sun-tracking has proven to be an effective way to increase the gain of solar panels. As compared with a traditional fixed panel, the extra benefits from tracking the sun were about 20–40% in the collectible radiation or output power [8]. Dual-axis tracked panels performed best in term of the solar gain, but a complicated tracking system was required. Thus, single-axis tracking systems were technically and economically more attractive in practical applications of non-concentrating solar devices. An experimental investigation by Abdallah [12] indicated that the v-axis tracking system performed better for energy collection of PV panels in comparison with the horizontal south-north single-axis tracking system, but such a conclusion was drawn based on the experimental measurements on particular days at a specific site and the long-term performance of such tracked solar panels was not investigated. On the other hand, the annual solar gain captured by a v-axis tracked solar panel would be affected by its tilt-angle relative to the horizon for a given site, and such works were rarely found in the literature. The objective of this work is to investigate the optical performance of such tracked solar panels as compared with full two-axis tracked and fixed panels in term of the annual collectible radiation based on monthly horizontal radiation data, and yearly optimal tilt-angles for maximizing their annual collectible radiation.

2. Mathematical procedure to estimate solar gain of tracked panels

Daily collectible radiation on fixed solar panels with tilt-angle, β_0, from the horizon can be calculated by summing the contribution of the beam radiation, the component of sky diffuse radiation, and the radiation reflected from the ground as follows:

$$H_{0,\mathrm{day}} = \int_{-t_0}^{t_0} f(\theta_0) I_\mathrm{b} \cos\theta_0 \,\mathrm{d}t + H_\mathrm{d} R_\mathrm{d} + \rho H_\mathrm{h}(1-\cos\beta_0)/2 \tag{1}$$

where ρ is the albedo of ground, H_d and H_h are the daily sky diffuse and total radiation on a horizontal surface, respectively; R_d stands for the ratio of the daily sky diffuse radiation on unit area of the tilted surface to that on unit area of the horizontal surface, and is determined by the distribution of sky diffuse radiation over the hemisphere. If the distribution of sky diffuse radiation is isotropic, then $R_\mathrm{d} = (1+\cos\beta_0)/2$. However, the actual distribution of sky diffuse radiation is not isotropic, as observed by Hamilton [13] who found that 63% of sky diffuse radiation came from the southern part of sky dome in the northern hemisphere, and recommended R_d as

$$R_\mathrm{d} = (2+\cos\beta_0)/3 \tag{2}$$

I_b in Eq. (1) is the instantaneous beam radiation intensity on a surface normal to radiation. The incidence angle of solar rays on a south-facing tilted surface, θ_0, is determined by

$$\cos\theta_0 = \cos\theta_z \cos\beta_0 + (\cos\delta\ \cos\omega\ \sin\lambda - \sin\delta\ \cos\lambda)\sin\beta_0 \tag{3}$$

The incidence angle of solar rays on the horizon, θ_z, is determined by

$$\cos\theta_z = \cos\delta\ \cos\lambda\ \cos\omega + \sin\delta\ \sin\lambda \tag{4}$$

where ω and λ are the hour angle and site latitude, respectively, δ is the declination of the sun and determined by

$$\sin\delta = -\sin 23.45\ \cos[360(n+10)/365.25] \tag{5}$$

where n is the day number counted from the first day of a year. $f(\theta_0)$ in Eq. (1) is a control function, being one for the case of $\cos\theta_0 \geq 0$ and otherwise zero, implying that solar rays cannot strike the solar panels at this moment; t_0 is the sunset time on the horizon and determined by

$$t_0 = \tau_\mathrm{day}\omega_0/2\pi \quad \text{and} \quad \mathrm{d}t = \tau_\mathrm{day}\,\mathrm{d}\omega/2\pi \tag{6}$$

where τ_day is the day length in seconds (24×3600 s), and the sunset hour angle, ω_0, is determined by

$$\cos\omega_0 = -\tan\delta\ \tan\lambda \tag{7}$$

Similarly, daily collectible radiation on full two-axis tracked panels can be estimated by

$$H_{2,\mathrm{day}} = \int_{-t_0}^{t_0} [I_\mathrm{b} + I_\mathrm{d}(2+\cos\beta_2)/3 + \rho I_\mathrm{h}(1-\cos\beta_2)/2]\mathrm{d}t \tag{8}$$

