

### Motivation

$$w \sim \mathcal{N}(\mu, \sigma^2)$$

Stochastic Networks

**It works!**

$$w \sim \mathcal{N}(\mu, 0)$$

Usual Networks

$$w \sim \mathcal{N}(0, \sigma^2)$$

This paper: Variance Nets

We can store information using variances only!

### Stochastic Deep Neural Networks

How to train:

1.  $\hat{W} \sim q(W | \phi)$  e.g., Gaussian  $q(W | \phi) = \mathcal{N}(W | \phi)$
2.  $\nabla_{\phi} L \cong \nabla_{\phi} (-\log p(Y | X, \hat{W}) + R(\phi))$
3. Update  $\phi$  and repeat until convergence

How to predict:

1. Weight Scaling Rule (WSR) (**heuristic**)

$$p(y | x) \approx p(y | x, \mathbb{E}W)$$

- + Fast and usually works well in practice
- May yield arbitrarily bad predictions!

2. Monte-Carlo estimate (**proper way**)

$$p(y | x) \approx \frac{1}{K} \sum_k p(y | x, \hat{W}_k), \quad \hat{W}_k \sim q(W | \phi)$$

- + Produces a correct unbiased estimate
- Requires to compute the output K times

### Variance Networks

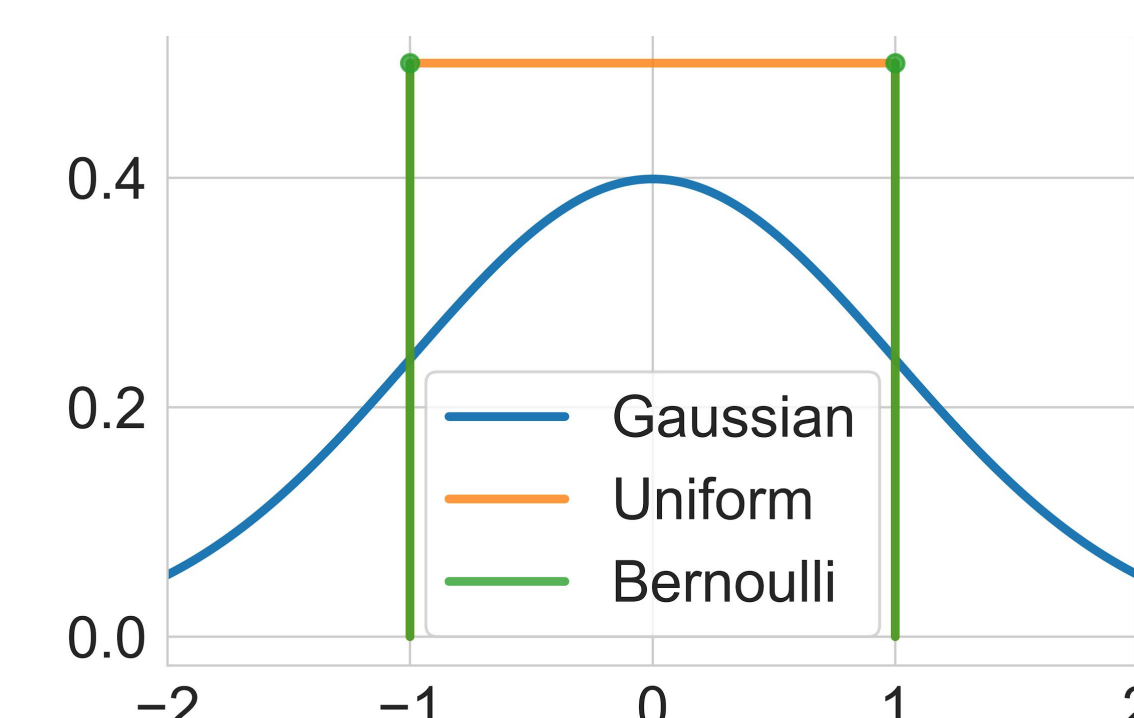
Variance layer has a symmetric weight distribution:

$$q(W | \phi) = q(-W | \phi)$$

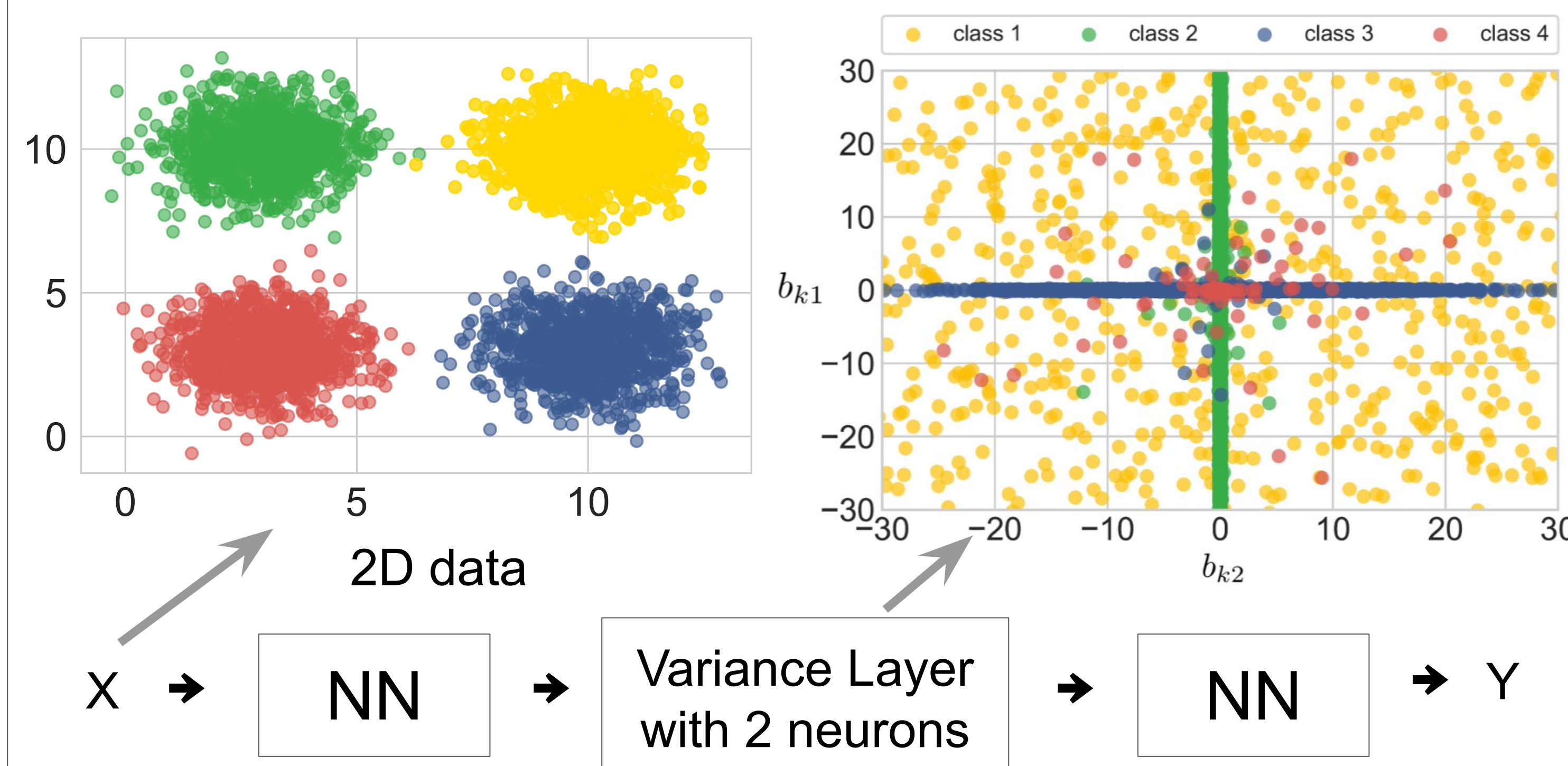
Examples of variance layers:

1. Gaussian:  $w_{ij} \sim \sigma_{ij} \cdot \varepsilon_{ij}$ ,  $\varepsilon_{ij} \sim \mathcal{N}(0, 1)$
2. Bernoulli:  $w_{ij} \sim \phi_{ij} \cdot (2\varepsilon_{ij} - 1)$ ,  $\varepsilon_{ij} \sim \text{Bernoulli}(\frac{1}{2})$
3. Uniform:  $w_{ij} \sim \phi_{ij} \cdot \varepsilon_{ij}$ ,  $\varepsilon_{ij} \sim \mathcal{U}(-1, 1)$

$\mathbb{E}W \equiv 0$ , hence WSR fails for such distributions!



### How it works? A toy example



### Classification results

Architecture	Dataset	Network	Accuracy (%)		
			1 samp.	Det.	20 samp.
LeNet5	MNIST	Dropout	99.1	99.4	99.4
		Variance	98.2	11.3	99.3
VGG-like	CIFAR10	Dropout	91.0	93.1	93.4
		Variance	91.3	10.0	93.4
VGG-like	CIFAR100	Dropout	77.5	79.8	81.7
		Variance	76.9	5.0	82.2

We achieve the same performance as usual networks!

