Image Denoising Using Sparse Representation via Compressive Sensing, Block Matching, Segmentation, and Redudant Representations

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***Abstract* – In this paper, the importance of denoising images will be discussed along with analyzing different aspects of sparse representation. This paper will also be exploring different methods others have proposed to ensure that sparse representation is constantly being improved and modified to achieve the best enhanced and noise-free image possible. A variety of techniques will be discussed which include compressive sensing, block matching, segmentation, redundant representations, learning/adaptive dictionaries, and general equations that were modified to incorporate the methods mentioned. This paper will also include figures and tables that will depict the result of the proposed methods as well as indulge in some of the limitations that it may pose.**

Keywords *– adaptive learning, block matching, dictionaries, redundant representation, segmentation, sparse representation*

I. Introduction

Images can display many important information if the details are easily visible. Images, in general, can become blurry and hard to see with little to no effort. For instance, an image can be blurry or unclear if the person taking the picture was not steady, the zooming of the images was overdone, resolution issues, and the list goes on. Although, general images that are taken day-to-day can pass as being unclear, images involving one's internal organs or structure should be clear and precise allowing for professionals and physicians to accurately identify the problem one may be having. One major issue that causes an image to be "grainy" is due to unwanted noise. This unwanted noise can occur from unwanted movement of the instrument being used to record an area of the body or the patient moving around. For this reason, a lot of methods, filters, and programs were created to help mitigate inaccurate images along with various techniques.

Images can be analyzed both in the spatial and frequency domain each having their own advantages and disadvantages. Depending on what information is required or needed, the appropriate domain can be used. For instance, observe the following system:

$$\xrightarrow{x(t)} \boxed{h(t)} \xrightarrow{y(t)}$$

Where x(t) is the input image, y(t) is the output image, and h(t) denotes the filter being applied. The filter being applied must be accurate and efficient allowing the output image to be as clear and accurate as possible. For this reason, sparsity for many years was a common method used for denoising images. However, it was found that when using sparse representations with other techniques of denoising, the effectiveness is greatly increase and will be discussed further in this paper. In the figure below, Figure 1, a comparison between an original image and an additive gaussian noise image, with a mean of 0.5 and a variance of 0.5, is shown to display the importance of how noise can completely distort an image and remove all its details and features.

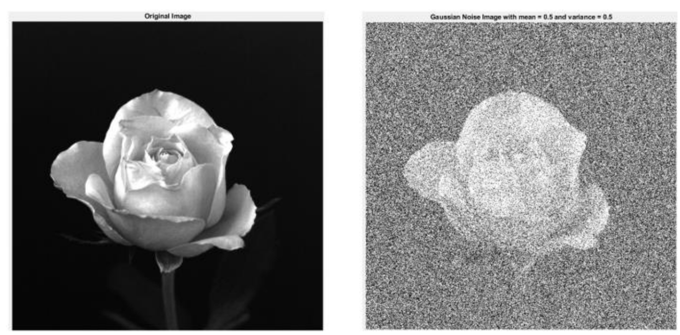
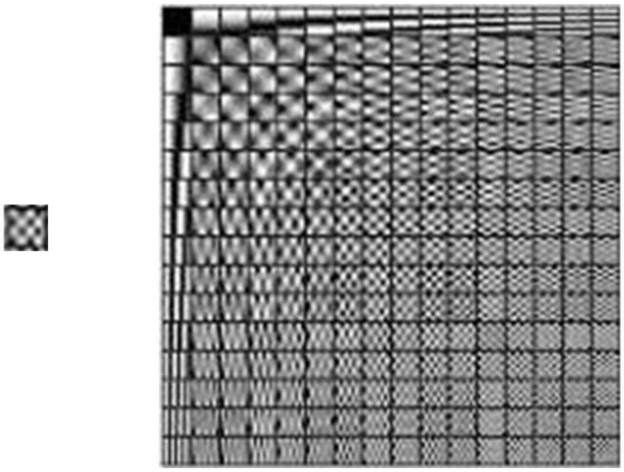


Figure 1: An original image compared to an additive gaussian noise image with a mean and variance of 0.5

II. Sparsity Representation

When it comes to denoising images, sparse is superior to a lot of the other methods out there as it helps solve systems of linear equations. The way sparse works is by finding patterns in images that are 'regular' and 'uniform'. Essentially, this means that sparse tries to identify locations in the image where no noise is present and tries to copy these non-noise-like patterns to other areas of the image where noise is present. By doing this, sparse can denoise the image.