where I_d and I_h are instantaneous sky diffuse and globe radiation on a horizontal surface, respectively, and the tilt-angle of full two-axis tracked panels from the horizon, β_2, is time dependent and equal to the zenith angle of the sun, i.e. $\beta_2 = \theta_z$ in this case. Daily collectible radiation on the v-axis tracked panels can be expressed by:

$$H_{1,\mathrm{day}} = \int_{-t_0}^{t_0} I_\mathrm{b}\cos\theta_1\,\mathrm{d}t + H_\mathrm{d}(2+\cos\beta_1)/3 + \rho H_\mathrm{h}(1-\cos\beta_1)/2 \tag{9}$$

where β_1 is the tilt-angle of v-axis tracked solar panels relative to the horizon. To be convenient for the calculation of incidence angle of solar rays on the tracked panel, θ_1, the coordinate system with the X-axis normal to the horizon and pointing to the top of sky dome, Y-axis pointing to east and Z-axis pointing to due north (see Fig. 1) was employed to position the sun. In this coordinate system, the unit vector from the earth to the sun can be expressed by [6]:

$$\mathbf{n}_\mathrm{s} = (\cos\theta_z, -\sin\theta_z\ \sin\phi_\mathrm{s}, -\sin\theta_z\ \cos\phi_\mathrm{s}) \tag{10}$$

where ϕ_s is the azimuth angle of the sun measuring from due south to west. To keep solar panels tracking the sun, the v-axis must be rotated in such a way that makes the azimuth angle of the tracked panels equal to that of the sun at any moment. Therefore, the unit vector of the normal of the v-axis tracked solar panels can be expressed by:

$$\mathbf{n}_\mathrm{c} = (\cos\beta_1, -\sin\beta_1\ \sin\phi_\mathrm{s}, -\sin\beta_1\ \cos\phi_\mathrm{s}) \tag{11}$$

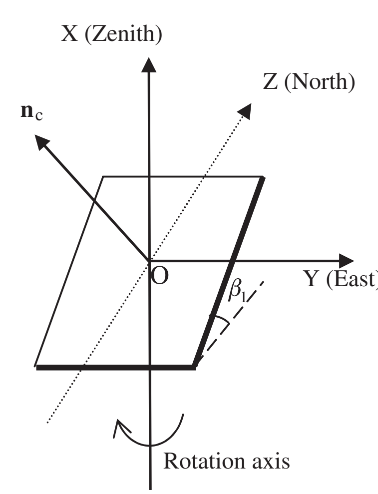

Fig. 1. Geometry of v-axis tracked solar panels.

The incidence angle θ_1 is obtained from the dot product of $\mathbf{n}_c$ and $\mathbf{n}_s$:

$$\cos\theta_1 = \mathbf{n}_s \cdot \mathbf{n}_c = \cos(\theta_z - \beta_1) \quad (12)$$

Knowing the variations of I_b, I_d and I_h with time in a day, daily collectible radiation on fixed or tracked panels could be obtained. Summing daily collectible radiation on fixed or tracked panels in all days of a year gives the annual collectible radiation. However, time variations of horizontal and diffuse radiation in any day of a year are not available in most places, and the most widely used and available data in solar calculations is the monthly horizontal radiation.

Given monthly horizontal radiation for a month, the monthly average daily radiation on the horizon, H_h, can be simply calculated by dividing the value by days of the month, and the monthly average daily diffuse radiation, H_d, can be estimated by the correlation suggested by Collares-Pereira and Rabl [6] as follows:

$$H_d/H_h = 0.755 + 0.347(\omega_0 - \pi/2) - [0.505 + 0.261(\omega_0 - \pi/2)]\cos[2(K - 0.9)] \quad (13)$$

where $K = H_h/H_0$, is the monthly average atmospheric clearness index for the month; H_0 is the monthly average daily solar radiation on an extraterrestrial horizontal surface and calculated by following expressions:

$$H_0 = \tau_{day} I_0 \cos\lambda \cos\delta(\sin\omega_0 - \omega_0 \cos\omega_0)/\pi \quad (14)$$

$$I_0 = 1373[1 + 0.033\cos(360n/365.25)] \quad (15)$$

To calculate H_d, H_0, and K, the middle day of a given month is taken as the representative day of the month. On obtaining H_d, the instantaneous sky diffuse radiation, I_d, and horizontal radiation, I_h, can be estimated by following expressions [6]:

$$I_d = r_d H_d \quad (16a)$$

$$I_h = r_h H_h \quad (16b)$$

$$I_b = (I_h - I_d)/\cos\theta_z \quad (16c)$$

$$r_d = \frac{\pi(\cos\omega - \cos\omega_0)}{\tau_{day}(\sin\omega_0 - \omega_0\cos\omega_0)} \quad (17a)$$

$$r_h = (a + b\cos\omega)r_d \quad (17b)$$

$$a = 0.409 + 0.5016\sin(\omega_0 - \pi/3) \quad (18)$$

$$b = 0.6609 - 0.4767\sin(\omega_0 - \pi/3) \quad (19)$$

In consequent calculations of the daily collectible radiation on a fixed or tracked solar panel in any day of a given month, daily horizontal radiation, H_h, and daily diffuse radiation, H_d, were taken to be their monthly average daily values, and sunset time is determined based on the declination of the sun in the day.

3. Methodology

For a specific site, the annual collectible radiation on a full two-axis tracked panel, S_2, is a constant statistically for many years, but the annual collectible radiation on a traditional fixed south-facing panel, S_0, depends on its tilt-angle and an optimal tilt-angle can be obtained by repeatedly calculating S_0 for different tilt-angles until a maximum annual collectible radiation, $S_{0,max}$, is found with the aid of computer simulations. Similarly, the annual collectible radiation on a v-axis tracked solar panel, S_1, is a function of its tilt-angle, and a yearly optimal tilt-angle, $\beta_{1,opt}$, which corresponds to a maximum annual collectible radiation, $S_{1,max}$, can be obtained by the same method as searching for optimal tilt-angles of fixed solar panels.

In the consequent calculations, time step, dt, for numerical calculations of daily collectible radiation is taken to be 60 s, steps of β_0 and β_1 for finding their optimal values were set at 0.1°, the albedo of the ground was taken to be 0.22, and monthly horizontal radiation data used in this work was obtained by averaging monthly measurements from 1961 to 1977 in key sites of China [14].

4. Results and discussions

Results obtained by numerical calculations based on the mathematical procedure developed in this work are listed in Table 1. It is seen that the yearly optimal tilt-angle, $\beta_{1,opt}$, of the v-axis tracked solar panels for maximizing annual collection of radiation, depending on the local climatic condition and site latitude where solar panels were installed, was much higher than the site latitude and optimal tilt-angle of traditional fixed solar panels, and the maximum annual collectible radiation on the v-axis tracked panels was about 95–96% of that on dual-axis tracked panels (see the column of $S_{1,max}/S_2$ in Table 1). These results indicated that v-axis sun-tracking was an effective tracking technique to boost the radiation collection of solar panels.

As compared with traditional fixed solar panels inclined at the yearly optimal tilt-angle $\beta_{0,opt}$, the increase in the annual collectible radiation on solar panels due to the use of v-axis sun-tracking technique, strongly depending on local solar resources, was between 13% and 28% (see the column $S_{1,max}/S_{0,max}$ in the Table 1). For the areas with abundant solar resources, such as Tibet and

Table 1
Results obtained by numerical calculations.

Locations	λ	$\beta_{0,opt}$	$\beta_{1,opt}$	$S_{1,max}/S_{0,max}$	$S_{1,max}/S_2$	$\Delta\beta_{1,opt}$
Beijing	39.95	39	55.2	1.2472	0.9633	−0.315
Harbin	45.75	44.3	58.5	1.2573	0.9662	0.915
Changchun	43.87	43	57.6	1.2548	0.9654	0.347
Shenyang	41.6	40	55.7	1.243	0.9649	0.478
Urumqi	43.78	40.2	57	1.2875	0.9639	0.877
Xining	36.58	37.4	54.5	1.2558	0.9607	−2.247
Lanzhou	36.02	34.9	53	1.2439	0.9609	−1.185
Xi'an	34.25	30.1	49.2	1.1958	0.9637	1.233
Yinchuan	38.42	38.1	55.2	1.2665	0.9611	−1.51
Huhehot	40.82	40.2	56.5	1.2749	0.9626	−0.936
Taiyuan	37.92	37	54.2	1.2462	0.962	−0.901
Shijiazhuang	38.07	36.9	54	1.2416	0.9624	−0.584
Tianjin	39.1	38.7	55.2	1.252	0.9624	−0.979
Jinan	36.88	34.8	52.9	1.242	0.9615	−0.413
Zhengzhou	34.72	32.3	51	1.2176	0.9619	−0.2
Hefei	31.88	28.7	48.6	1.2013	0.9619	−0.019
Shanghai	31.2	27.6	47.3	1.1801	0.9636	0.75
Nanjing	32.07	29.3	48.6	1.1944	0.9628	0.13
Hangzhou	30.33	27.3	46.9	1.1768	0.964	0.471
Nanchang	28.67	24.4	45.5	1.173	0.9638	0.574
Wuhan	30.63	25.8	46.8	1.1878	0.9627	0.805
Changsha	28.25	22.1	44	1.1628	0.9649	1.746
Chengdu	30.67	23.3	42.7	1.1309	0.9717	4.936
Chongqing	29.5	19.4	41.2	1.1328	0.9714	5.522
Guiyang	26.57	19.2	41.5	1.1405	0.9681	2.934
Kunming	25.03	27.5	47.9	1.2048	0.9586	−4.669
Lhasa	29.72	32.6	52.7	1.2838	0.9551	−5.806
Fuzhou	26.08	21.8	44	1.1664	0.9732	0.051
Guangzhou	23	21.7	43.9	1.1662	0.9621	−2.255
Nanning	22.8	19.1	42.6	1.1605	0.9626	−1.111
Taipei	25.03	19.2	41.9	1.1485	0.9657	1.331