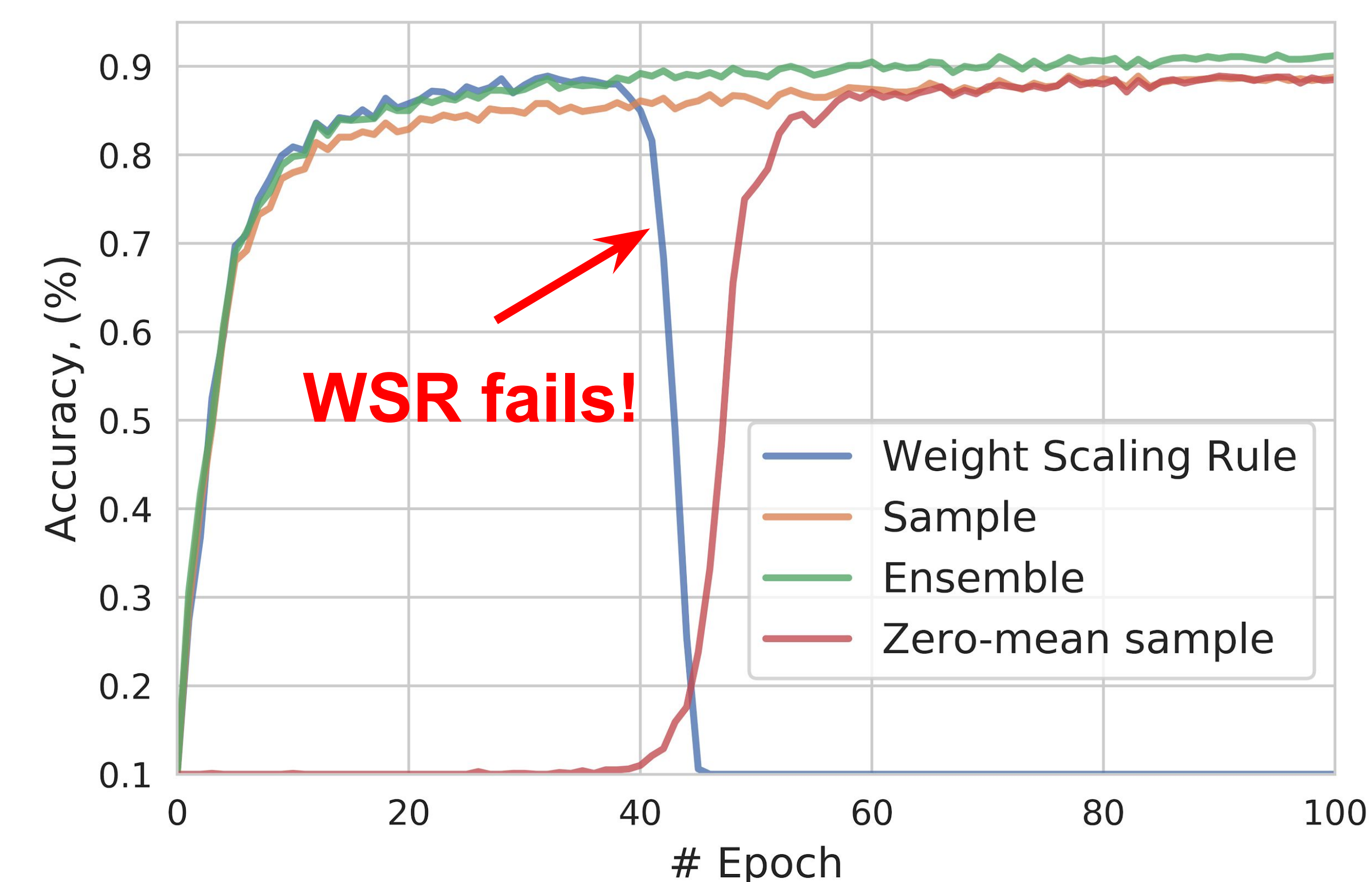
### Variational Dropout → Variance Networks

Variational Dropout:

$$\underbrace{-\mathbb{E}_{q(W | \phi)} \log p(Y | X, W)}_{\text{Data-term (e.g. cross-entropy loss)}} + \underbrace{D_{\text{KL}}(q(W | \phi) \| p(W))}_{\text{Regularizer}} \rightarrow \min_{\phi}$$

In practice the Variational Dropout model converges to variance networks!

$$\mathcal{N}(\mu_{ij}, \alpha \mu_{ij}^2) \xrightarrow{\alpha \rightarrow \infty} \mathcal{N}(0, \alpha \mu_{ij}^2)$$



### Better ELBO

	layer-wise $\mathcal{N}(\mu_{ij}, \alpha \mu_{ij}^2)$	neuron-wise $\mathcal{N}(\mu_{ij}, \alpha_j \mu_{ij}^2)$	weight-wise $\mathcal{N}(\mu_{ij}, \alpha_{ij} \mu_{ij}^2)$	additive $\mathcal{N}(\mu_{ij}, \sigma_{ij}^2)$
ELBO	-9.4	-11.0	-287.4	-227.9
Data term	-9.04	-8.34	-21.13	-31.2
KL term	0.36	2.66	266.25	196.74
Mean prop. acc.	11.3	11.3	96.6	99.2
Test-time averaging	99.3	99.2	99.4	99.2

Less flexible posterior approximations result in much better ELBO!