Sparse representation has a common connection with two commonly used terms – atom and dictionary. In sparse, an atom is a block/pattern, either pre-defined or adaptive, which will be explained in Section V, and a collection of atoms are referred to as a dictionary. In Figure 2, (a) depicts an atom where (b) illustrates a dictionary.



(a) (b)

Figure 2: (a) a block/pattern referred to as an atom, (b) a collection of atoms referred to as a dictionary

As mentioned earlier, spare representation is commonly used for solving systems of linear equations thus, when dealing with geometric shapes, this can cause some problems. [1] For this reason, a prebuilt dictionary is used to overcome these challenges. A common pre-built dictionary used to denoise images is the Discrete Cosine Transform (DCT) dictionary which converts data like pixels and waveforms into frequency components. The following denoising techniques, when paired with sparsity, will be discussed in this paper: compressive sensing, block matching, segmentation, redundant representation, and learning/adaptive dictionaries.

III. Compressive Sensing and Block Matching Paired with Sparse Representation

In digital image processing, unwanted noise and/or blurred edges are two prime challenges that one faces. Unwanted noise or blurred edges can come from a variety of different factors – the most common being moving artifacts. These artifacts are not controllable and must be removed using image processing techniques. These techniques help correct defects that were produced during the acquisition and transmission process. Denoising is used in modern time to help gain a clearer and higher quality image from a blurry or not clear image. This paper thoroughly explains a new method for how these noises can be filtered to achieve a greater image quality. This new method consists of older methods along with different approaches. With current denoising processes, there are various disadvantages to each of them. For instance, Elad and Aharon created a dictionary to denoise images using the K-SVD algorithm – uses sparse representations. This algorithm recovers a noisy image using the dictionary that includes redundancy in a global Bayesian objective and sparsity. [2] The Bayesian optimization works by having an input signal that takes the form of a random function and comparing it with a signal that one believes, a.k.a. prior, to be the correct signal. Once the prior function is compared and evaluated with the random function an acquisition function is constructed which can then be used to calculate other data on that acquisition function. [2, 3] The article then continues to emphasize that due to having a prior, one's belief of what the function should be, this can cause biases and inaccurate results if the acquisition function received is incorrect. In addition, another disadvantage for this Elad and Aharon's method is that overfitting occurs which means that the model has "memorized" the data that was given while neglecting to learn the relationship between the features and labels. [4] For this reason, this article indulges in a new method that incorporates the Bandelet transform, denoising based on compressive sensing, and block matching algorithms. The Bandelet transform is beneficial a beneficial tool that helps identify edges and texture in images. This transform can identify the geometric flow of vectors by using the gray levels in a local area as can be seen in Figure 3 below.

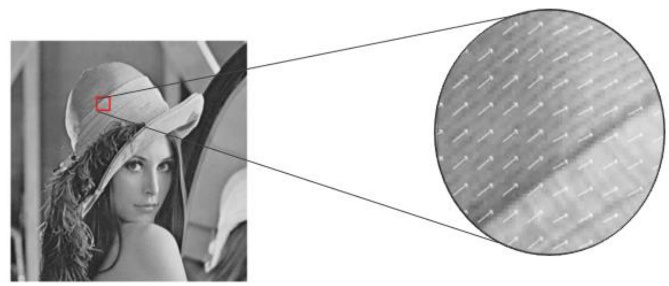


Figure 3: A local area of the woman's hat illustrates the geometric flow based on the grey level intensities

In Figure 3, when the woman's hat is zoomed in and the local area of interest is analyzed, the circular image, it is seen that based on the grey level intensities of the image, the geometric flow is determined. The left side of the image can be seen to have a darker shade than the right side of the image. Due to this grey level intensity discrepancy, the Bandelet transform identified that the geometric flow is going from left to right in an upward motion.