Xinjiang regions, the increase in annual solar gain was above 28%, whereas for the areas with poor solar resources, such as Sichuan Province, Chongqing Municipality, Hunan Province, and Guizhou Province, the increase was less than 16%. This indicates that the sun-tracking technique was suitable to be employed in the areas with abundant solar resources for boosting energy collection of solar panels.

The effect of site latitudes on the yearly optimal tilt-angles of a v-axis tracked solar panel is presented in Fig. 2. It is seen that the yearly optimal tilt-angle of v-axis tracked solar panels increased generally with the increase of the site latitude, and an empirical correlation for a quick estimation of the optimal tilt-angles of such tracked solar panels was obtained by linearly fitting these data as follows:

$$\beta_{1,\mathrm{opt}} = 23.68 + 0.7811\,\lambda \ \text{(in degrees)} \tag{20}$$

Deviations of optimal tilt-angles of v-axis tracked solar panels expected by Eq. (20) from those obtained by numerical calculations, $\Delta\beta_{1,\mathrm{opt}}$, are given in the last column of Table 1. It is seen that the optimal tilt-angles expected by Eq. (20) were in good agreement with actual calculations with the deviation less than 3° for most places in China except in the areas with abundant or poor solar resources. For the areas with abundant solar resource, such as Lhasa, the expected $\beta_{1,\mathrm{opt}}$ was about 5° lower than the actual calculation, whereas for the areas with poor solar resources, such as Chengdu and Chongqing, the expected $\beta_{1,\mathrm{opt}}$ was about 5° higher than actual calculations. Another exception was Kunming, a site climatically characterized by sunny winters and rainy summers, and the expected $\beta_{1,\mathrm{opt}}$ was 4.7° lower than the actual calculation.

Effects of tilt-angle of v-axis tracked solar panels on the annual collectible radiation in terms of the ratio, $S_1(\beta_1)/S_1(\beta_{1,\mathrm{opt}})$, the annual solar collectible radiation on the panels with any tilt-angle β_1, to that with the optimal tilt-angle $\beta_{1,\mathrm{opt}}$, are shown in Fig. 3. Fig. 3 shows that 3° deviation of tilt-angle from its optimal value resulted in the reduction of annual collectible radiation on v-axis tracked solar panels less than 0.3%, indicating that the empirical expression, Eq. (20), was reasonable to estimate the optimal tilt-angle of v-axis tracked solar panels for most places in China except for the areas with abundant or poor solar resource.