A. *Compressive Sensing*

Compressive Sensing (CS) theory is used to help denoising an image. This theory takes a noise-free image and subtracts it with the noisy image while adding the sparsity of the denoised image in a basis. The following formula is assuming that "a natural image has a compact representation when projected in an appropriate basis:"

$$\arg\min_{u} \frac{1}{2}\|\phi\, u - \phi v\|^2 + \lambda\|\psi u\|_1$$

Where,
u = noise free image
v = noisy image
ψu = basis of image
ϕ = operator that separates noise

From this equation, compressive sensing becomes easier when the noise-free and noisy image are known as when subtraction between them occurs, the noise present is eliminated resulting in a clearer image. Compressive sensing, just like other methods is used for denoising images, and when paired with sparse representation, the results are improved. There are two algorithms for compressive sensing: Total Variation-based (TV) algorithms and Wavelet-based algorithms. [5] The TV method occurs when, both, ϕ and ψ are identical matrices and gradient operator, respectively. This method was proposed by Durand and Froment where they explored that when the gradient vector of an image is sparse, the TV method is a viable option for denoising purposes. In the wavelet-based algorithm, it states that when both, ϕ and ψ the matrices and the wavelet transform, respectively, are identical then the TV method is extended as an approximation technique. Using the wavelet-based transform, it acts as a soft thresholding tactic for denoising images.

B. *Block Matching*

Block matching is a denoising technique that finds similar grey level blocks around an image to help with edge identification and preservation. Once these blocks are identified a variety of grouping techniques can be used such as vector quantization, k-means clustering, fuzzy clustering, and self-organizing maps. [5] An image of how different areas of an image is grouped can be seen in Figure 4.

Figure 4: Grouping Similar Blocks of an Image using Block Matching

From Figure 4, the 4 letters – a, b, c, and d, represent a group. These groups are formed by identifying patterns in an image. There are various forms of grouping such as vector quantization, k-means clustering, fuzzy clustering, and self-organizing maps. [6] In the image above, the ‘a’ blocks are all on the shader of the house as it shares a general pattern. The same can be said for the ‘b’ block where a shadow and L-like shape of the wall can be seen, etc. These blocks/atoms all share the same characteristic and pattern thus, they are grouped together.

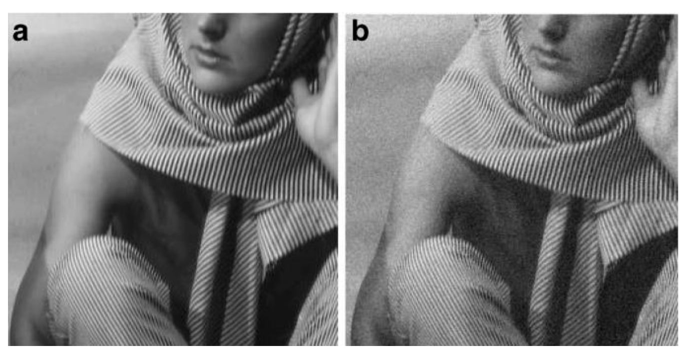
C. *Proposed Method*

Hamid and Seyede proposed a new method that included all 3 methods and combines them to create a detailed denoising scheme. The first step is to use block matching technique to determine where similarities are in the image. This will allow the algorithm to find all the similar blocks and patterns in the image to accurately denoise areas of the same pattern. Next, the grouping technique they proposed to use was the k-means clustering method where it is calculated to find the indices for each group patch. This allows the algorithms to know where all the patches of the specified group are, so it can be used when needed. In addition, vector $\hat{a}_g$ is calculated along with the dictionary using the following formula:

$$\arg\min_{u,a_g} \lambda \|\phi\, u - \phi v\|_2^2 + \sum_g u_g \|a_g\|_0 + \sum_g \sum_{ij} \left\|D a_g - R_g^{ij} u\right\|_2^2$$

Where,
g = group index
R_g^{ij} = n x N matrix to extract g^{th} group
D = Dictionary
a_g = sparse vector corresponding to g^{th} group

Lastly, Hamid and Seyede mention to use an overcomplete DCT (Discrete Cosine Transform) dictionary by using the TwIST (two-step iterative shrinkage/thresholding) algorithm which helps in determining the denoised image u. The DCT dictionary is greatly used in data compression and converting data, like pixels and waveforms, into frequency components. The TwIST algorithm is ideal for optimizing images as it used values of u that are from the previous and current iterations as oppose to IST (Iterative shrinkage/thresholding) algorithm that is takes in inputs from the current iteration process. From this we can denoise images accurately while simultaneously creating a trained dictionary. The results can be seen in Figure 5.



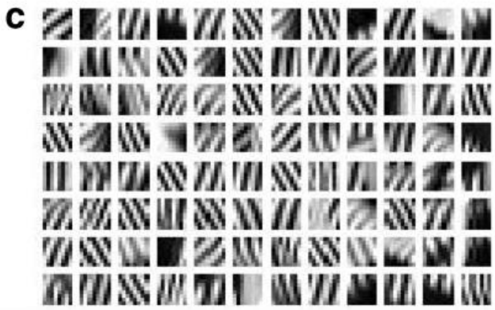


Figure 5: (a) Noise-free Image of Barbara, (b) Noisy Image of Barbara, (c) Trained Dictionary

It is worth mentioning that the trained dictionary follows patterns that are seen in images (a) and (b) in Figure 5 and that these patterns were used to denoise the image accurately and efficiently.

They also went in extra step and compared their proposed methods other existing methods such as 3D block matching (BM3D), compressive sensing using bandelet transform (BSBT) and sparse representation over learned dictionaries (SROLD). [5] In Figure 6, an image of a couple is denoised using different methods along with Hamid's and Seyede's proposed method.

Figure (6): (a) Original Image, (b) Noisy Image (σ = 20), result of (c) SROLD, (d) BM3D, (e) CSBT, (f) Proposed Method

From Figure 6, it is evident that although all the images are denoised to a certain extent and the couple image is clear, a way to determine which method is more superior than the other is by measuring the PSNR, peak signal-to-noise ratio, and SSIM, structural similarity. The PSNR is essentially the measurement of an image's quality where the SSIM measure how closely the enhanced image is to another. The tables below summaries both the PSNR and SSIM values found, by Hamid and Seyede, while portraying that their proposed method denoises images better than other methods.

TABLE I. PSNR RESULTS OF VARIOUS DENOISING METHODS

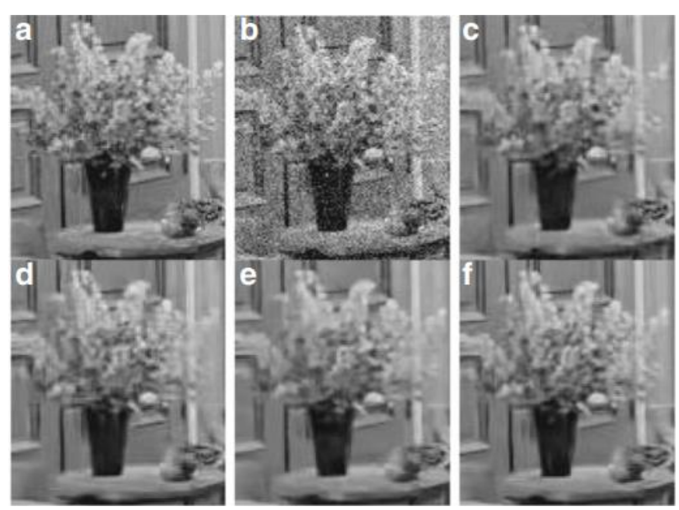
Method	**Standard Deviation (σ)**							
	10	20	30	40	50	60	70	80
Original PSNR	28.14	22.11	18.59	16.09	14.15	12.57	11.23	10.07
CSBT	33.2	29.26	27.98	26.21	25.59	23.77	23.51	22.62
SROLD	33.53	30.01	27.96	26.32	25.27	24.48	23.90	23.31
BM3D	34.04	30.76	28.87	27.48	26.46	25.66	25.00	24.42
Proposed Method	**35.52**	**31.76**	**29.89**	**27.74**	**26.13**	**26.13**	**25.81**	**24.86**

TABLE II. SSIM RESULTS OF VARIOUS DENOISING METHODS

Method	**Standard Deviation (σ)**							
	10	20	30	40	50	60	70	80
Original SSIM	0.7162	0.4559	0.3125	0.2278	0.1715	0.1350	0.1104	0.0926
CSBT	0.8871	0.7993	0.7372	0.6910	0.6447	0.6163	0.5860	0.5589
SROLD	0.8987	0.8150	0.7446	0.6819	0.6278	0.5902	0.5634	0.5363
BM3D	0.9086	0.8452	0.7924	0.7435	0.7058	0.6675	0.6399	0.6075
Proposed Method	**0.9261**	**0.8655**	**0.8098**	**0.7584**	**0.7195**	**0.6730**	**0.6467**	**0.6163**

It is important to note that the bolded values in the tables display the methods that was most effective. It is seen that the proposed method was most effective where block matching had very similar values. The BM3D method will be explored further in the implementation section, Section VI.