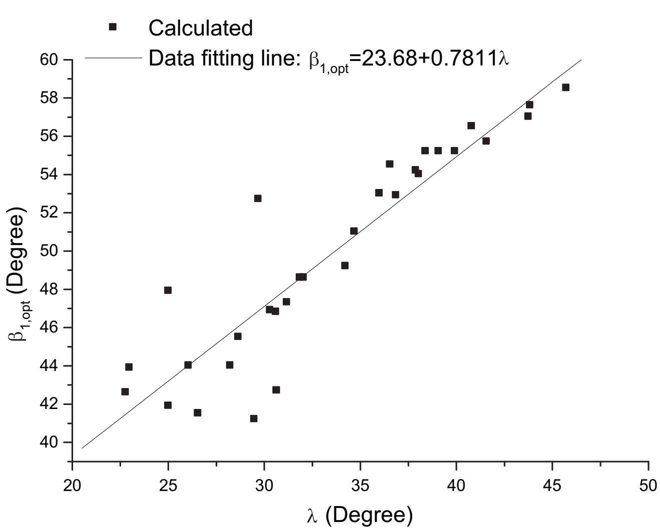

Fig. 2. Effect of site latitudes on the yearly optimal tilt-angle of v-axis tracked solar panels.

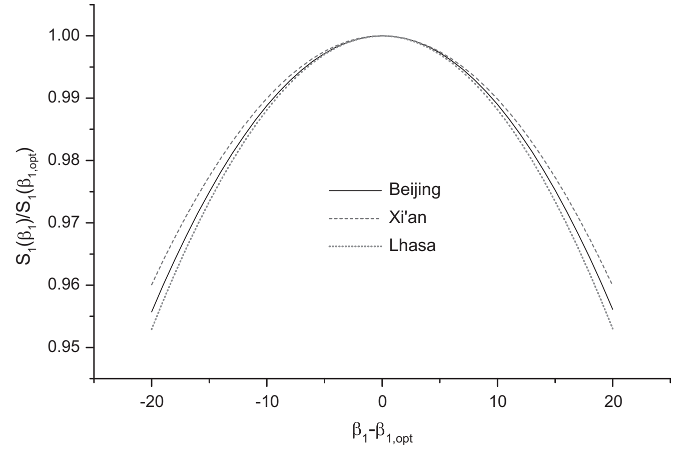

Fig. 3. Effects of tilt-angle of v-axis tracked solar panels on the annual collectible radiation.

5. Conclusions

Results obtained by numerical calculations based on the mathematical procedure proposed in this work show that the annual collectible radiation on a solar panel was related to local solar resource and sun-tracking strategies. For solar panels tracking the sun about the v-axis, the yearly optimal tilt-angle of the tracked solar panels, depending on the site latitude and local climatic condition, was much higher than that of traditional fixed solar panels, and the maximum annual collectible radiation was about 96% of that on dual-axis tracked panels, indicating that such sun-tracking strategy was effective to boost energy collection of solar panels. But this does not imply that dual-axis sun-tracking technique is necessary in all cases. For some applications, such as solar systems with a high concentration ratio, dual-axis sun-tracking is necessary; whereas for non-concentrating solar panels, single-axis sun-tracking might be more attractive economically and technically due to the use of a simple sun-tracking system. As compared with the traditional fixed solar panels inclined at the yearly optimal tilt-angle, the increase in the annual collectible radiation due to the use of such sun-tracking technique strongly depended on the local solar resources, for the areas with abundant solar resources, such increase was above 28%, whereas for the areas with poor solar resources, the increase was less than 16%, indicating that the v-axis sun-tracking technique was suitable to be employed in the regions with abundant solar resources. Comparisons of optimal tilt-angles between actual calculations and those expected by the empirical correlation proposed in this work showed that the proposed correction for estimating the yearly optimal tilt-angle of v-axis tracked solar panels was reasonable for most places in China except in the areas with abundant or poor solar resources.

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Nomenclatures

H: daily radiation, J/m^2
I: instantaneous radiation intensity, W/m^2
K: monthly average atmospheric clearness index, dimensionless
n: day number counted from the first day of a year or unit vector
R_d: ratio of diffuse radiation on a tilted surface to that on the horizon, dimensionless
S: annual collectible radiation, J/m^2
t: solar time, seconds

Greek letters
β: tilt-angle of solar panels from the horizon, degree
δ: declination of the sun, degree
$\Delta\beta_{1,opt}$: difference of optimal tilt-angle of v-axis tracked solar panels between expected and calculated
ϕ: azimuth angle, degree
λ: site latitude, degree
θ: incidence angle of solar rays on any surface, degree
ρ: reflectivity of the ground, dimensionless
τ_{day}: day length, second
ω: hour angle, radian

Subscripts
0: fixed solar panels, sunset, extraterrestrial
1: v-axis tracked solar panels
2: dual-axis tracked solar panels
b: beam radiation
d: sky diffuse radiation
h: hemisphere radiation
max: maximum value
opt: optimal value
s: sun