Moreover, a closer analysis on edge preservation and noise reduction was conducted and the results are shown below.



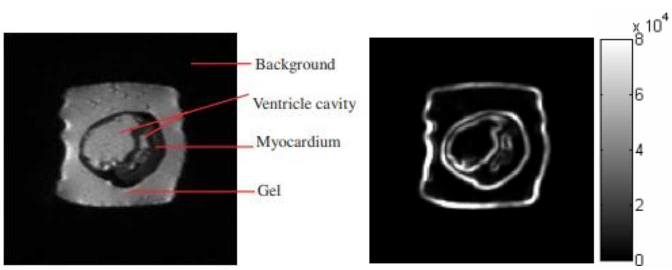
This image indicates that using different methods of denoising techniques does not yield the same results. It was seen that SROLD, (c), had greater capability for preserving edges but the over clarity of the image was not the best. This can also be seen in Table 1 where the PSNR was 33.53 where the best method resulted in a PSNR of 35.52. The block matching technique, (d), was successful for removing noise however small rippling effects were added to the image when closer looked at.

IV. Denoising Cardiac Diffusion Tensor Magnetic Resonance (DT-MRI)

In the medical field, denoising images can positively impact in determining problems in the human body. Due to motion artifacts being one of the highest chances of producing unwanted noise, image denoising becomes very useful for removing those noises. In a paper, it explains how denoising is used to remove unwanted noise in a cardiac diffusion tensor magnetic resonance images (DT-MRI) using sparse representation combined with segmentation. With the heart constantly pumping, and no way of making the heart not move, motion artifacts are bound to be introduced thus, creating noise. Furthermore, this paper introduces a method that will generate a dictionary that entails the features of the image which will then be passed through a segmentation algorithm to help manipulate the atoms that were selected in the dictionary to be more adaptive of the image's features. [7]

A. *Segmentation*

Segmentation is used to make atoms more adaptive in the dictionary. There are a variety of segmentation methods, however the one used in this paper, by J Bao, M Zhu, Y Liu, and others is edge detection by thresholding. By using this method of segmentation, one can identify the different regions and edges in an image by using NSD – Non-stationarity detection. The purpose of NSD is that it allows for all the components in the image to become independent of one another allowing for the program or observer to easily identify the regions and edges.



(a) *(b)*

Figure 7: (a) Original Image, (b) NSD Image

From Figure 7, it can be seen that the original image consists of a background, ventricle cavity, myocardium, and gel. However, they are all grouped toghther. For this reason, the NSD is used to make all of these componetns are independent of each other allowing the algortihm to detect the edges and determine the regions.

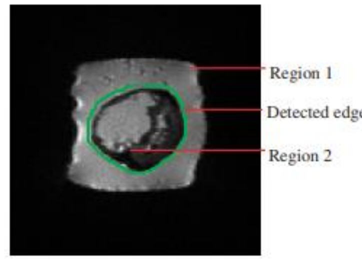


Figure 8: Regions and Edges Detected after Segmentation and NSD

It can be seen in Figure 8, that the 2 regions are determined as well as the edge is detected as it is outlined in green.

In segmentation an approximation of the what the denoised image should be, using the atoms, is gradually cycled until denoising is achieved. The paper indicates that the outcome of the simulated image along with the real cardiac DT-MRI images performed better than current techniques. Current techniques being used is finding the partial-differential-equation (PDE) and wavelet filter. Although the PDE filter is good for preserving an image's edges, the major disadvantage is that it does not work well when dealing with high noise levels as the contrast is degraded and makes the image hard to see clearly. The wavelet-based technique used for denoising has yielded improvements in the overall image quality, like the Fourier transform and the discrete cosine transform, however, the elements that need to be observed and the noise are convolved with the basis function resulting in the image to not be as clear as needed. Furthermore, when using the wavelet-based denoising technique, determining a threshold can cause problems as it is an estimate affects the accuracy of the final image. Sparse representation-based denoising (SPDN) however has proven to show more accurate results than the classical denoising approaches mentioned above.

B. *Segmenaation-guided Sparse Representation-Based Denoising (SPDN)*

Sparse representation-based denoising (SPDN) is a better method to use for denoising because the sparse model aids in finding underlying details of the original image as if noise were not present. [7] Due to pixels exhibiting strong dependencies on each other in images, the sparse representation identifies these dependencies to have strong correlation in terms of carrying important information that can help in identifying the structure of the object. Due to the atoms being present in the dictionary that sparse representation uses, the noisy image's structural features can still be approximated with suitable atoms. This paper goes into discovering the denoising features using sparse representation on a DT-MRI image.

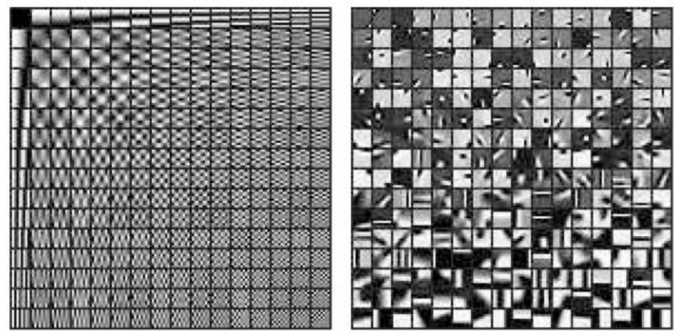
The paper talks about using a segmentation – guided SPDN approach where an appropriate base or pattern can be chosen in the dictionary to allow the program/approach to be more adaptive and efficient. This method will also utilize the nonstationary degree (NSD) method that is known to work well with images that have noise present. Using the NSD detector and multiple bases from the dictionary, it allows for more accurate results. In the paper, they proposed that there should be 3 parameters that can be adjusted which are the factor, particle size, and filter window size. The factor will be denoted by C and should be greater than one as this affects the denoising results. A thing to keep in mind when choosing a C value is that when the factor C is too large, this can cause over-filtering making the image seem more darker or brighter. In the paper they found that C values between 1.05 and 1.15 yielded relatively similar denoising results however, they found that when they change the C value to 1.13, they got more clearer results. For the patch size, in image processing, patch sizes typically come in various sizes like 8x8, 16x16, and 32x32. [7] In this paper, they used an 8x8 patch size as it saves computation time and proved to be a good denoising patch size. Lastly, for the window size, it will be denoted by L, they utilized a 3x3 window size because they discovered that any size higher than that could cause blurry edges and is not the best for edge enhancement for the image being observed. The proposed method was executed on both small and large data size and the results were superior in both cases.

V. Image Denosing via Sparse and Redundant Representation Over Learned Dictionaries

In this paper, they use image denoising methods on images that have gaussian additive noise. The general formula used to represent additive noise to an image is:

$$y = x + v$$

Where the y, x, and v are the output image, input image, and gaussian additive noise, respectively. Images are often corrupted due to gaussian noise and using sparse representation can help remove that noise. This paper explains that Elad and Aharon had proposed an approach called patch-based image processing that would analyze the image and get patches from a dictionary that can help in clearing up an image. They proposed that the patch denoising dictionary is fixed and it has a size of $\sqrt{m}$ x $\sqrt{m}$ that would be centered on the image at some index *i*. An important aspect to know about the patches is that a proper dictionary must be used to get relatively appropriate results. Elad and Aharon's method would average the estimates received by the patch and average them before outputting the denoised image. [8] Typically, a DCT (Discrete cosine transform) dictionary would be used for all images however this can cause discrepancies for some areas of an image. Due to this reason, a new modified method was proposed where Roth and Black, wanted to create a method that implemented dictionary learning. Therefore, this method is a "generic" method for denoising images. The advantage of implementing dictionary learning is that it allows an image to be adaptive regardless of the noise. The patch of the images will be learned by the dictionary allowing images to keep their edges to some extent and remove the unwanted noise present in the image. Essentially what dictionary learning does is that it tries to learn patterns from the patches in the images. This type of approach is called a "global" method for denoising images. This paper explained that although using predefined dictionaries gets the job done adequately, dictionary learning seems to be the more stable option. [8] They explained how using a DCT dictionary and a learned dictionary patches can vary. The images below help display a visual representation of what a DCT dictionary looks like as oppose to a global dictionary.



(a) *(b)*

Figure 9: (a) Predefined Dictionary, (b) Dictionary Learning

From Figure 9 illustrates the difference between a pre-defined dictionary and an adaptive learning dictionary. In Figure 9 (a), it is evident that an overall pattern is followed whereas in (b) the dictionary is much more randomized as the patterns of the atoms in the dictionary correspond more closely to the image that is being observed. In other words, even though the patterns are randomized, in Figure 9 (b), the patches will output a more accurate image than a pre-defined DCT dictionary. Furthermore, the paper introduced gaussian noise to an image and denoised it is using the predefined and adaptive learning dictionary to get a better grasp of the visual differences that can be seen.

(a) (b)

(c) (d)

Figure 10: (a) Original Image, (b) Additive Gaussian Noise Image, (c) Adaptive Learning Dictionary, (d) DCT Dictionary

In addition, the Figure above shows an actual image that was processed using both the dictionary learning and the predefined dictionary. Figure 10 (a) portrays the original image whereas the image beside it, (b), introduced additive gaussian noise. Figure 10 (c) used the trained dictionary, and it is evident that the colour and the image quality remained relatively the same, but in the second image of the second row, (d), the image varies in color. Although sometimes it can be hard to truly know if an image is clearer or better than another, the signal-to-noise ratio (SNR) was calculated. In Figure 10 (c), had an SNR of 28.8528 dB and (d) had an SNR of 30.8295 dB. From these values, it can be seen that when using adaptive learning dictionaries for denoising purposes, it is superior to the overcomplete DCT dictionary.

VI. IMPLEMENTATION OF BLOCK MATCHING

For the implementation portion of this paper, a MALAB code was found that implemented the block matching algorithm. This was done by Tampere University of Technology. They implemented the Block Matching algorithm along with other filters to achieve the best denoised image possible. This program computes the PSNR of both the noisy image along with the denoised image. [9] This will allow the observer to identify the quantity of how different the two images are.

In the MATLAB code, there were a lot of embedded functions that used other functions from other files. For this reason, when I tried to implement sparsity into the file, I was constantly running into function errors or compiling errors. In addition, I also tried to add a dictionary output function that I was able to find from other sources to help display the different atoms being used when denoising the image.

The beauty of the MATLAB code was that Tampere University of Technology allowed the user to input his or her function or use the default values that were coded already. They had a variety of images to choose from and although the images were hard to find and incorporate, I managed to add these folders and names to a zip file which was handed in through D2L.

This code was reproduced based on an article that moderately resembled a paper that was reviewed in this report called "Image Denoising by Sparse 3D Transform-Domain Collaborative Filtering." The paper explored the effects and efficiency of denoising when additive white gaussian noise is added to a greyscale image. The program created a block matching algorithm for attenuation this additive gaussian noise.

The function name was BM3D which took in a y, z, sigma, profile, and print_to_screen. The y and z variables represent a noise-free image and noisy image with a matrix of M x N, respectively. The sigma variable represents the standard deviation that will be imposed on the image. The intensities can range from 0-255 even if the range of z is [0,1]. In the code, they have 2 profiles: "np" and "lc" which means Normal Profile and Fast Profile, respectively. For the following images that will be produced, the Normal Profile will be used. Finally, the screen-printing function is an optional feature for if the user wants output additional information of the image being processed.

Like mentioned earlier, the function will use pre-defined parameters if not specified by the user. The figure below will display how different images can be used to get different information about the image.

```
image_name = [
%      'montage.png'
%      'Cameraman256.png'
%      'boat.png'
      'Lena512.png'
%      'house.png'
%      'barbara.png'
%      'peppers256.png'
%      'fingerprint.png'
%      'couple.png'
%      'hill.png'
%      'man.png'
    ];
```

Figure 11: Lena512 Image Selected

By using this code orientation, the user can specify what image he/she wants to use and enhance by simply removing the comment. For the following example, we will use the Lena512 image as seen in Figure 11.

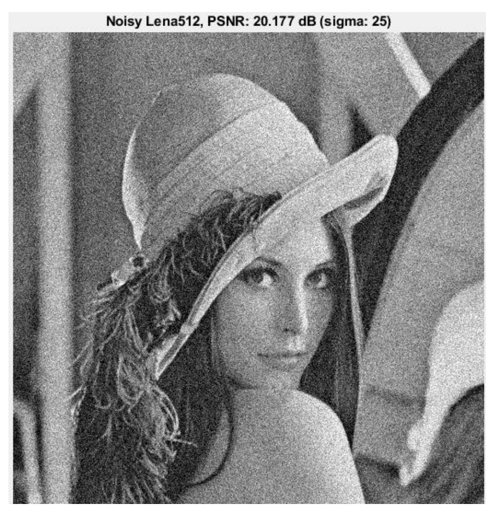


Figure 12: Gaussian Noise Image

Figure 13: Denoised Image using Block Matching

From the output images, as seen in Figure 12 and 13, the PSNR is 20.177 and 32.077, respectively, with a standard deviation of 25. It is evident that Figure 13 displays a better image quality than Figure 12 which is also indicated by the PSNR. Block matching found various blocks in the noisy image and tried to mimic the pattern on other areas of the image to result in an overall crisp, clearer, and denoised image. The next image that will be analyzed will be Barbara.

```
image_name = [
%     'montage.png'
%     'Cameraman256.png'
%     'boat.png'
%     'Lena512.png'
%     'house.png'
     'barbara.png'
%     'peppers256.png'
%     'fingerprint.png'
%     'couple.png'
%     'hill.png'
%     'man.png'
    ];
```

Figure 14: Usage of Different Images

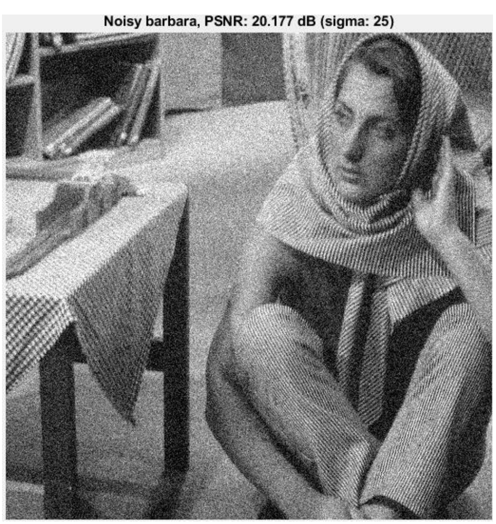


Figure 15: Gaussian Noise Image

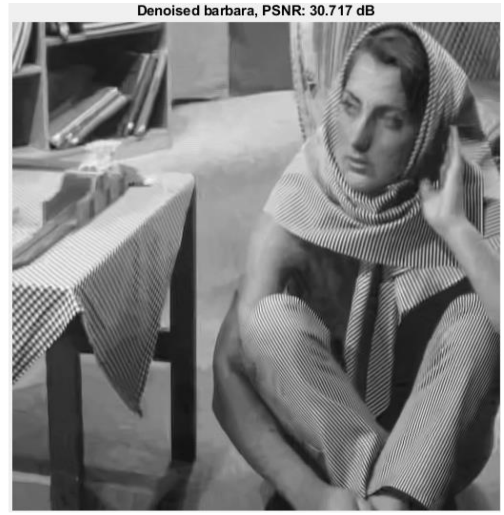


Figure 16: Denoised Image using Block Matching

From Figure 15 and 16, the improvement for denoising has been accomplishing using the block matching algorithm where the PSNR is 20.177 and 30.717, respectively. Lastly, I was unable to implement sparsity into this code however, I did manage to find an image where the denoised image was outputted along with the adaptive dictionary as seen in Figure 17. [10]

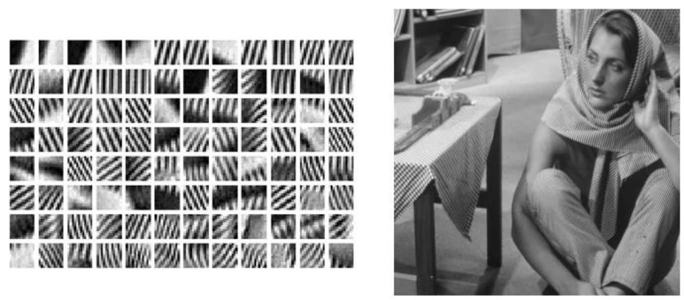


Figure 17: Outputted Denoised Image and Adaptive Learning Dictionary

As mentioned in Section V, adaptive learning dictionaries yield better results than pre-defined dictionaries. In Figure 17, the adaptive dictionary is outputted, and patterns of Barbara's pants, scarf, tablecloth, and other areas of the image are grouped and stored in the dictionary.

VII. Conclusion

In conclusion, denoising images have shown significant importance for identifying details, features, or diagnosis purposes. From this report, it was seen that sparsity is a common technique used for denoising images, however, when sparsity is paired with other techniques such as compressive sensing, block matching, segmentation, and learning/adaptive dictionaries, the denoising results increase drastically. Therefore, constantly adapting current methods of denoising algorithms is needed to ensure that noise from an image is being reduced efficiently and accurately.

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